ALGEBRAIC TOPOLOGICAL METHODS IN COMPUTER SCIENCE (ATMCS) III

Paris, France July 7-11, 2008

July 7

8:00	8:50	refreshments and orientation
8:50	9:00	opening remarks
9:00	10:00	invited talk *Trace spaces: Organization, calculations, applications* (Martin Raussen)
10:00	10:15	break
10:15	11:15	invited talk (Timothy Porter)
11:15	11:30	break
11:30	12:00	*Geometric databases* (David Spivak)
12:00	12:30	*Simulation hemi-metrics for timed Systems, with relations to ditopology* (Uli Fahrenberg)
12:30	14:30	lunch
14:30	15:30	invited talk *Computation and the periodic table* (John Baez)
15:30	15:45	break
15:45	16:45	invited talk *Structure discovery, routing and information brokerage in sensor networks* (Leonidas Guibas)
16:45	17:00	break
17:00	17:30	*Topological methods for predicting the character of response surface $z = f(x, y)$ while elastomer systems studying* (Ilya Gorelkin)
17:30	18:00	*Extremal models of concurrent systems, the fundamental bipartite graph, and directed van Kampen theorems* (Peter Bubenik)

The end of the booklet contains all available abstracts, in alphabetical order of presenter.

July 8

8:30	9:00	refreshments
9:00	10:00	invited talk (speaker Dmitry Kozlov)
10:00	10:15	break
10:15	11:15	invited talk *Locally ordered spaces as sheaves* (Krzysztof Worytkiewicz)
11:15	11:30	break
11:30	12:30	*A survey on homological perturbation theory* (Johannes Huebschmann)
12:30	14:30	lunch
14:30	15:30	invited talk *Path categories* (Rick Jardine)
15:30	15:45	break
15:45	16:45	invited talk *Topology of motion planning algorithms* (Michael Farber)
16:45	17:00	break
17:00	17:30	*A basic coherence theorem formulated in rewriting theory* (Paul-André Mellis)
17:30	18:00	*Persistent homology and detection of disease clusters with arbitrary shapes* (Andrew Blumberg)

July 9

8:30	9:00	refreshments
9:00	10:00	invited talk *Building databases for global dynamics* (Konstantin Mischaikow)
10:00	10:15	break
10:15	11:15	invited talk *New perspectives on shape comparison by size theory* (Claudia Landi)
11:15	11:30	break
11:30	12:00	*Digitally continuous maps, multivalued retractions and parallel thinning algorithms* (Antonio Giraldo)
12:00	12:30	*Computation and application issues in multidimensional shape description* (Daniela Giorgi)
12:30	14:30	lunch
14:30	15:45	poster session featuring current list of confirmed presenters: (1) *Applications of size theory in shape comparison* (Andrea Cerri) (2) *Persistent homology methods in longitudinal shape analysis* (Jennifer P. Gamble) (3) *Fragment Consistent Kleene Models, Fragment Topologies, and Positive Process Algebras* (Cyrus Nourani) (4) *Algebra of queues: difference between two queues* (Przemyslaw R. Pochec) (5) *Morse matchings: cancelling critical simplices* (José Antonio Vilches) (6) *Classification of micro-calcifications using Betti numbers at various scales* (Reyer Zwiggelaar)
15:45	16:45	invited talk *Topology and the game of twenty questions* (Robin Forman)
16:45	17:00	break
17:00	18:00	invited talk *Cubes, homotopy and process algebra* (Philippe Gaucher)

Sometime between 18:00 to 19:00, Guy Cousineau, President of Paris VII, will address us for twenty minutes. Shortly afterwards, interested participants can assemble for an optional social event. Details will be forthcoming during the conference.

July 10

8:30	9:00	refreshments
9:00	10:00	invited talk *Constructive algebraic topology* (Francis Sergeraert)
10:00	10:15	break
10:15	11:15	invited talk *Homology of higher categories* (Francois Metayer)
11:15	11:30	break
11:30	12:00	*Directed spaces generated by directed cubes* (Lisbeth Fajstrup)
12:00	12:30	*Reeb graphs for shape analysis and synthesis* (Silvia Biasotti)
12:30	14:30	lunch
14:30	15:30	invited talk *Topics in persistent homology* (Herbert Edelsbrunner)
15:30	15:45	break
15:45	16:45	invited talk *Directed algebraic topology and concurrency* (Emmanuel Haucourt)
16:45	17:00	break
17:00	17:30	*Implementation of a library to study the asymptotics of plane sections of periodic surfaces* (Roberto De Leo)
17:30	18:00	*Parametric Morse theory and sequential image anaysis* (Neza Mramor)

July 11

8:30	9:00	refreshments
9:00	10:00	invited talk *Topology and data* (Gunnar Carlsson)
10:00	10:15	break
10:15	11:15	invited talk *Topology and anyonic topological quantum computation* (Louis Kauffman)
11:15	11:30	break
11:30	12:00	*Digital objects, chain homotopies and discrete Morse theory* (Pedro Real)
12:00	12:30	*Combinatorics of surface triangulations* (Simuel Vidal)
12:30	14:30	lunch
14:30	15:30	invited talk *The fundamental symmetric cubical category of a directed Space* (Marco Grandis)
15:30	15:45	break
15:45	16:45	invited talk (Sanjeevi Krishnan)
16:45	17:00	break
17:00	17:30	*Algebraic topological models for octrees* (Helena Molina-Abril)
17:30	18:00	*Higher-dimensional categories with finite derivation type* (Philippe Malbos)
18:00	18:15	Closing remarks (Maurice Robin, Director General of DIGITEO)

Abstracts

John Baez, *Computations and the periodic table*

By now there is an extensive network of interlocking analogies between physics, topology, logic and computer science, which can be seen most easily by comparing the roles that symmetric monoidal closed categories play in each subject. However, symmetric monoidal categories are just the $n = 1, k = 3$ entry of a hypothesized "periodic table" of k-tuply monoidal n-categories. This raises the question of how these analogies extend. We present some thoughts on this question, focussing on how symmetric monoidal closed 2-categories might let us understand the lambda calculus more deeply.

Silvia Biasotti (joint work Daniela Giorgi, Michela Spagnuolo and Bianca Falcidieno), *Reeb graphs for shape analysis and synthesis*

Reeb graphs are compact shape descriptions that convey topological information by analysing the evolution of the level sets of a function defined on the shape.

Their definition dates back to 1946, and finds its root in Morse theory. Reeb graphs have been proposed as shape descriptors to solve different problems arising in Computer Graphics, and nowadays they play a fundamental role in the field of computational topology for shape analysis.

This talk provides an overview of the mathematical properties of Reeb graphs and reconstructs their history in the Computer Graphics context. We will also discuss the enrichment of the Reeb graph with geometrical information about the shape, which give an abstraction of the main shape features. Finally, directions of future research will be discussed.

Andrew Blumberg (joint work with Gunnar Carlsson and Michael A. Mandell), *Persistent homology and detection of disease clusters with arbitrary shapes*

Detection of disease clusters is a problem of central importance in public health. The problem is to find clusters of disease cases which are "unusually dense", given the background population and some model of the baseline disease incidence. Such clusters can indicate developing disease outbreaks and hence locations for public health intervention. The standard method is to apply *circular scan statistics*, due to Kulldorf and collaborators; this method applies a likelihood ratio test to circular subregions of the region under investigation. Variants of the circular scan statistic methodology are currently in use by public health departments in the United States.

Recently, Wieland, et al. introduced a new method based on Euclidean minimum spanning trees for disease cluster detection. This method corrects a central defect of the circular scan statistic: the circular scan statistic is optimized for detecting circular clusters. The minimum spanning tree method has dramatically improved perception of noncircular disease clustering data. However, relating the natural test statistic in this method to the underlying statistical model of the disease distribution is difficult, and in particular certain standard statistical variants are hard to handle (e.g. covariates). Also, a key step in the algorithm is extremely computationally demanding.

We introduce a new method for disease cluster detection based on persistent homology. Our algorithm is based on the observation that the "potential clusters" Wieland, et al. derive from minimum spanning trees are in fact precisely persistent components.

Thus, we proceed by computing persistent components and then computing a likelihood ratio for the associated simplicial complex. This geometric interpretation of the clustering algorithm permits extremely useful refinements, notably the incorporation of "density" parameters in terms of the presence of higher-dimensional simplices. This allows the algorithm to be tuned via a geometrically meaningful integer parameter (a dimension constraint) to bias towards dense, circular clusters.

Experimental results show that our new method achieves superior perception to both the circular scan statistic and the Euclidean minimum spanning tree algorithm.

Peter Bubenik, *Extremal models of concurrent systems, the fundamental bipartite graph, and directed van Kampen theorems*

Analyzing concurrent programs, in which multiple processes use shared resources, is notoriously difficult. One approach is to model the state space of such a program using a directed space, and then study its topology. The fundamental category of this space describes the execution space, but it is typically uncountable. Approaches to reducing this category include using components (Fajstrup, Goubault, Haucourt, Raussen) and using injective and projective models (Grandis). I use Grandis' future retracts and past retracts to construct extremal models of the fundamental category. Minimal extremal models are typically finite. In addition, the fundamental bipartite graph, which captures the homotopically distinct execution paths, is an invariant of the extremal model. Furthermore, extremal models can be constructed in a piece-by-piece manner using a van Kampen-type theorem.

Gunnar Carlsson, *Topology and data*

I will survey various ways of using topological methods and points of view to study data sets from various scientific areas. In particular, I will discuss persistent homology and its generalizations, as well as topological methods for mapping out data sets.

Andrea Cerri, *Applications of size theory in shape comparison*

Size Theory was proposed in the early 90's as a geometrical/topological approach to the problem of shape comparison. The main idea is to translate the task of comparing two objects in a database (e.g. images, 3D models or sounds) into the one of comparing two suitable topological spaces $\mathcal{M}$, $\mathcal{N}$ (non-empty, compact and locally connected Hausdorff spaces), endowed with two continuous functions $\varphi : \mathcal{M} \to \mathbb{R}$, $\psi : \mathcal{N} \to \mathbb{R}$ that are chosen according to the applications. These functions are called *measuring functions* and can be seen as descriptors of the features considered relevant for the comparison. The pairs $(\mathcal{M}, \varphi)$, $(\mathcal{N}, \psi)$ are said to be *size pairs* and provide a representation of the considered shapes: In Size Theory, such pairs can be compared by *size functions*, whose role is to capture *qualitative* aspects of a shape and represent them in a *quantitative* way. The idea is to study the pairs $(\mathcal{M}\langle\varphi \le x\rangle, \mathcal{M}\langle\varphi \le y\rangle)$, where $\mathcal{M}\langle\varphi \le t\rangle$ is defined by setting $\mathcal{M}\langle\varphi \le t\rangle = \{P \in \mathcal{M} : \varphi(P) \le t\}$ for $t \in \mathbb{R}$: The size function $\ell_{(\mathcal{M},\varphi)} : \{(x, y) \in \mathbb{R}^2 : x < y\} \to \mathbb{N}$ is then the function that takes each point (x, y) of the domain into the number of connected components of $\mathcal{M}\langle\varphi \le y\rangle$ containing at least one point of $\mathcal{M}\langle\varphi \le x\rangle$ [1]. By means of Size Theory, we can then model a shape by a size pair, and describe it by considering the associated size function: As a consequence, the comparison of two shapes can be translated into the simpler task of comparing two functions from the half–plane $\{(x, y) \in \mathbb{R}^2 : x < y\}$

to the natural numbers. However, a common scenario in applications is to deal with multidimensional information: Indeed, a shape can be more thoroughly characterized by means of a set of real functions, each investigating specific features of the shape under study. This problem can be faced by observing that size functions are modular descriptors: In order to study different properties of a shape, we only need to change the measuring function. Since its introduction, Size Theory has been studied and applied in quite a lot of applications: An example is given by [2], where the authors propose an automatic retrieval system for trademark images based on size functions, to support human labor in guaranteeing copyright policy. Other examples on the use of Size Theory in applications can be found in several fields, ranging from leukocyte classification in medical context [3] to image retrieval in the World Wide Web [4]: This work proposes to be an overview on some meaningful experimental results, in order to show the capability and the flexibility of this theoretical framework in dealing with concrete applications.

References

[1] P. Frosini, and C. Landi, *Size Theory as a Topological Tool for Computer Vision*, Pattern Recogn. Image Anal. **9(4)** (1999), 596–603.
[2] A. Cerri, M. Ferri, and D. Giorgi, *Retrieval of trademark images by means of size functions*, Graphical Models **68** (2006), 451–471.
[3] M. Ferri, S. Lombardini and C. Pallotti, *Leukocyte classification by size functions*, In Proc. of the 2^{nd} IEEE Workshop on Applications of Computer Vision, IEEE Computer Society Press, Los Alamitos, CA (1994), 223–229.
[4] A. Cerri, M. Ferri, P. Frosini, and D. Giorgi, *Keypics: free–hand drawn iconic keywords*, International Journal of Shape Modelling **13(2)** (2007), in press.

Roberto De Leo, *Implementation of a library to study the asymptotics of plane sections of periodic surfaces*

The problem of asymptotics of plane sections of periodic surfaces was first introduced in Fifties by solid state physicists.

In the quantum theory of metals electron's quasi-momenta are periodic and bound to lie on a constant-energy periodic surface (Fermi Surface, FS) which characterizes the metal's electronic properties; the introduction of a magnetic field obliges the orbits of quasi-momenta to lie on planes perpendicular to the magnetic field and therefore all of their possible orbits are exactly the intersections between those planes and the FS. A physical quantity called magetoresistance depend qualitatively on the asymptotics of those orbits and so that their topology is somehow experimentally measureable.

The rich topological structure of the problem was discovered only in the last twenty years by S.P. Novikov and his pupils as a concrete case of his extension of the Morse theory to closed 1-forms and ultimately led to the conclusion that in the most interesting cases the asymptotics of FS's plane sections are described by a fractal in the space of directions.

In this talk we present our implementation of the algorithms needed to find numerically the asymptotics of plane sections once a periodic surface and a (magnetic field) direction are given; the main tasks are the following: 1. producing the full intersection between a (integer) plane and a periodic surface; 2. extracting the critical sections, i.e. those produced by planes tangent to the surface; 3. evaluating the homological class of the critical sections in the 3-torus and on the surface. Finally we shall shortly present the most interesting results obtained numerically so far.

Herbert Edelsbrunner, ***Topics in persistent homology***

Uli Fahrenberg, ***Simulation hemi-metrics for timed systems, with relations to ditopology***

During recent years, interest has emerged in quantitative, or robust, notions of (bi)similarity for probabilistic and timed systems. One motivation for this are some issues pertaining to modeling and implementability; real-world implementations are subject to imprecision, hence model-checking should be robust with respect to such imprecision.

For real-time systems, several notions of metrics on timed words can be found in the literature, and also a few notions of bisimulation metrics and simulation hemi-metrics. In this paper, we introduce some new ones, to complete the picture and arrive at a somewhat systematic treatment. We are especially interested in cases where one can introduce simulation metrics on timed transition systems and metrics on timed languages such that the semantics mapping from one to the other is a distance-preserving continuous function.

Metrics on timed languages are generally uncomputable, and for bisimulation metrics, computability is open in most cases, though with one notable example of a computable bisimulation metric for timed automata, as shown by Henzinger, Majumdar, and Prabhu in 2005.

When turning from bisimulation to simulation, the right notion to use is a hemi-metric, inducing a structure of directed space. One interesting feature here is that the topology encodes the (metric) distance, whereas the directed structure encodes the simulation, hence there is a certain separation of information. Again, we expose distance-preserving dimaps from timed transition systems to timed languages.

The work presented here is in progress; we are especially interested in working out decidability issues. The relations to directed topology are, we hope, interesting, and could open up for new application areas of directed topology.

Lisbeth Fajstrup, ***Directed spaces generated by directed cubes***

Directed topology is studied in many different settings: Locally partially ordered spaces, d-spaces, cubical sets,... The category $\mathbf{d-Space}$, of topological spaces with a selected subset of the set of paths denoted the directed paths or dipaths has nice properties for directed homotopy, e.g. a Van Kampen theorem proved by M. Grandis. However, it seems to be too big for some purposes. In particular directed covering theory, or delooping from the computer science point of view is not well understood in such a general setting. In "A convenient category for Directed Topology", F. and J.Rosicky, we introduce the subcategory $\mathbf{d-Space}_{\mathcal{B}}$ of d-Spaces, which are generated by directed cubes in the following sense:

A directed cube is a product $B = I_1 \times \cdots \times I_n$, where I_j is the unit interval with either the discrete order, $a \leq b \Leftrightarrow a = b$ or the interval with the standard order. A directed path is a continuous map $\gamma : I \to B$, where I has the standard order, respecting the order in each coordinate.

$\mathcal{B}$ is the full subcategory of $\mathbf{d-Space}$ of such cubes.

A d-Space X is in $\mathbf{d-Space}_{\mathcal{B}}$ if $U \subset X$ open if and only if $\phi^{-1}(U)$ is open in B for all d-maps $\phi : B \to X$ where B is in $\mathcal{B}$. Moreover, the dipaths in X should be the image of dipaths under $\phi : B \to X$ or concatenations of such.

The category $\mathbf{d-Space}_{\mathcal{B}}$ contains geometric realization of cubical complexes, which are needed in applications - as models for Higher Dimensional Automatafor instance.

And it avoids some of the pathological examples of d-Spaces, so one should expect this to be a convenient category. The analogy with compactly generated spaces is obvious, and a study of directed mapping spaces in this setting should be fruitful. Some of the properties already discovered are:

- $\mathbf{d}-\mathbf{Space}_{\mathcal{B}}$ is locally presentable.
- The inclusion $I : \mathbf{d}-\mathbf{Space}_{\mathcal{B}} \to \mathbf{d}-\mathbf{Space}$ has a right adjoint, the Boxification functor, which takes a d-Space X to the d-Space $B(X)$ with topology given as above by the d-maps of cubes into X ($B(X)$ has more opens than X) and dipaths the same as in X. For d-map f, $B(f) = f$ as maps of sets.
- A dicovering, $p : Y \to X$ in d-Space is a d-map with unique lift of dipaths and dihomotopies of dipaths with fixed initial point. A *pointed* space $X \in \mathbf{d}-\mathbf{Space}_{\mathcal{B}}$, $x \in X$ is *wellpointed* if all points in X are reachable by a dipath initiating in x. For a wellpointed space X, x, there is a *universal dicovering* $\pi : \tilde{X} \to X$ with $\tilde{X} \in \mathbf{d}-\mathbf{Space}_{\mathcal{B}}$ wellpointed, and in fact $\pi : I(\tilde{X}) \to I(X)$ is universal for dicoverings of $I(X)$ in pd-Space, the category of pointed d-Spaces.

Michael Farber, *Topology of motion planning algorithms*

I will discuss problems of algebraic topology motivated by the task of designing motion planning algorithms in robotics. A motion planning algorithm is a rule associating with any pair (A, B) of states of the system a continuous motion of the system starting at the state A and ending at the state B. The complexity of the problem of constructing a motion planning algorithm is measured by a numerical invariant $TC(X)$ where X denotes the configuration space of the system. The notation TC stands for topological complexity. $TC(X)$ depends only on the homotopy type of the configuration space X of the system and can be estimated and computed using various tools of modern algebraic topology including cohomology algebras and cohomology operations. In the talk I will present results about computing the topological complexity $TC(X)$ for several practically important problems: (1) Simultaneous control of many objects; (2) Collision free control of multiple objects moving in $\mathbb{R}^2$ or in $\mathbb{R}^3$; (3) Collision free control of multiple objects moving along a fixed graph; (4) Motion planning algorithms for collision free control of multiple objects in the presence of several moving obstacles.

Robin Forman, *Topology and the game of twenty questions*

In the usual game of 20 questions, one player tries to determine a hidden object by asking a series of "yes or no" questions. A wide variety of binary searches have this general form. In applications, one is usually limited to a predetermined set of questions, and one is not required to determine the hidden object precisely, but rather only up to some equivalence. This is this game we will examine.

We will assume that one can complete the task if one is permitted to ask all of the allowable questions. The question we will investigate is - Is it possible to do better? That is, can one complete the task without asking all of the allowable questions? If not, the problem is called /evasive/.

Our approach is to restate the problem in a more topological form. We will then define a new homology theory, generalizing simplicial homology, that captures the difficulty of solving this problem. The link between the homology theory and the original search problem is provided by a generalization of "Discrete Morse Theory." This work is an extension and refinement of a line of work beginning with the paper of Kahn, Saks and Sturtevant "A topological approach to evasiveness."

Jennifer Gamble (joint work with Dr. Giseon Heo) *Persistent homology methods in longitudinal shape analysis*

In the field of statistical shape analysis, groups of objects are compared in order to determine whether their mean shapes differ in a statistically significant way. In this context, "shape" refers to geometric information that is invariant to translations, rotations and scaling. A number of statistical and mathematical methods have been developed to facilitate this "comparison" between groups. Common areas of application are medicine and biology, with the objects of interest represented in the form of landmark configurations. A usual method is to transform these configurations into single points in some "shape space", and then perform statistical analyses on the corresponding tangent space approximations (using a pooled mean shape as the pole). In this setting, performing longitudinal analysis to compare multiple groups over time can be difficult, as the number of variables is often much larger than the number of subjects, with complicated covariance structure. In this poster we will explore the use of algebraic topological methods, particularly persistent homology, to analyze shape changes in groups over time. An orthodontic dataset involving three treatment groups will be used as an example. Using the tangent space coordinates we will use the software PLEX to analyze the relationship between our treatment groups over time. Specifically, we will contrast the relationships between the groups (with each subjects configuration represented by a point/vertex in the tangent space) at multiple time points, by analyzing the differences in their persistent homology features. In cases where there are large deformations between the different shapes, tangent space approximations can be inappropriate. We will also discuss alternative distance measures directly in the shape space, and methods of dimensionality reduction.

Philippe Gaucher, *Cubes, homotopy, and process algebra*

In directed algebraic topology, the concurrent execution of n actions is abstracted by a full n-cube. Each coordinate corresponds to one of the n actions. This n-cube may be viewed as a representable presheaf of the category of precubical sets, as a topological n-cube equipped with some continuous paths modelling the possible execution paths up to homotopy, and as a commuting n-cube, i.e usually the small category associated with the poset of vertices of the n-cube. In fact, we have to remove the identity maps for various reasons, e.g., because the full n-cube does not contain any loop. In this talk, all these points of view are related to one another by considering Milner's calculus of communicating systems (CCS). All operators of this process algebra are given a higher dimensional interpretation. The restriction to dimension 1 corresponds to the usual structural operational semantics.

Daniela Giorgi, *Computation and application issues in multidimensional shape description*

In Computer Graphics and Vision, computational-topology offers a theoretical framework for the formalization and solution of problems related to shape analysis, description and comparison. Methods that make use of the properties of real functions defined on the shape [1] are often well suited to describe objects that are non-rigidly related to each other. The role of these functions, that we may call measuring functions, is to measure quantitative geometric properties of the shape, while taking into account its topology.

Recent advances in Size Theory have shown that it is possible to derive a concise and informative geometrical-topological shape descriptor also in the case of multivariate measuring functions, that is functions taking values in $\mathbb{R}^k$ [2]. The idea of using k-dimensional measuring functions arises from the observation that the shape of an object can be better characterized by means of a set of functions, each investigating specific features of the shape under study. For instance, scientific simulations of real phenomena typically require the analysis of a huge and composite amount of data. At the same time, there are properties that are naturally k-dimensional: a first example is colour, which lives in the 3-dimensional RGB space. The possibility of working from the beginning with k-dimensional functions, instead of merging the information of k separate functions a posteriori, allows one to produce a single descriptor containing the information of the k functions at the same time. In other words, k different functions concur to produce a single descriptor.

In this talk, we will deal with the main issues related to the application in a discrete setting of the concepts introduced in Multidimensional Size Theory, in particular multidimensional size functions and multidimensional matching distances. A computational scheme coherent with the mathematical model will be given, highlighting that the technique proposed is able to deal with different model representations, such as simplicial complexes and digital spaces. Experimental results will be provided so as to illustrate the feasibility of the approach, in terms of storage space, computation time and efficacy of description. Finally, different families of measuring functions will be analyzed, in the light of their capability to capture salient shape features.

References

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[2] S. Biasotti, A. Cerri, P. Frosini, D. Giorgi, C. Landi: Multidimensional size functions for shape comparison. Journal of Mathematical Imaging and Vision, in press (DOI 10.1007/s10851-008-0096-z).

Antonio Giraldo, *Digitally continuous maps, multivalued retractions and parallel thinning algorithms*

In a recent paper ("Digitally continuous multivalued functions": In "Discrete Geometry for Computer Imagery", Lecture Notes in Computer Science **4992** (2008) 81-92) we have introduced a notion of continuity in digital spaces which extends the usual notion of digital continuity. Our approach, which uses multivalued maps, provides a better framework to define topological notions, like retractions, in a far more realistic way than by using just single-valued digitally continuous functions. In particular, we characterized the deletion of simple points, one of the most important processing operations in digital topology, as a particular kind of retraction.

A simple point of a binary image is defined as a point whose deletion does not alter the topology of the image. However, it is well known that the parallel deletion of simple points needs not to preserve topology (the simple set being the middle points in a 3x2 rectangle). A set whose deletion does not change the topology is called a deletable set.

In this work we deepen into the properties of this family of continuous maps, now concentrating on parallel deletion of simple points and thinning algorithms, seeing them as digital (deformation) retractions.

We show that if $D \subset X$ is deletable, then there exists a digitally continuous multivalued function $F : X \longrightarrow X \setminus D$ such that $F(x) = \{x\}$ if $x \in X \setminus D$ (i.e. a multivalued retraction from X to $X \setminus D$). Although the converse is not true, in general, we give conditions for it to hold.

In order to guarantee that the parallel deletion of simple points preserve the topology, several strategies have been developed in the literature. We show how some of the more extended ones are also related or even can be characterized as retractions in terms of our notion of continuity.

Ilya Gorelkin (joint work with Ivan Agayants), *Topological methods for predicting the character of response surface $z = f(x, y)$ while elastomer systems studying*

The methods of graph theory are widely used in nowadays theoretical chemistry. The main advantage of this methods are simplicity of representation and obviousness of objects and its relations picturing [1]. The latter became the origin of idea of using the graphs as mathematical structures for describing the whole spectrum of contour composition-property diagrams used in technology of elastomer processing [2].

While solving optimization tasks for elastomer treatment processes, it may happen that one cannot vary factors in the whole range of researcher interest. The limits can appear in the recipe or technological factors (component concentrations, solubility, equipment possibilities etc) and when data from various references is used. Thus, there are experimentally unexamined areas.

To predict a character of response isolines arrangement, the method of splitting the factor space into square areas (in projection on horizontal plane) was used. On the basis of information about singular points of the surface this areas are transformed to graphs of certain view: square graph has vertexes in corners, on sides and inside; in order to distinguish the type of singular points, different valences are assigned to the vertexes; oriented edges are used to show if a singular point is a maximum or minimum point [2].

The algorithm of experimentally uncovered areas graphs searching was developed. It takes into consideration a range of rules and limitations and consists of several steps including: determination of the sought after graph vertexes which are adjacent with known graphs vertexes; determination of vertexes presence on sides, that (sides) are not adjacent with known graphs; determination of vertex presence inside the sought after square graph; searching of graph vertexes connection variants. In result one or several graph variants appears, giving an indication of what kinds of response surface can be in unexamined area. The algorithm to be reported was realized with the aid of Scilab scientific software package [3].

References

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[3] http://scilab.com

Marco Grandis, *The fundamental symmetric cubical category of a directed space*

The recent domain of directed algebraic topology studies 'directed spaces', where paths and homotopies can not generally be reversed. The general aim is modelling non reversible phenomena, but the present applications are mostly concerned with the theories of concurrent processes and rewrite systems. At the place of the classical fundamental groupoid of a topological space, a directed space has a fundamental category, whose applications to concurrency have already been studied in many papers. Here, we want to study an infinite dimensional version of the fundamental category, of a cubical type and more precisely a symmetric cubical one, because symmetries will simplify the coherence problems. We start from the singular cubical set of a directed space, its transpositions (which permute variables) and its concatenation laws in the various directions. These partial operations only behave well up to reparametrisation mappings, which are invertible in the case of the associativity and interchange constraints (even strict for interchange), but non invertible for left and right unitarity. The whole structure forms thus a 'symmetric u-lax cubical category', where the term 'u-lax' refers to this peculiar aspect of laxity. Applying a formal strictification procedure, we get a strict symmetric cubical category and then its n-dimensional symmetric cubical versions.

Leonidas Guibas, *Structure discovery, routing, and information brokerage in sensor networks*

Wireless sensor networks have recently come into prominence because they hold the potential to revolutionize many segments of our economy and life, from environmental monitoring and conservation, to manufacturing and business asset management, to automation in the transportation and health-care industries, to battlefield awareness and other defense applications. Since sensor networks are embedded systems, it is attractive to consider network algorithms motivated by how one might approach problems in the continuous physical world the nodes live in. Geographic routing is an example. The challenge, however, is how to exploit powerful ideas from geometry and topology in a setting where the imperfect sampling of the environment and the volatile nature of wireless links create many difficulties in going from such continuous high-level ideas to robust discrete and distributed algorithms. This talk will consider some tools recently developed towards this goal, in the area of network structure discovery, routing, and information brokerage, delivery, dissemination, and aggregation in sensor networks.

Emmanuel Haucourt, *Directed algebraic topology and concurrency*

Static analysis is a branch of computer science which often uses graphs to represent programs. While this approach suffices in dealing with purely sequential processes, the analyses of multi-tasking programs leads one to study the Cartesian product of such graphs. Then problems arise from the fact that the path functor (i.e. left adjoint to the forgetful functor from Cat to Grph) does not preserve Cartesian products. We attempt to fix the problem by replacing graphs by their geometric realizations in the category PoTop of pospaces, and the path category functor by the fundamental category functor from PoTop to LfCat, a certain reflexive subcategory of Cat. This fundamental category functor is an analog of the classical fundamental groupoid functor from Top to Grpd. We then arrive at the beginnings of Directed Algebraic Topology (DAT). After giving a quick overview of the computer-scientific context, I will give some categorical properties of PoTop, as well as a description of the component category of a pospace, which generalizes the notion of connected components of a topological space. Note: We take "graph" to mean "presimplicial set of dimension 1", or in other words, a set

A of arrows, a set V of vertices, and two functions s (source) and t (target) from A to V. In particular, there might be several arrows from a vertex v to v'.

Johannes Huebschmann, *A survey on homological perturbation theory*

Higher homotopies are nowadays playing a prominent role in mathematics as well as in certain branches of theoretical physics. Homological perturbation theory (HPT), in a simple form first isolated by Eilenberg and Mac Lane in the early 1950s, has become a standard tool to handle algebraic incarnations of higher homotopies. A basic observation is that higher homotopy structures behave much better relative to homotopy than strict structures, and HPT enables one to exploit this obser- vation in various concrete situations. In particular, this leads to the effective calculation of various invariants which are otherwise intractable. The formulas which result from HPT-constructions are recursive, and the calculation of a specific object, e. g. a certain group cohomology group or Lie algebra cohomology group can be carried out, at least in principle, in finitely many steps. Experts on computational algebraic topology (e.g. F. Sergeraert and his collaborators) have already successfully carried out such calculations. Higher homotopies and HPT-constructions abound but they are rarely recognized explicitly and their significance is hardly understood; at times, their appearance might at first glance even come as a surprise, for example in the Kodaira-Spencer approach to deformations of complex manifolds or in the theory of foliations. A basic tool in HPT is the perturbation lemma. Starting from a chain deformation retraction, after introduction of a perturbation of the differential on the big object, this lemma yields recursive formulas for the constituents of a new chain deformation retraction from the perturbed big object onto a perturbed small object. In group cohomology, the perturbation could, for example, correspond to perturbing an abelian group structure to a non-abelian one. At each recursive step, the resulting terms can be determined by explicit programmable algorithms. Sometimes explicit numerical calcu- lations can then be carrried out on the small object. Another basic tool is a lemma providing, under suitable circumstances, a recursive construction for a twisting cochain or, equivalently, solution of the master equation, deformation equation, or Maurer-Cartan equation. Again at each recursive step, the resulting term can be determined by an explicit programmable algorithm. The talk will illustrate, with a special emphasis on the compatibility of perturbations with algebraic structure and on effective calculation, how HPT may be successfully applied to various mathematical problems arising in group cohomology, algebraic K-theory, deformation theory, differential geometry, physics, etc. More information about HPT can be found in my survey article arXiv:0710.2645 (in: Gerstenhaber-Stasheff Festschrift, Birkhauser, to appear) and on my home page.

Rick Jardine, *Path categories*

The path category $P(X)$ of a simplicial set X is the quotient of the free category on the graph $\mathrm{sk}_1(X)$ subject to the requirement that the boundary of each 2-simplex of X gives a commutative triangle in $P(X)$. The functor $X \mapsto P(X)$ is left adjoint to the nerve functor.

Calculations of the morphism sets for path categories $P(X)$ will be displayed for very simple simplicial sets X.

Such calculations are difficult in general. In particular, the morphisms $x \to y$ in the path category $P(|K|)$ of the triangulation of a higher dimensional automaton (or cubical complex) K are the execution paths between the states x and y in the concurrency model associated to K, and it is the fundamental problem of this form of concurrency theory to distinguish one such path from another.

Path categories are a basic tool for the study of quasi-categories, and they occur naturally in higher category theories in general. I shall give a sufficient description of Joyal's quasi-category model structure for simplicial sets to show that all quasi-category weak equivalences $X \to Y$ of simplicial sets induce categorical equivalences $P(X) \simeq P(Y)$ of their associated path categories. In a different context, homotopy coherence resolutions of small categories can be used to give resolutions of path categories in categories enriched in simplicial sets. Examples of this phenomenon will be displayed.

Louis Kauffman, *Topology and anyonic topological quantum computation*

Recently, Freedman, Kitaev and their collaborators have shown how braiding operators in certain topological quantum field theories are universal for quantum computation. In particular, one can focus on the topoloogical quantum field theory called Fibonacci Anyons. (There are two basic particles call them 1 and 0. The only non-trivial interaction is $1 + 1 \to 0$ or 1. The corresponding recoupling theory is intricate. The braiding is non-trivial and can model quantum computation.) The purpose of this talk is to give a simple model for the Fibonacci Anyons in terms of $q = e^{i\pi/5}$ deformed spin networks, and to show how the structure of the model proceeds from the structure of the bracket model of the Jones polynomial.

The point of view of this talk allows discussion of the relatiohship of quantum information theory and quantum computing with the knot polynomials. The use of spin networks in these models suggests a deeper dialogue with quantum gravity. In the first place, the result about Fibonacci anyons shows that a deformation of the classical spin networks to a nearby root of unity allows the generation of arbitrarily good approximations to unitary group transformation in the braiding representations associated with these spin networks. This means that at the mathematical level there is a unification of the generation of background space (spin geometry) and the generation of quantum mechanical evolutions. In a sense this is a background for a possible unified quantum geometry.

The talk will also discuss quantum algorithms for computing the Jones polynomial and colored Jones polynomials from this point of view.

Sanjeevi Krishnan (joint work with Eric Goubault and Emmanuel Haucourt), *Directed covering space theory and static program analysis*

The state space of a machine often admits a coherent preordering of its open subsets. A covering map pulls back such structure to define a "locally monotone map" of "locally preordered spaces." In order to analyze concurrent systems exhibiting cyclical behavior, we identify homotopy-theoretic criteria for a locally preordered space to admit a cover whose open subsets are partially ordered. We compare our covering space theory with others in the literature, and explore some potential interactions between directed topology, monoid theory, partially ordered group theory, and static program analysis.

Claudia Landi, *New perspectives on shape comparison by size theory*

Philippe Malbos (joint work with Yves Guiraud),
Higher-dimensional categories with finite derivation type

Craig Squier has proved that, if a monoid admits a presentation by a finite, convergent (confluent and terminating) word rewriting system, then it has finite derivation type: this is a homotopical property of a 2-complex associated to the word rewriting system. An open question, until now, was to extend this result in higher dimensions: for a n-category, does having a presentation by a finite and convergent $(n+1)$-polygraph implies finite derivation type?

For that, we define the finite derivation type property for n-categories and show that Squier's result does not hold in general: we give a 2-category presented by a finite convergent 3-polygraph which does not have finite derivation type. However, we study additional sufficient conditions for a finite convergent 3-polygraph to have finite derivation type.

Paul-André Mellis, *A basic coherence theorem formulated in rewriting theory*

In this talk, I will explain how to establish the coherence theorem for braided monoidal categories using rewriting theory – this involving the construction of a category of fractions in the sense of Gabriel and Zisman. This approach clarifies the combinatorial nature of this basic coherence theorem. I will then compare this proof to a series of more conceptual proofs developed in the literature.

Francois Metayer, *Homology of higher categories*

Rewriting systems generate a whole space of computations naturally endowed with a structure of strict higher dimensional category. We introduce a simple geometric invariant of these spaces, polygraphic homology. Starting with a concrete, combinatorial description, we eventually interpret our constructions in the abstract framework of model categories, based on recent joint work with Yves Lafont and Krzysztof Worytkiewicz.

Konstantin Mischaikow, *Building databases for global dynamics*

Helena Molina-Abril (joint work with Nicolas Jesson, Jean-Luc Mari, Pedro Real), *Algebraic topological models for octrees*

In this paper we deal with the homology computation of binary digital volumes expressed by an octree structure. As a first step, we obtain a simplicial complex topologically equivalent to the octree volume. Finally we generate a simplicial field, which provides us essential homological information about the object (homology groups, Euler characteristic, Betti numbers, homology generators, cycle homological classification...).

Nezra Mramor (joint work with Jure Zabkar, Gregor Jerse),
Parametric Morse theory and sequential image analysis

In this paper, we present three applications of parametrized discrete Morse theory to image analysis. Discrete Morse theory, introduced by Foreman, represents a discrete analogue of classical smooth Morse theory. It enables an analysis of the critical points of a smooth function f, sampled in a finite number of points, and a decomposition of the domain of f into descending and ascending regions which simulates the Morse-Smale decomposition.

Given a regular cellular decomposition K of the domain, the sampled values of f can be extended to a discrete Morse function on K, that is, a function associating a value to every cell of K which is monotone with respect to dimension in almost all cases. The exceptions to this rule determine a pairing of the cells, called a discrete vector field, defined on a subset of K. Cells which remain unpaired are the critical cells of the discrete Morse function and correspond to critical points of a smooth function in smooth Morse theory.

A parametric discrete Morse function, as considered by King, Knudson and Mramor, is a collection of discrete Morse functions $f_{t_i}, i = 1, ..., m$ defined on K, and a pairing of the cells that occur in adjacent slices. It produces a bifurcation diagram for the births and deaths of critical cells of the functions f_{t_i} with increasing t.

Parametric discrete Morse functions can be successfully applied to analyze of various types of image sequences. In our first application, we consider a sequence of meteorological radar images. Sequential radar images show the process of precipitation intensity over a fixed region which is viewed as a parametric discrete Morse function. Accompanied with additional information from other data sources (e.g. temperature profiles, wind speed and direction etc.) the bifurcation diagram could be used to study the development of thunderstorms. In the second example, we analyze a spatial sequence of CT scans of a human abdomen. Each image represents a slice of the abdomen at a different height.

The third application tackles a robotic domain in which the robot observes a red and a blue ball with an on-board camera. Its task is to learn the notion of object occlusion, that is, the robot learns from examples that objects should partially overlap before total occlusion. The model is represented in the form of a finite automaton.

Cyrus Nourani, ***Fragment Consistent Kleene Models, Fragment Topologies, and Positive Process Algebras***

Starting with Infinite language categories the author since 1995 had explored the algebraic topologies on infinite languages. The specific computing applications range from basic computability to models based on fragments are presented. Positive categories and Horn categories are new fragment categories defined and the applications to a Positive Process algebraic computing (Nourani 2005) is outlined. For example, the author defined the category LP,w to be the category with objects positive fragments and arrows the subformula preorder on formulas to present models. The model bases are Fragment Consistency Models where new techniques for creating generic models are defined. Infinitary positive language categories are defined and infintary complements to Robinson consistency from the authors preceding papers are further developed to present new positive omitting types techniques and infinitary positive fragment higher stratified computing categories. Further model-theoretic consequences are presented in (Nourani 2005a). Fragment consistent model techniques (Nourani 1996-2005) are applied to generate Kleene models. Generic diagrams (Nourani 1980s) allow us to define canonical models with specific functions and primitive extensions. String ISL algebra (Nourani 2006) is a Sigma-algebra with an additional property that the signature Sigma has a subsignture Omega that cotains only on 1-1 functions. Specific applications to Kleene algebras with ISL is presented. Positive Process Fragment algebras are defined and applied with the above to obtain a model by defining a language category over the signature. The preorder category has language fragment sets as objects and is preorded . Examples on regular expression languages might be presented.

1. Language Fragments Structures

Let w be the countable ordinal omega. Let $Lw1, F$ be the least fragment of $Lw1, 1$ which contains $L < A >$. Each formula ϕ in $K < A >$ contains only finitely many c in C. This implies when raking leaves on the trees, there are only finite number of named branches claimed by constant names. The infinite trees are defined by function names, however. The functions define the model with the constants. From the functorial view what follows resembles to a Cosmic Archeology, scooping out the model theoretic specifics for a functorial model theory. Lemma 1. For a chain A alpha;, $\alpha < \beta$ of models, $UA\alpha$ is the unique model with universe $UAalpha$; which contains each $A\beta$ as a submodel. ; A functor $F : (Lw1, F)^{\mathrm{op}} \rightarrow \mathrm{Set}$ can be defined by sets Fi, where the Fis are defining a free structure on some subfragment of $Lw1, F$. To be specific we can define the subfragment models $A(Fi)$ straight from the omega-inductive definition of the Infinitary fragment. $F0$ assigns names to the Set members, for example. $F1$ can define 1-place functions and relations, so on and so forth. Functorial Limit Chain models as follows. We shall refer to it by FLC-models (Nourani 1996). Let A and B, be models for Fi and $Fi + 1$, respectively. Let $A == LB$ (A elementarily equivalent to B as models), and $f : A < LB$ mean the L-reduct of A and B are elementarily equivalent and that f is an elementary embedding of $A|L$ into $B|L$. Let $AFLC$ model be the limit model defined by the elementary chain on the L-reducts of the models defined by the Fis. A specific FLC model is defined by the following theorem.
Theorem 1 There is an elementary chain FLC model for L, where L is $Lw1, F$.

2. Fragment Consistent Kleene Models

Definition A preorder $<<$ on a Σ-algebra A is said to be morphic iff for every functions σ in signature $\Sigma < s1, s1...sn >$ and ai, bi in Asi, and Bsi, respectively, if $ai << bi$ for i in $[n]$ then, $\sigma < A > (a1, ..., an) << \sigma < B > (b1, ..., bn)$. A Kleene algebra, (Koz90) is an algebra $A = (A, +, 0, ;, 1, *)$ such that $(A, +, 0)$ and $(A, ;, 1)$ are monoids, with $+$ commutative and idempotent, and satisfying
(1) $a(b + c) = ab + ac$
(2) $a0 = 0$
(3) $1 + aa* <= a*$
(4) $1 + a * a <= a*$
(5) $(a + b)c = ac + bc$
(6) $ab <= b - - > a * b <= b0a = 0$
(7) $ba <= b - - > ba* <= b$

Denote by KA the class of models of these axioms, and write Horn(KA) and Eq(KA) for the Horn and equational theories of KA respectively.

Proposition 1. Kleene structures can be granted with an initial model characterization with morphic preorders.

Well apply fragment consistent model techniques to generate Kleene models. A tree game degree is the game state a tree is at with respect a model truth assignment, e.g. to the parameters on the Boolean functions on a game tree.

3. ISL Algebras

A String ISL algebra (Nourani 2005) is a Σ-algebra with an additional property that the signature Σ has a subsignture Ω that is only on 1-1 functions. The algebra was developed to apply to agent computing models where the 1-1 property allows the model to treat agent computing processes apart from everyday algebraic computing signatures. We can define specific ISL algebras based on specific signatures. For example, a Kleene ISL algebra is an algebra A $(A, +, 0, ., 1, *)$ such that $(A, +, 0)$ and $(A, ;, 1)$ are monoids, with + commutative and idempotent, and satisfying the Kleene conditions.

Lemma 2 String ISL algebra extending a Kleene algebra A $(A, +, 0, ., 1, *)$ such that $(A, +, 0)$ and $(A, ;, 1)$ are monoids, with + commutative and idempotent, is Kleene.

Lemma 3 String ISL algebra homomorphically extending a algebra A $(A, +, 0, ., 1, *)$ such that $(A, +, 0)$ and $(A, ;, 1)$ are monoids, with + commutative and idempotent, is Kleene.

Theorem 2. Let T be a ISL language theory. T is (a) a sound logical theory iff every axiom or proof rule in T preserves the tree game degree; (b) a complete logical theory iff there is a function-set pair $< F, S >$ defining a canonical structure M such that M has a generic diagram definable with the functions F.

4. Computing Product Models

From Pratt, let $L = L0, L1, L2, ..., Lw$ be the set of languages of the form $Li = xj|j < i$ for $i = 0, 1, 2, ..., w$; over an alphabet whose single symbol is x. This set is closed under the standard regular operations and has constants 0 and 1, namely $L0$ and $L1$ respectively, making it a subalgebra of the set of all languages on that alphabet and hence a model of REG. The above are example computing motivations to the following proposition. A specific equational action logic Equ(ACT) (Pratt 1990), Nourani (2006) is an example application area. The preliminary statement on the applications of the preceding section to computing, for example above, is the following proposition.

Proposition 2 (Positive Process Fragments) We can apply fragment consistency to obtain a model for Equational product languages by defining a language category over the signature. The preorder category has language fragment sets as objects and is preorded with the preimplication ordering.

Furthermore, we can obtain product models that apply the fragment consistency techniques the reduced products on the language sequents above up to positive Horn sentences, for example, to obtain product models.

Theorem 3. There is a fragment consistency positive process fragment model obtained for the language sequent Gamma that is a REG Model.
Theorem 4 There is a fragment consistency positive process fragment model that is a Equ(ACT) Model.

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Przemyslaw R. Pochec, *Algebra of queues: difference between two queues*

Queues as observed physical objects consist of elements and have a length. These characteristics present a possibility of using queues in set, or in algebraic, operations.

One such operation, a subtraction between two queues, may have practical applications in modeling and predicting performance of parallel systems. The difference between two queues can be seen as another queue, and in one case of two modified M/M/1 queues is an M/M/1 queue.

In my poster presentation I would like to present a summary of results concerning the difference queues.

Martin Raussen, *Trace spaces: Organization, calculations, applications*

A topological approach to the study of concurrency phenomena has led to the definition of Higher Dimensinoal Automata: The main interest, both mathematically and in concurrency, is in the study of *directed* (d)-paths in a pre-cubical complex up to *directed* homotopy. These paths can, in general, not be reversed; hence algebraic topological invariants of the spaces of d-paths in a d-space may vary with the selection of start and end points.[1]

Apart from the number of components of such a trace space that often can be calculated using a Seifert-van Kampen type theorem proved by Marco Grandis, not much has been known in general about the topology of d-path or trace spaces. We use a version of the Vietoris-Begle theorem due to Smale to arrive at additional information, e.g.:

Decomposition: Suppose given *closed d-convex* "layers" $L_1, \dots, L_{n-1} \subset X$ in a d-space X with additional properties (satisified by the geometric realization of a pre-cubical complex). If the layers are *unavoidable* in the given order for each trace (or each one in given homotopy classes) from x to y, then the space $\vec{T}(X)(x,y)$ of traces in X from x to y is weakly homotopy equivalent to a fibered product of the spaces $\vec{T}(X)(L_i, L_{i+1})$.

Reachable sequences: If, moreover, $\vec{T}(X)(x_i, x_{i+1})$ is either empty or (weakly) contractible for every $x_i \in L_i, x_{i+1} \in L_{i+1}$, then $\vec{T}(X)(x,y)$ is weakly homotopy equivalent to the subspace of "reachable" sequences $(x_1, \dots, x_{n-1}) \in X^{n-1}$ connecting points in these layers. In particular, for a nice space X, the infinite

[1]For technical reasons, it is often advisable to divide out (weakly) increasing reparametrizations; the quotient objects are the so-called traces – as in the title of the talk.

dimensional trace spaces can often be identified with a subspace of a finite dimensional space. Both results above can be generalized to d-paths along *digraphs* of layers.

Piecewise linear traces: If X is the geometric realization of a precubical complex, one may consider the *piecewise linear* (geodesic) d-paths in X that are linear on each cell (including its boundary). The space of piecewise linear traces $\vec{T}_l(X)$ is weakly homotopy equivalent to the space of all traces (connecting given end points). It can be subdivided into cube paths containing all PL-d-paths that follow a sequence of cubes. Such a cube path is a product of simplices, and $\vec{T}_l(X)$ comes with a prodsimplicial combinatorial structure. Since cube paths can be ordered by their length in "rounds", (connectivity) computations as in the work of Herlihy et al. seem promising.

If time permits, the following topics can be sketched as well:

Arc length: of d-paths in a pre-cubical complex and natural parametrization: A technically important notion. Remark that arc length is preserved under a d-homotopy!

Variation of invariants: A categorical organisation of the algebraic topological invariants of trace spaces with varying end points.

Pedro Real, *Digital objects, chain homotopies and discrete Morse theory*

In this paper, using a cellular analogous continuous, a nD digital object is seen from an geometric-algebraic point of view as a special chain homotopy acting on every cell of the object. Discrete Morse Theory can be integrated to this framework in a straightforward and algorithmic manner. We show some examples in $3D$ of the power of this approach in (co)homology computation.

Francis Sergeraert, *Constructive algebraic topology*

"Standard" Algebraic Topology is not appropriately designed for computations, for example the usual spectral sequences are most often *not* computable from the data usually available. From a theoretical point of view, standard Algebraic Topology is *not* constructive. To make it constructive, it suffices to replace the usual spectral sequences by the so-called Basic Perturbation "Lemma". Algorithms are then easily obtained, and by an interesting feedback process, the usual spectral sequences then become constructive as well.

David Spivak, *Geometric databases*

The theory of databases is central to computer science. Many formulations of databases have been given over the years, though most references seem to avoid rigorous mathematical definitions for any of them. In this paper, we define a new category of databases called the category of geometric databases. Among the formulations currently in use, geometric databases are most similar to relational databases.

Given data types DT, the Cech complex $C(DT)$ provides a classifying object for databases on DT. It is a simplicial set together with a sheaf of data called the universal data bundle. A database on DT is a pair (X, O_X), where X is a simplicial set and O_X is a sheaf of sets on X, equipped with a bundle map to the universal data bundle. Morphisms of databases

$$(f, f^{\#})\colon (X, O_X) \to (Y, O_Y)$$

are defined in a familiar way, e.g. they are defined analogously to morphisms of ringed spaces.

We show that any relational database can be reformulated as a geometric database. We further show that many of the typical operations and notions from the theory of relational databases (e.g. joins, projects, selects, cascading deletions, data striping, functional dependencies, etc.) have category-theoretic analogues in our setting. For example, the join of two databases corresponds to their union (i.e. colimit) under this correspondence.

Aside from providing a beautiful geometric picture of data organization, our theory has several advantages over the theory of relational databases. Most notably, queries of a geometric database always yield new geometric databases, whereas the corresponding statement for relational databases does not hold. One can take limits and colimits in the category of databases or calculate the homology of a database, and the results of these operations have meaningful "real-world" interpretations.

Samuel Vidal, ***Combinatorics of surface triangulations***

Triangulations of surfaces are one of the most basic building block of algebraic topology and they constitute an important data structure in computer graphics as they provide a handy discrete model of surfaces. From the point of view of computer science, the applications of surface triangulations are well known and numerous, they touch both practical and theoretical aspects of the discipline and they range from computer graphics to discret methods of solving partial differential equations. They also play a central role in manyalgorithms of computational geometry, a fast growing sub ject having an heavy industrial impact due to its use in computer aided design. One particularly interesting treat of the sub ject, apart from its broad range of applications, is precisely its ubiquity both in computer science, mathematical physics and even pure mathematics, providing generous range of fruitful exchange between seemingly remote parts of science. From the point of view of mathematics, the theory of combinatorial maps is also a venerable subject dating back to Cayley and Hamilton. Since those times, it generated an impressive amount of results of all sorts concerning the particular enumeration problem of counting the rooted combinatorial maps. Those results came from various communities of researchers, each with its own methods and tradition. Among them, enumerative combinatorists of course played a significant role, starting with pioneering works by Tutte [6] on rooted planar maps. Those works where at first motivated by the four color problem. Theoretical physicists also played a significant role, starting with the work by tHooft [5] on integration on random matrix spaces and Feynman diagrams. Pure mathematicians like Harer and Zagier [1] also have contributed to the theory in connection with cutting edge algebraic geometry problems concerning moduli spaces of Riemann surfaces. Last but not least, one must mention in mathematical physics the Witten-Kontsevich model of quantum gravity [2] using in a central fashion the higher combinatorics of triangular maps and trivalent diagrams. Although a lot is known concerning the theory of rooted combinatorial maps, very little is currently known about the outstanding problem of enumeration of unrooted combinatorial maps up to isomorphism, except for planar maps with the pioneering work of Liskovets [3]. It appears as a very difficult problem of combinatorics, which stayed barely untouched for almost 150 years. As a matter of fact, the only general result on those important ob jects were up to now contained in the recent paper by Mednykh and Nedela [4]. Using the Joyal theory of structure species, we describe a way to count both rooted and unrooted triangulations, up to diffeomorphism of the underlying surfaces. The generating series thus obtained are explicitly connected to the asymptotic expansion of the Airy function.

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Krzysztof Worytkiewicz, *Locally ordered spaces as sheaves*

Locally ordered spaces are manifolds with atlases consisting of partially ordered charts. They have been studied for some time since they are well suited to model the behavior of interacting computational processes in a way which captures the flow of time. Computational paths are modeled by continuous locally non-decreasing maps while meaningful homotopies among such paths are the non-decreasing ones. There are however some diffculties with the category of locally ordered spaces L, notably with colimits. The present work is about an approach to locally ordered spaces in terms of sheaves over a quite natural site (P, t). This appraoch rectifies some of the mentioned difficulties.The main technical developments discussed in this talk will be the characterisation of those sheaves which are in the image of an embedding $L \to \mathrm{Sh}(P, t)$, followed by some homotopy-theoretical remarks.

Reyer Zwiggelaar, *Classification of micro-calcifications using Betti numbers at various scales*

Mammographic screening involves a detailed visual search of breast X-rays for signs of cancer. It is expected that computer aided diagnosis (CAD), with a sufficiently high sensitivity and specificity, will lead to an improvement in readers' performance.

Micro-calcifications appear in mammographic images as groups of small bright blobs. Current CAD systems can detect more than 98% of malignant micro-calcifications, but at the same time detect non-malignant (i.e. benign) micro-calcifications and as such the specificity of these CAD methods needs to be improved.

The automatic classification of clusters of micro-calcifications tends to be based on features extracted from the distribution and individual micro-calcifications or other anatomical structures (e.g. linear structures such as ducts or vessels) associated with malignant micro-calcifications.

Here we want to investigate what role the connectivity between the individual micro-calcifications within a cluster can play in this classification process. A cluster of micro-calcifications can be regarded as a single object, which contains a number of segments (the individual micro-calcifications) and the connectivity between these segments can be determined through homology.

The number of components, tunnels and voids are captured by Betti numbers: β_0, β_1, and β_2. The Betti numbers (or homology groups) can be estimated by measuring the number of vertices, edges, faces, etc.. However, very simple computer vision techniques can also be used to determine the number of components (e.g. bright areas) and tunnels (dark areas within bright areas) in images. In effect, we want to determine how H_0 (or β_0) changes as a function of some other parameter (in this case related to the scale of objects). The values of this function form our feature space. Once feature vectors have been obtained, we use a classical nearest-neighbour (NN) classification approach. A simple Euclidean distance and a leave-one-out methodology were used.

The data used in our experiment are twenty 512×512 pixel image patches, each containing a cluster of micro-calcifications, taken from the Mammographic Image Analysis Society database. Mammographic diagnosis has been proved by biopsy and there are nine malignant and eleven benign calcification clusters. In each group, there are mammograms with a variety of breast backgrounds (glandular, fatty and dense). All the micro-calcifications have been annotated by an expert radiologist. All the malignant versus benign classification was estimated on an image basis.

The initial result indicate correct classification of 95% for benign versus malignant diagnosis, with an area under the receiver operating curve equal to 0.91. These results compare favorably to those published, which show A_z values that range from 0.69 to 0.87. It should be noted that all of the malignant cases were classified correctly.

From the initial results on the mammographic patches, it seems that the distribution of positions where micro-calcifications emerge are important to determine the probability of the clusters being malignant or benign. This makes the assumption that the centres of the micro-calcifications as determined by the described approach are close (identical) to the positions where these are initially started development, which is not a particular strong assumption because most micro-calcifications are spherical to elliptical in shape.