The Pearson $\chi^2_P(X_1 : X_2) = \int \frac{(x_2(x) - x_1(x))^2}{x_1(x)} \, d\nu(x)$ and Neyman chi square distances $\chi^2_N(X_1 : X_2) = \chi^2_P(X_2 : X_1)$ are generalized to higher-order chi (signed) divergences $\chi^k_P(X_1 : X_2) = \int \frac{(x_2(x) - x_1(x))^k}{x_1(x)} \, d\nu(x)$, and we show how to compute $f$-divergences from those chi-type divergences. For random variables of the same exponential family with affine natural space (like isotropic Gaussians, Poissons or multinomials), the Pearson/Neyman chi square distance between $X_1 \sim \mathcal{E}_F(\theta_1)$ and $X_2 \sim \mathcal{E}_F(\theta_2)$ is given by $\chi^2_P(X_1 : X_2) = e^{F(2\theta_2 - \theta_1) - (2F(\theta_2) - F(\theta_1))} - 1 = \chi^2_N(X_2 : X_1)$. For Poisson distributions, we get $\chi^2_P(\lambda_1 : \lambda_2) = \exp \left( \frac{\lambda_2^2}{\lambda_1} - 2\lambda_2 + \lambda_1 \right) - 1$ and for isotropic Gaussians $\chi^2_P(\mu_1 : \mu_2) = e^{(\mu_2 - \mu_1)^2} (\mu_2 - \mu_1) - 1$. For members of the same exponential family with affine natural parameter space, $\chi^k_P(X_1 : X_2) = \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} e^{F((1-j)\theta_1 + j\theta_2)}$, so that $\chi^k_P(\lambda_1 : \lambda_2) = \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} e^{(\lambda_1 - \lambda_2)^2} - (1-j)(\lambda_1 + j\lambda_2)$, and $\chi^k_P(\mu_1 : \mu_2) = \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} e^{2f(j-1)(\mu_1 - \mu_2)^2} (\mu_1 - \mu_2^2)$. $f$-Divergences can be expressed as $I_f(X_1 : X_2) = \int x_1(x) \sum_{i=0}^{\infty} \frac{1}{i!} f^{(i)}(\lambda) \left( \frac{x_2(x)}{x_1(x)} - \lambda \right)^i \, d\nu(x) = \sum_{i=0}^{\infty} \frac{1}{i!} f^{(i)}(\lambda) \chi^i_{\lambda,P}(X_1 : X_2)$. In particular, when $X_1$ is close to $X_2$, we have $I_f(X_1 : X_2) \sim \frac{f''(\lambda)}{2} \chi^2_P(X_1 : X_2)$. The Kullback-Leibler divergence is expressed as $KL(X_1 : X_2) = \sum_{j=2}^{\infty} \frac{(-1)^j}{j!} \chi^j_P(X_1 : X_2)$. Series truncation for approximation and upper bounds of the Taylor remainder are studied with experiments.

References