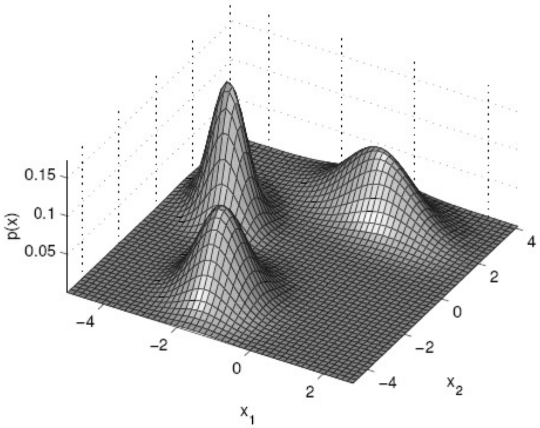
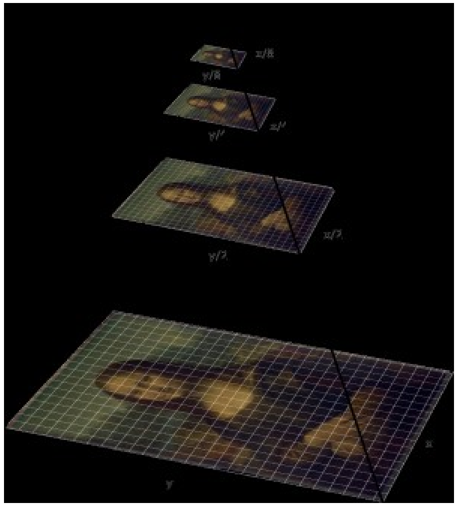
INF555

Fundamentals of 3D





Lecture 9:

Laplacian Image pyramids Expectation-Maximization

+ Overview of computational photography

Frank Nielsen
nielsen@lix.polytechnique.fr

30 Novembre 2011

Télécharger votre projet le 10 Décembre au soir

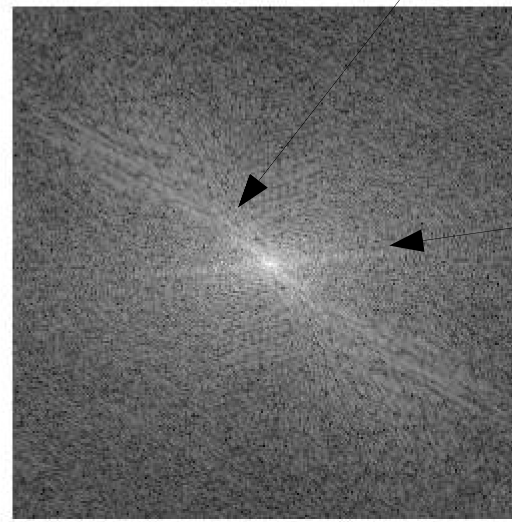
Examen 13 Décembre

Venez avec une clé USB (Projet, TDs)

→ Utilisez Processing.org+JAMA+JMyron (dans la mesure du possible)

Interpreting Fourier spectra

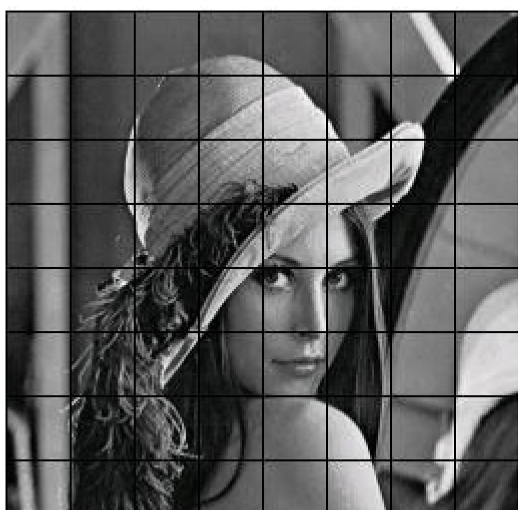
Stripes of the hat

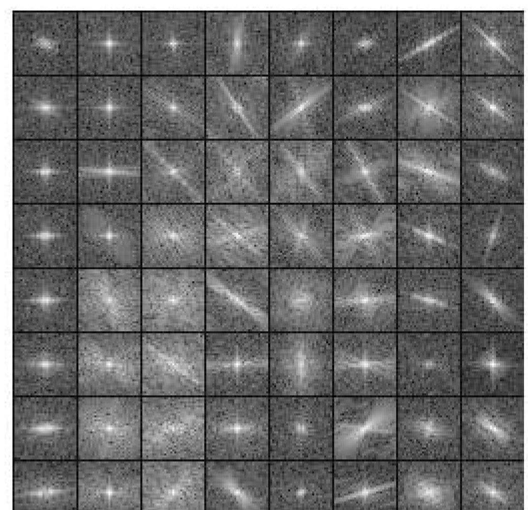


Stripes of the hair

90 degrees

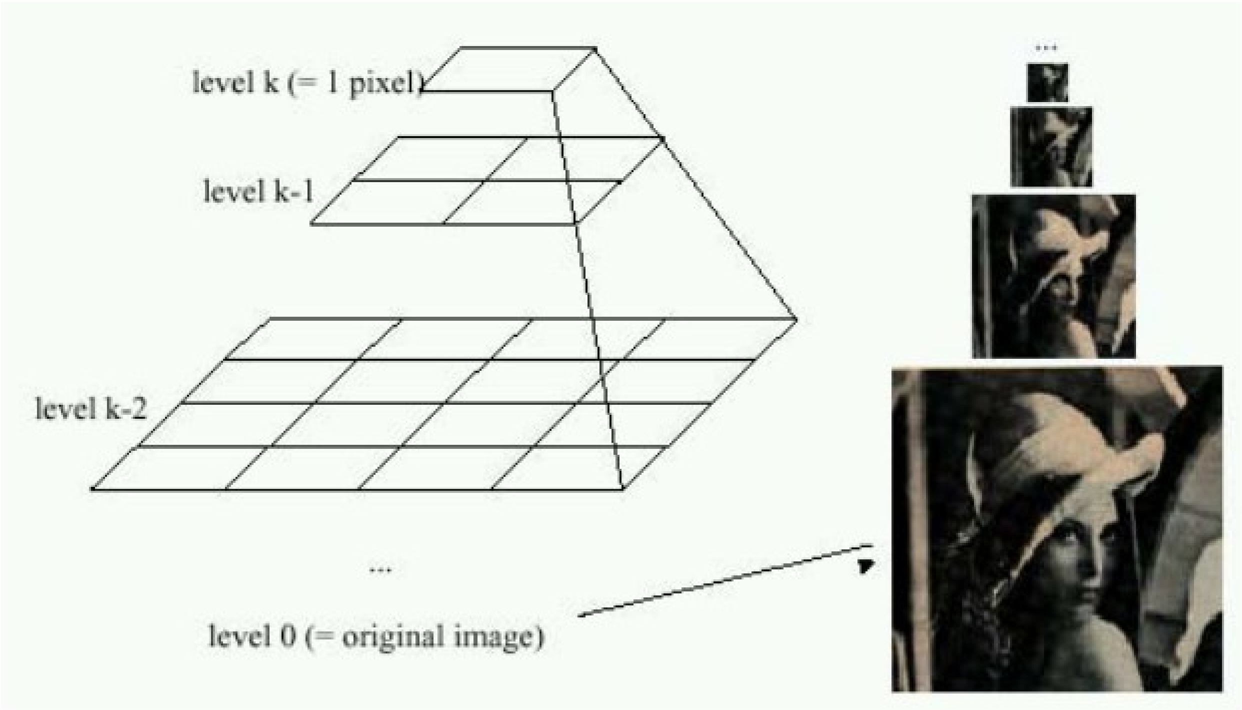
Fourier log power spectrum





Divide by blocks

Laplacian image pyramids



Used also in computer graphics for texturing (mipmapping)

Laplacian image pyramids

Gaussian. Blur and sample, and then

Laplacian. Interpolate and estimate

$$\mathbf{G}_1 = \mathbf{I}$$

$$\mathbf{G}_i = \mathrm{EXPAND}(\mathbf{G}_{i+1}) + \mathbf{L}_i.$$ Reconstruction

$$\mathbf{L}_i = \mathbf{G}_i - \mathrm{EXPAND}(\mathbf{G}_{i+1})$$ Residual

$\mathbf{L}_1 = \mathbf{G}_1 - \mathrm{EXPAND}(\mathbf{G}_2)$	$\mathbf{G}_4 = \mathbf{L}_4 + \mathrm{EXPAND}(\mathbf{G}_5)$
$\mathbf{L}_2 = \mathbf{G}_2 - \mathrm{EXPAND}(\mathbf{G}_3)$	$\mathbf{G}_3 = \mathbf{L}_3 + \mathrm{EXPAND}(\mathbf{G}_4)$
$\mathbf{L}_3 = \mathbf{G}_3 - \mathrm{EXPAND}(\mathbf{G}_4)$	$\mathbf{G}_2 = \mathbf{L}_2 + \mathrm{EXPAND}(\mathbf{G}_3)$
$\mathbf{L}_4 = \mathbf{G}_4$	$\mathbf{G}_1 = \mathbf{L}_1 + \mathrm{EXPAND}(\mathbf{G}_2) = \mathbf{I}$

$\rightarrow$ Precursors of wavelets

Laplacian image pyramids

Blurring is efficient for sampling as it removes high-frequency components. (sample at fewer positions.)

Gaussian kernel and resampling at a *quarter* of the image size.
Blurring and resampling is computed using a *single* discrete kernel.

Why Gaussians?

- Central limit theorem:
  (mean of random variables approach Gaussian distribution)

- Infinitely differentiable functions

- Fourier of Gaussians are Gaussians

- Human brain has neuronal regions doing Gaussian filtering

Laplacian image pyramids: Lossless multi-scale representation of images

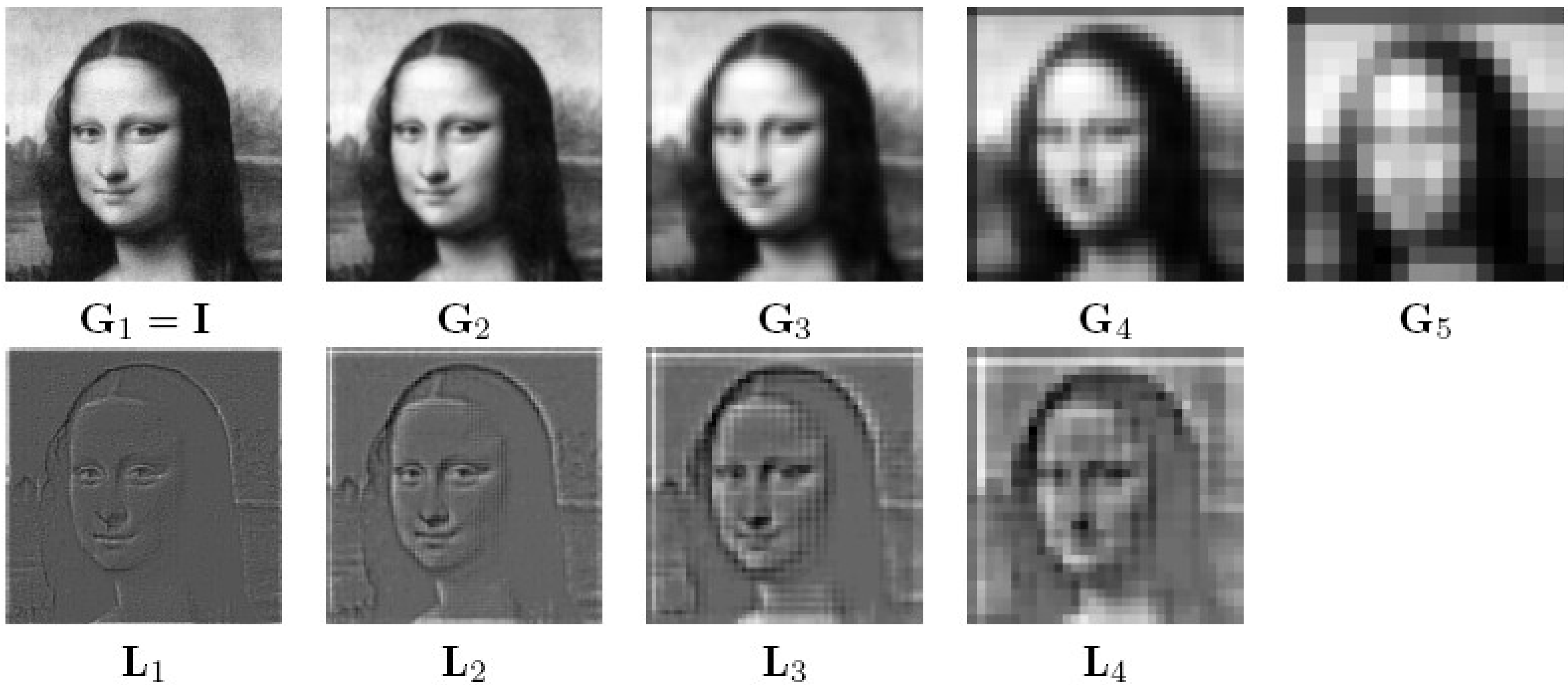
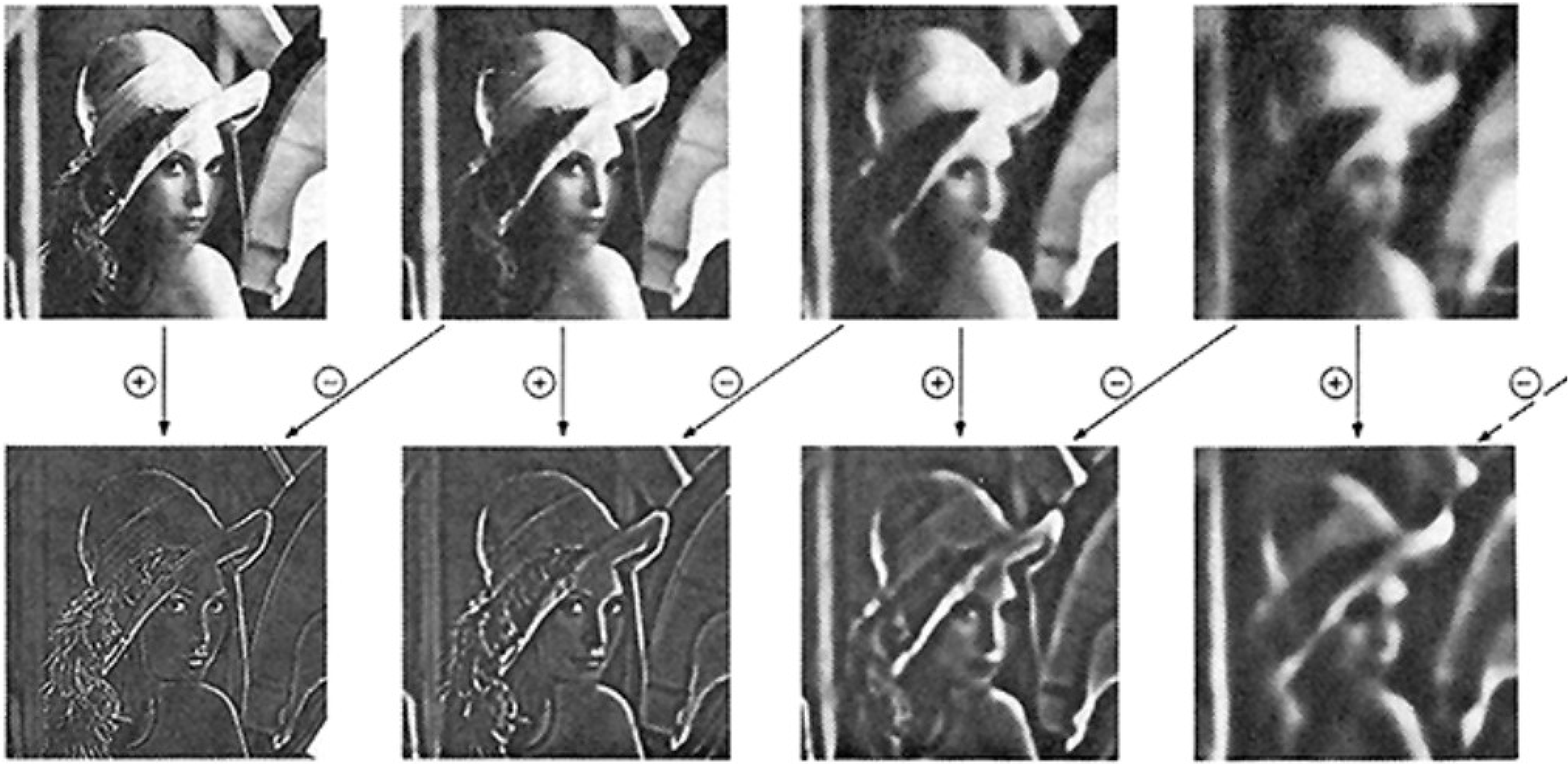


FIGURE 4.46 *Gaussian and Laplacian image pyramids: the original image* $\mathbf{I}$ *can be reconstructed without any error from the smallest image of the Gaussian pyramid* $(\mathbf{G}_5)$ *and the Laplacian image pyramid* $\mathcal{L} = \{\mathbf{L}_i\}_i$.

Laplacian image pyramids: Reconstruction process



Laplacian image pyramids: Application to blending

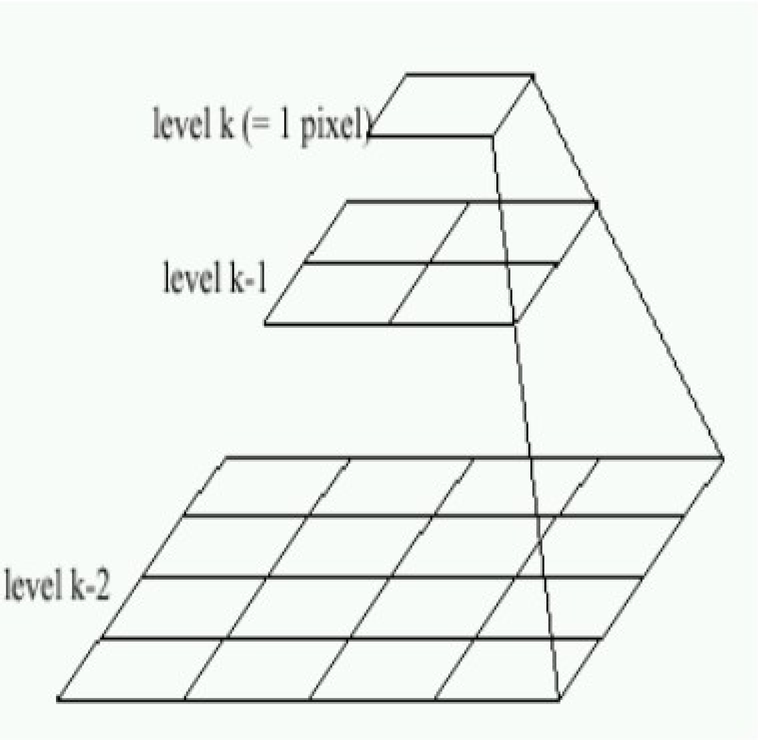
Multiband blending.

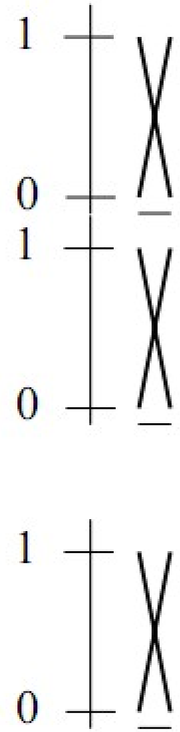
Blending two overlapping images using their pyramids

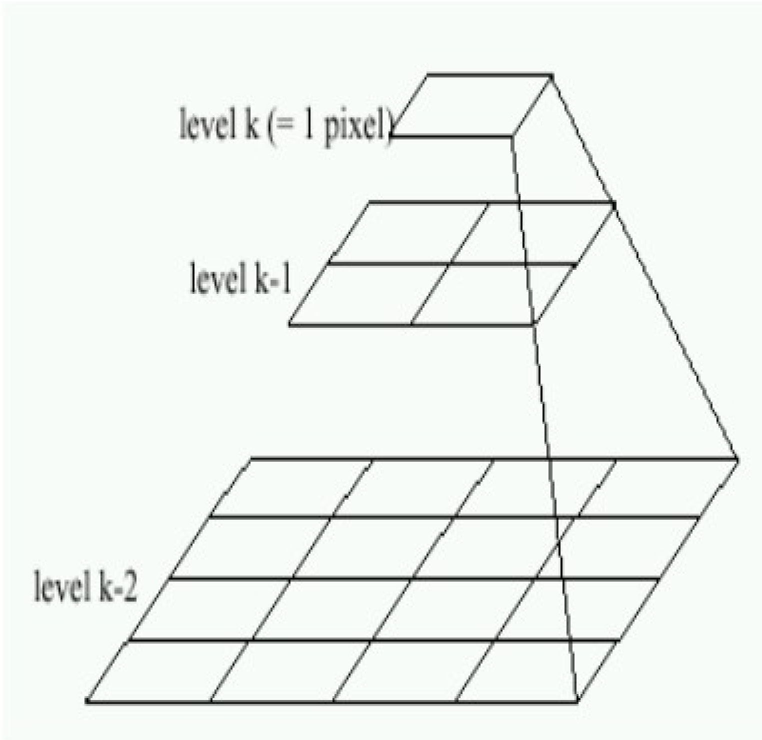
- Compute Laplacian pyramids *L*(**I**1) and *L*(**I**2) of **I**1 and **I**2.
- Generate a hybrid Laplacian pyramid *Lr* by creating for each image of the pyramid a 50%/50% mix of images, obtained by selecting the leftmost half of *L*(**I**1) with the rightmost half of *L*(**I**2).

- Reconstruct blended images from the Laplacian pyramid *Lr*.

Laplacian image pyramids: Application to blending





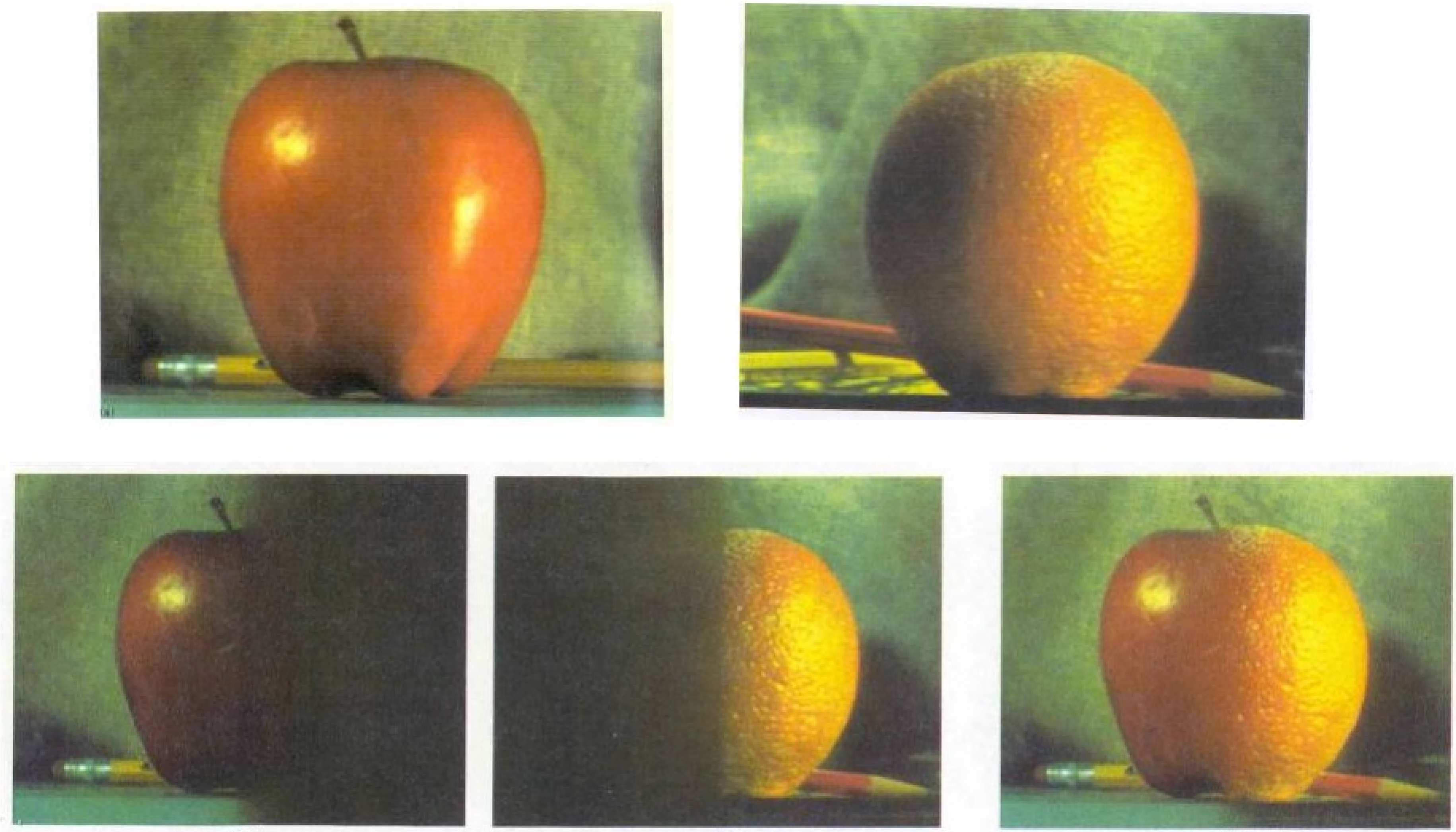


Left pyramid

blend

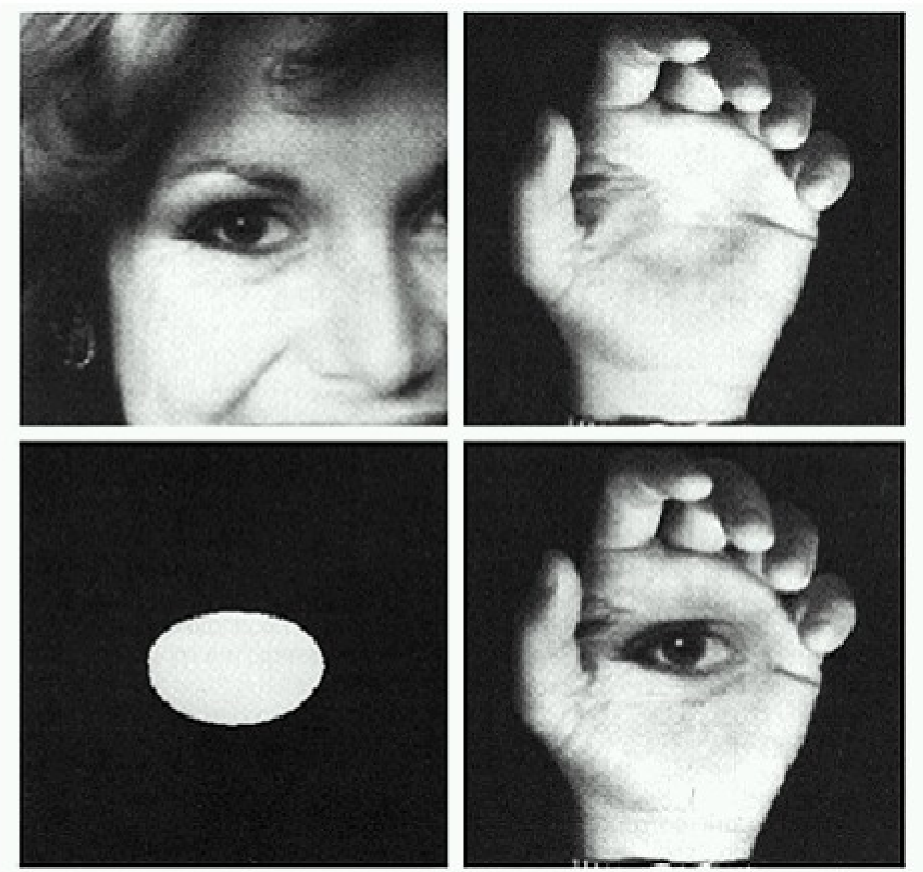
Right pyramid

Laplacian image pyramids: Application to blending

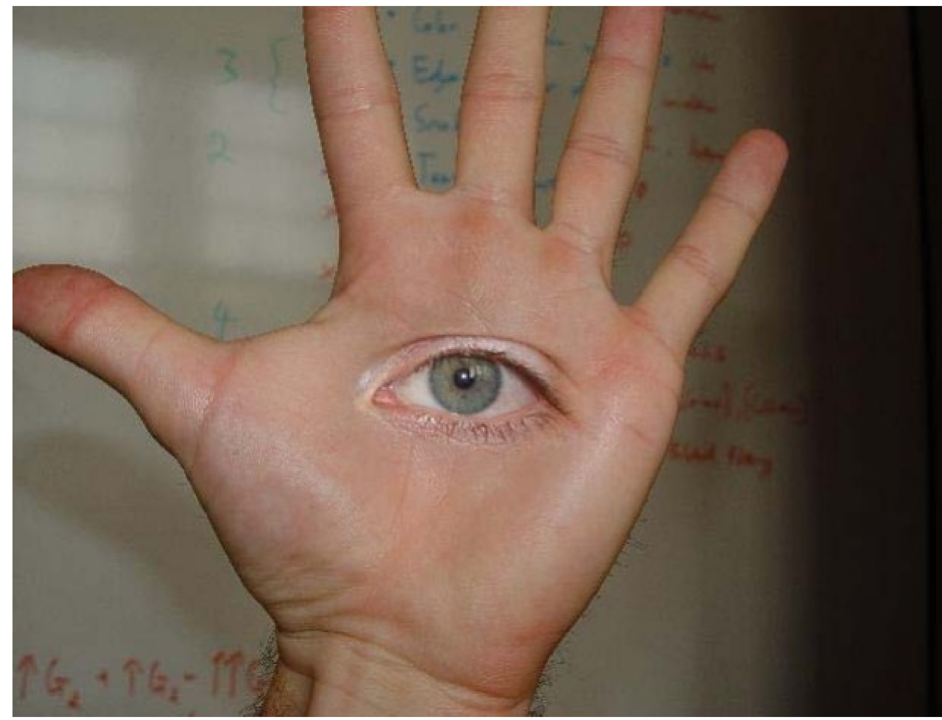


(d) (h) (l)

Laplacian image pyramids: Application to blending



Using a region mask



Nowadays, we better use **Poisson image editing** and gradient/image reconstruction

Soft clustering using Expectation-Maximization (EM)

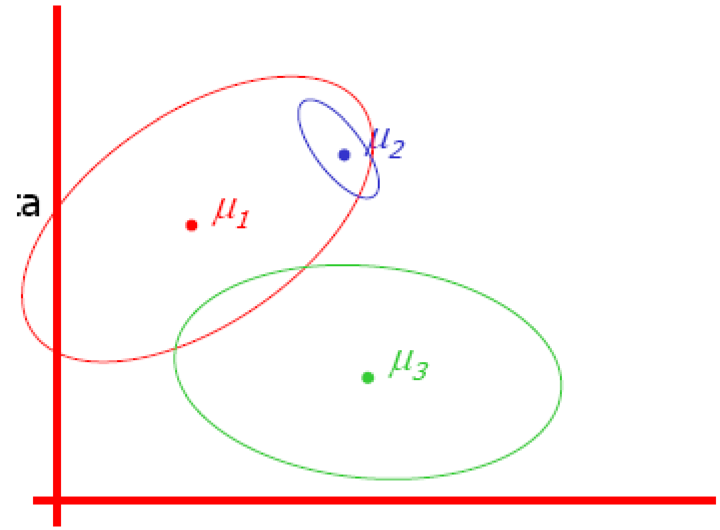
Generative statistical models

$$\mathbf{x}; \boldsymbol{\theta}_m \sim \mathcal{N}\left(\mathbf{x}; \boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m\right)$$

$$\mathcal{N}\left(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) = (2\pi)^{-d/2} \left|\boldsymbol{\Sigma}\right|^{-1/2} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}-\boldsymbol{\mu})\right\}$$

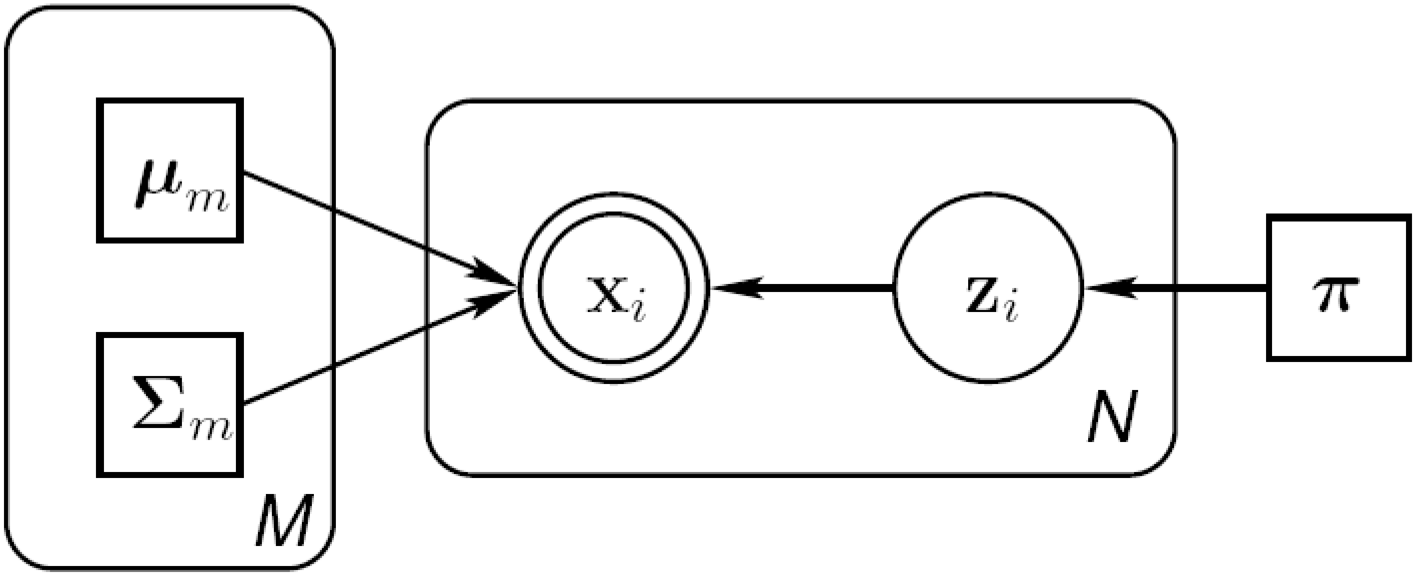
$$f(Y=y|\theta) = \sum_{j=1}^{k} \alpha_j \frac{1}{\sqrt{(2\pi)^d |\Sigma_j|}} \exp\{-\frac{1}{2}(y-\mu_j)^T \Sigma_j^{-1} (y-\mu_j)\}$$

GAUSSIAN MIXTURE MODELS (GMMs)



http://www.neurosci.aist.go.jp/~akaho/MixtureEM.html

Indicator variables z (also called latent variables)



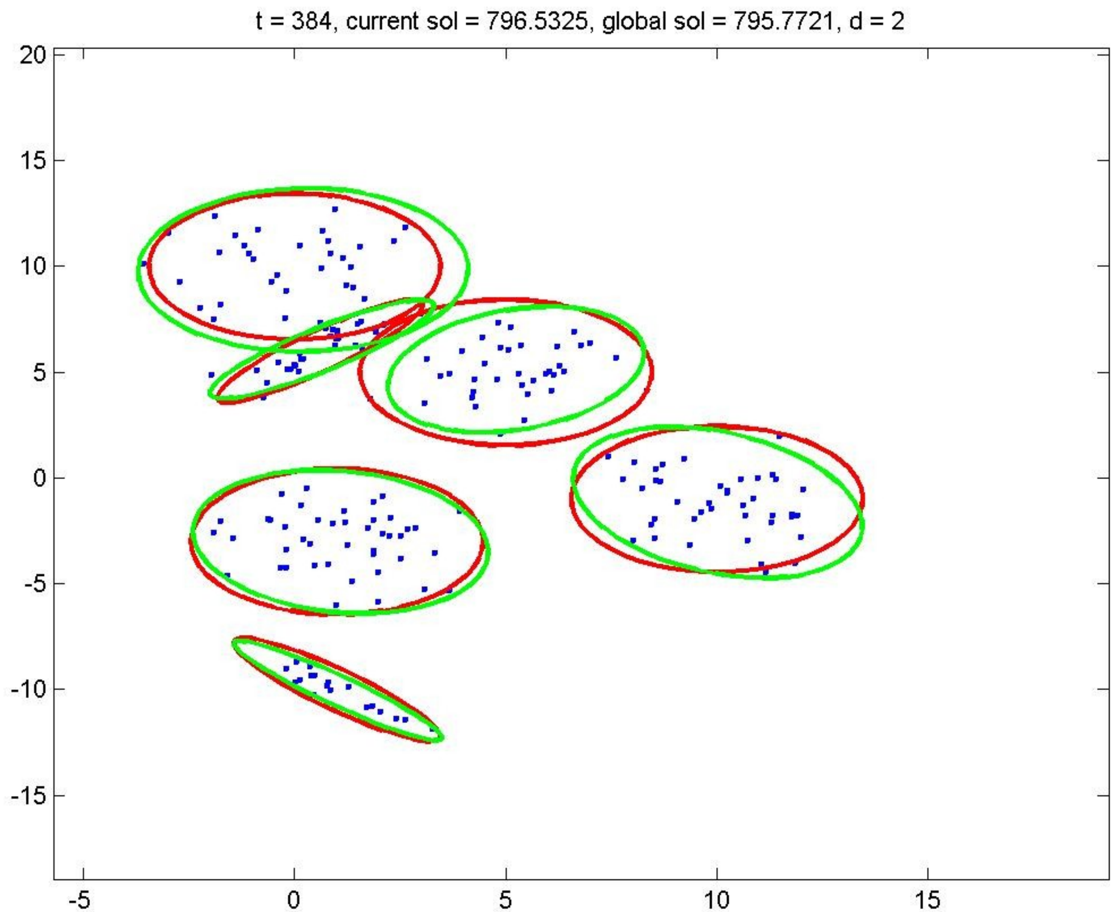
$$\mathbf{z}_i = \underbrace{\begin{bmatrix} 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 \end{bmatrix}^T}_{M \text{ elements}}$$

Multinomially distributed π_m

$$p(\mathbf{x}_i | z_{im} = 1; \boldsymbol{\theta}) = \frac{1}{(2\pi)^{d/2} |\boldsymbol{\Sigma}_m|^{1/2}} \exp\left\{ -\frac{1}{2} (\mathbf{x}_i - \boldsymbol{\mu}_m)^T \boldsymbol{\Sigma}_m^{-1} (\mathbf{x}_i - \boldsymbol{\mu}_m) \right\}$$

Generating samples from Gaussian Mixture Models (GMMs)

1: **for** $i = 1$ to N **do**
2: $\quad m \leftarrow$ index of one of the M models randomly selected according to the prior probability vector $\boldsymbol{\pi}$
3: $\quad$ Randomly generate $\mathbf{x}_i$ according to the distribution $\mathcal{N}(\mathbf{x}_i; \boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m)$
4: **end for**



Maximize the likelihood (incomplete)

$$\mathcal{L}(\boldsymbol{\theta}) = p(\mathbf{X};\boldsymbol{\theta})$$

$$\boldsymbol{\theta} = \{\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m, \pi_m\}_1^M$$

Maximize the likelihood (complete likelihood)

$$p(\mathbf{X}, \mathbf{Z};\boldsymbol{\theta})$$

joint distribution of $\mathbf{X}$ and $\mathbf{Z} = \{\mathbf{z}_i\}_1^N$

Expectation-Maximization algorithm: Iteration

EM iteration:

- Expectation step : Soft assignment to clusters

$$w_{tj} = p(x_t = j | y_t) = \frac{\alpha_j f(y_t | \mu_j, \Sigma_j)}{\sum_{i=1}^{k} \alpha_i f(y_t | \mu_i, \Sigma_i)}$$

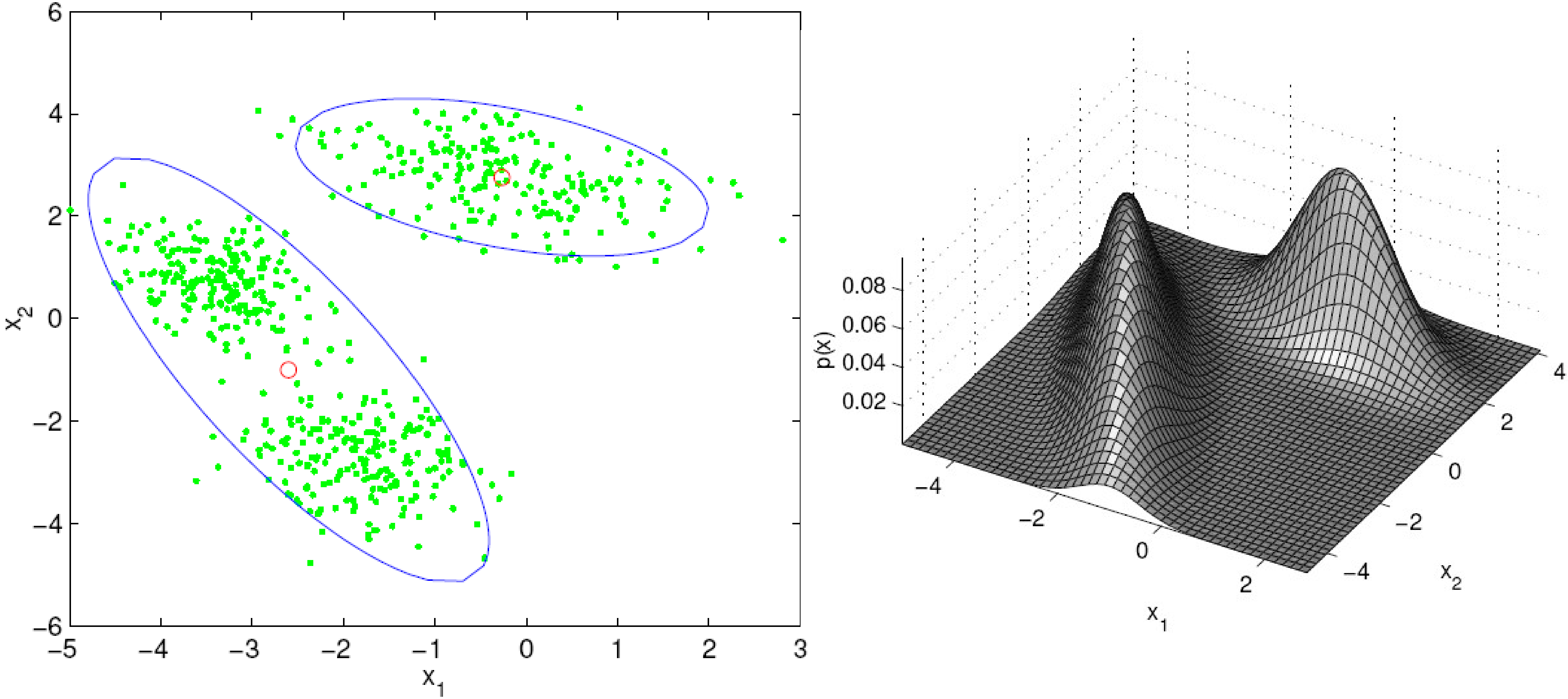
- Maximization step : Given assignments find best parameters

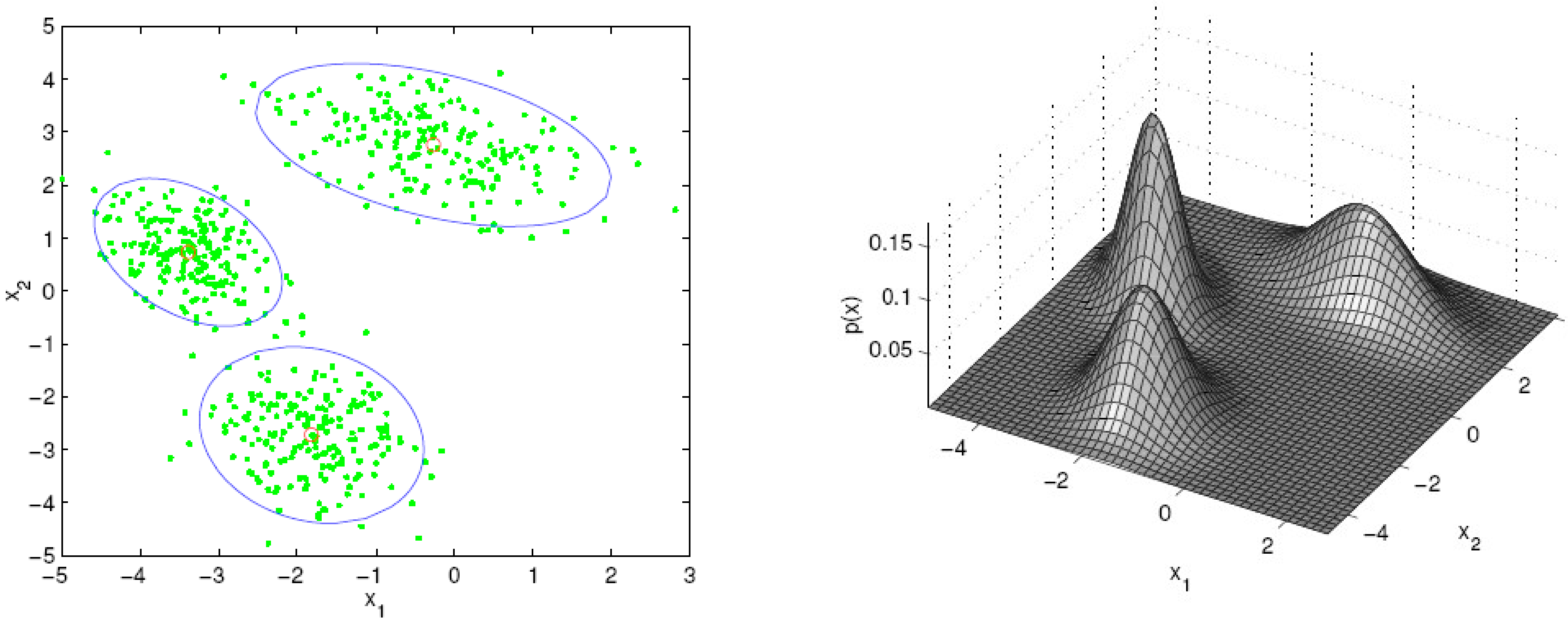
$$\hat{\alpha}_j \leftarrow \frac{1}{n} \sum_{t=1}^{n} w_{tj}$$

$$\hat{\mu}_j \leftarrow \frac{\sum_{t=1}^{n} w_{tj} y_t}{\sum_{t=1}^{n} w_{tj}}$$

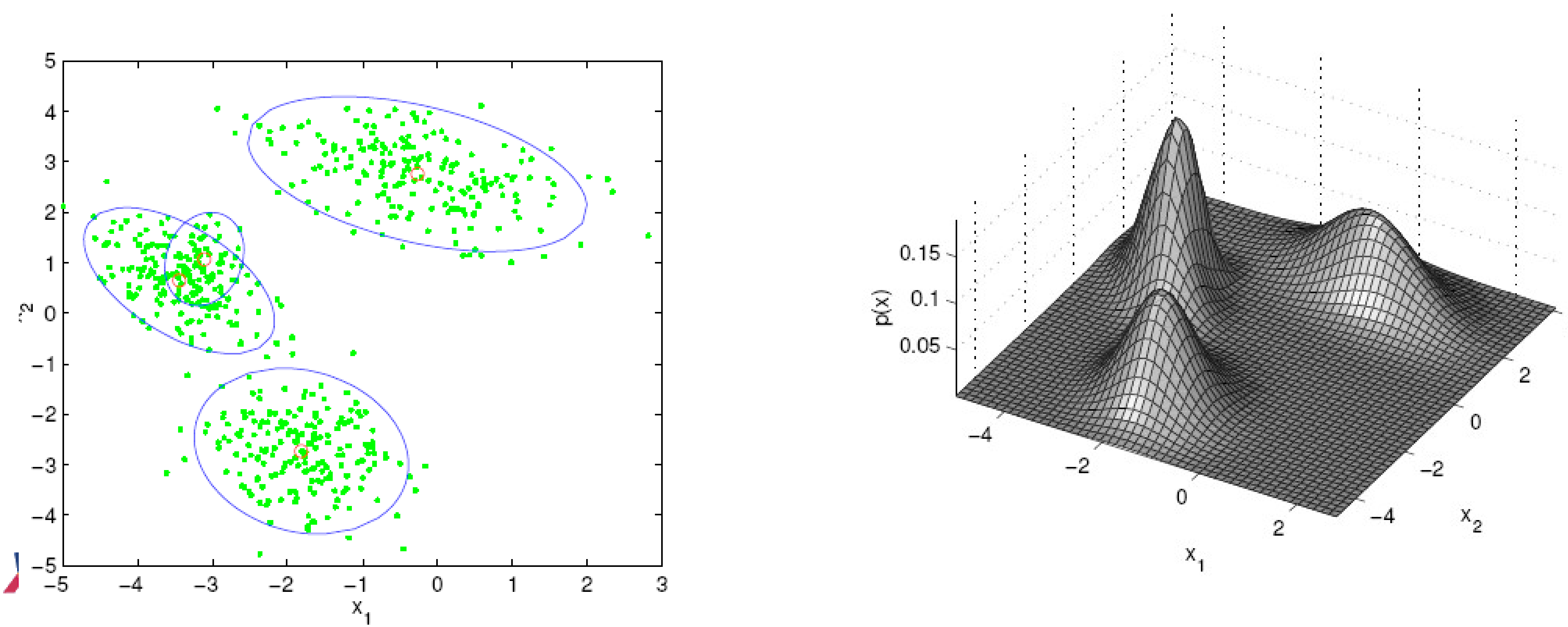
$$\hat{\Sigma}_j \leftarrow \frac{\sum_{t=1}^{n} w_{tj} (y_t - \hat{\mu}_j)(y_t - \hat{\mu}_j)^T}{\sum_{t=1}^{n} w_{tj}}$$

Initialize with k-means (or k-means++)

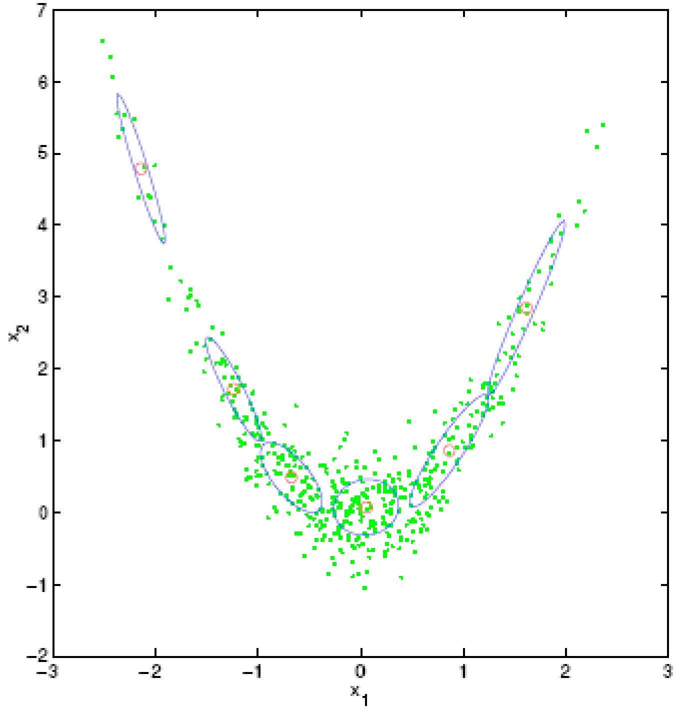

6
4
2
0
−2
−4
−6
x_2
−5
−4
−3
−2
−1
0
1
2
3
x_1
0.08
0.06
0.04
0.02
p(x)
−4
−2
0
2
x_1
4
2
0
−2
−4
x_2

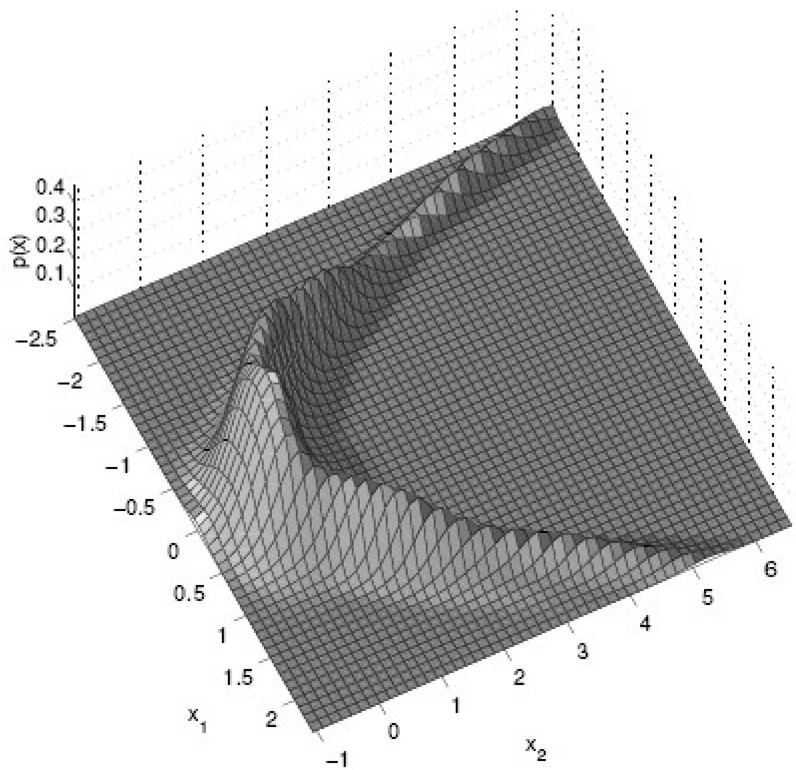


x1
x2
p(x)

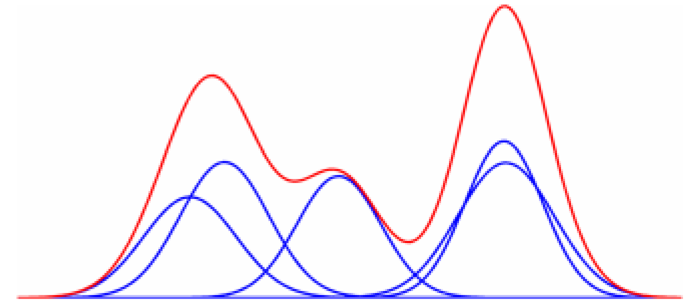


x1
x2
p(x)


x1
x2

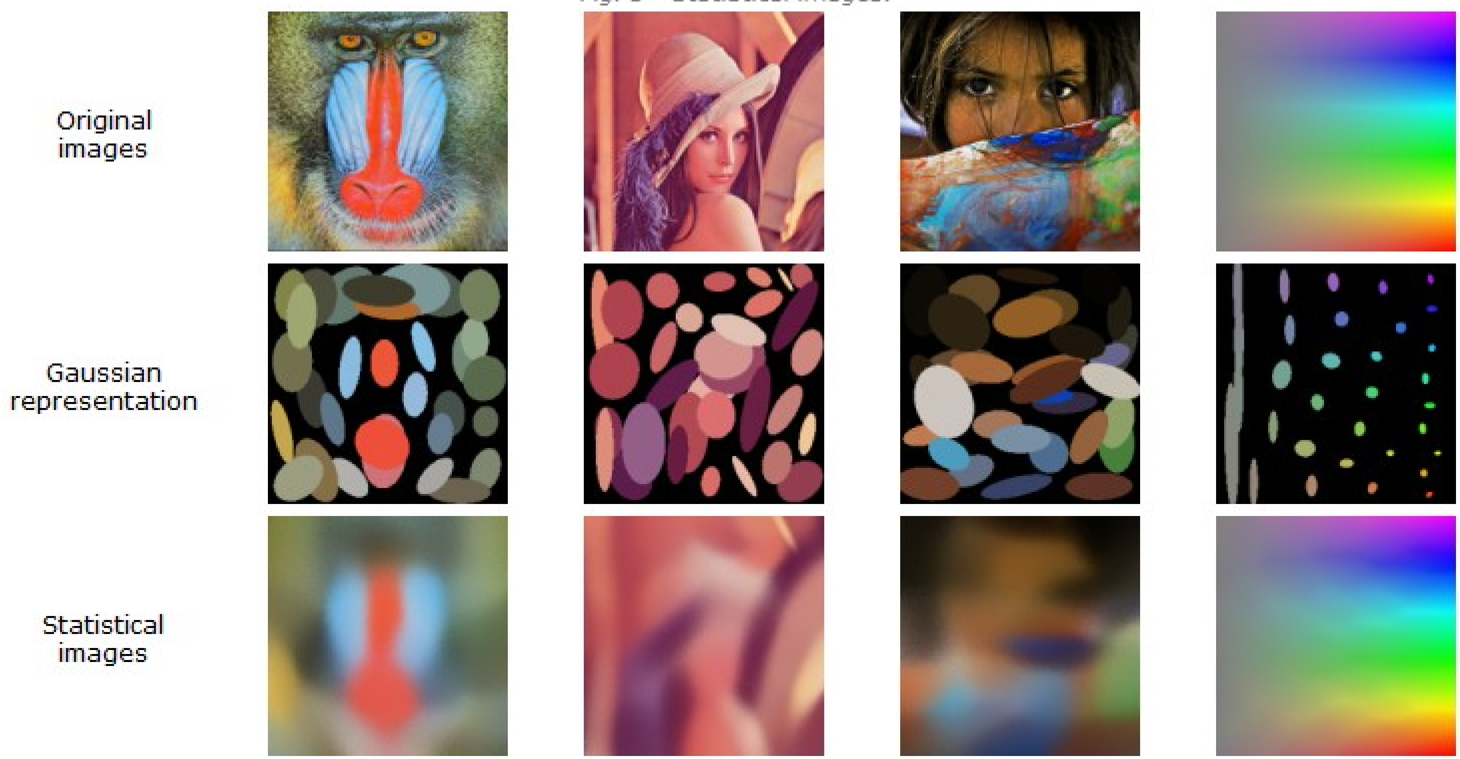

p(x)
x1
x2

Modeling images with Gaussian mixture models



RGB+XY= Point in 5D

Fig. 3 - Statistical images.



En Java, http://www.lix.polytechnique.fr/~nielsen/MEF/

En Python, http://www.lix.polytechnique.fr/~schwander/pyMEF/

Gaussian mixture models for image segmentation

Any smooth density function can be arbitrarily closely approximated by a Gaussian mixture model