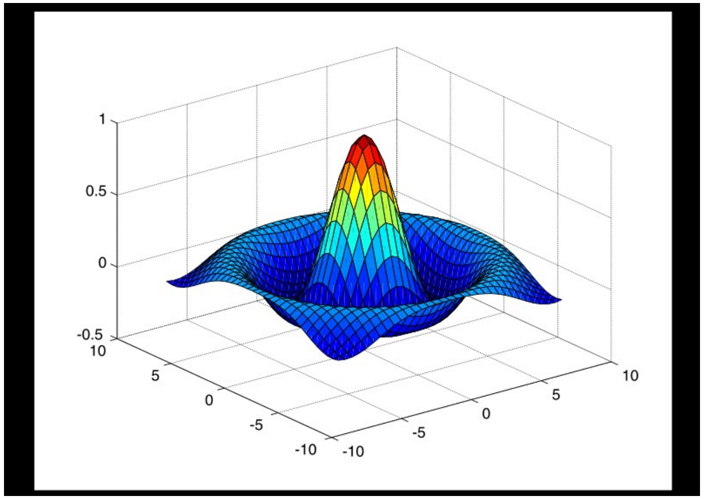
INF555

Fundamentals of 3D



Lecture 6:
Metric ball trees/Texture synthesis
Advanced coordinate pipelines
Fourier analysis/interpolation

Frank Nielsen
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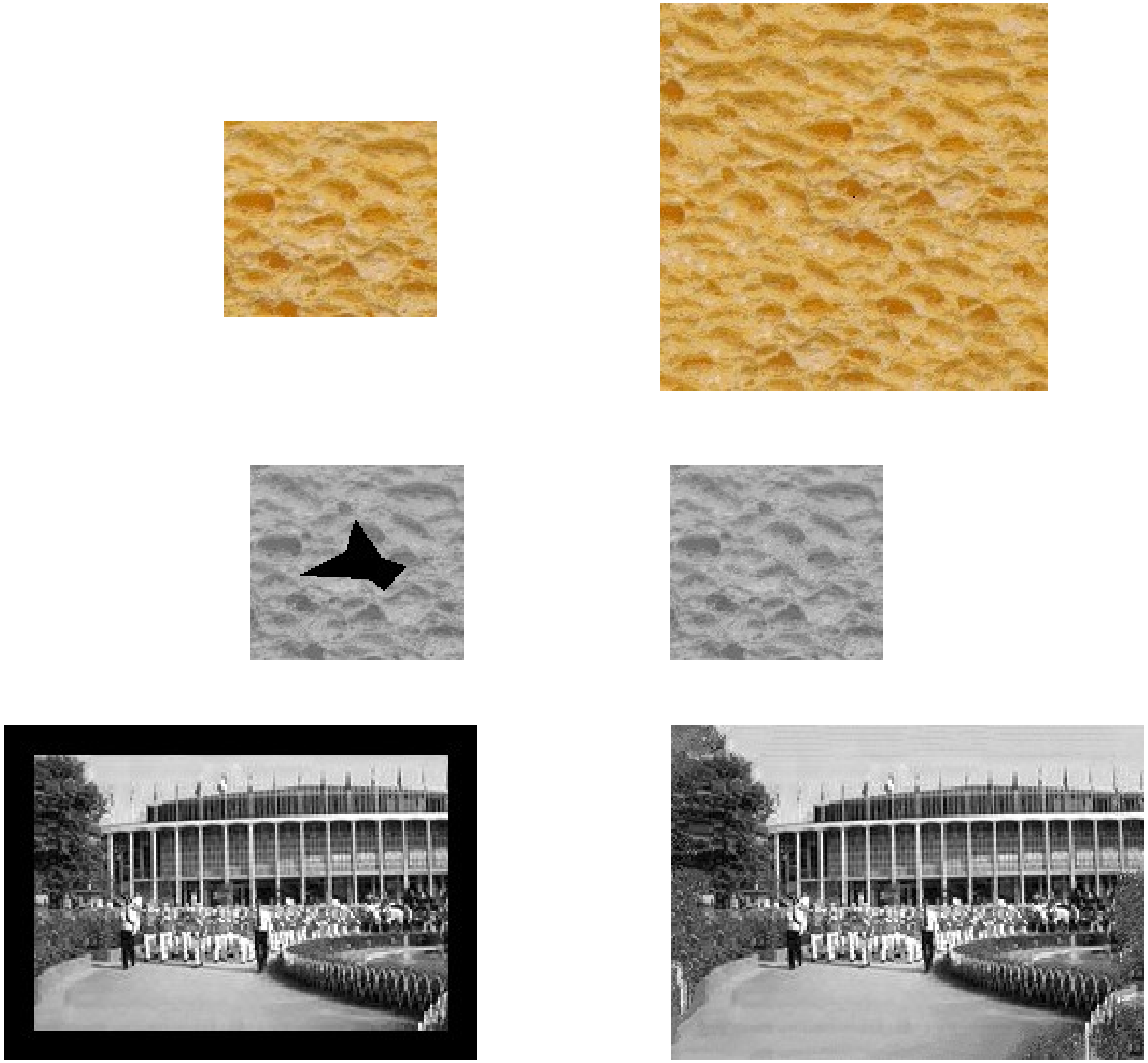
19th October 2011



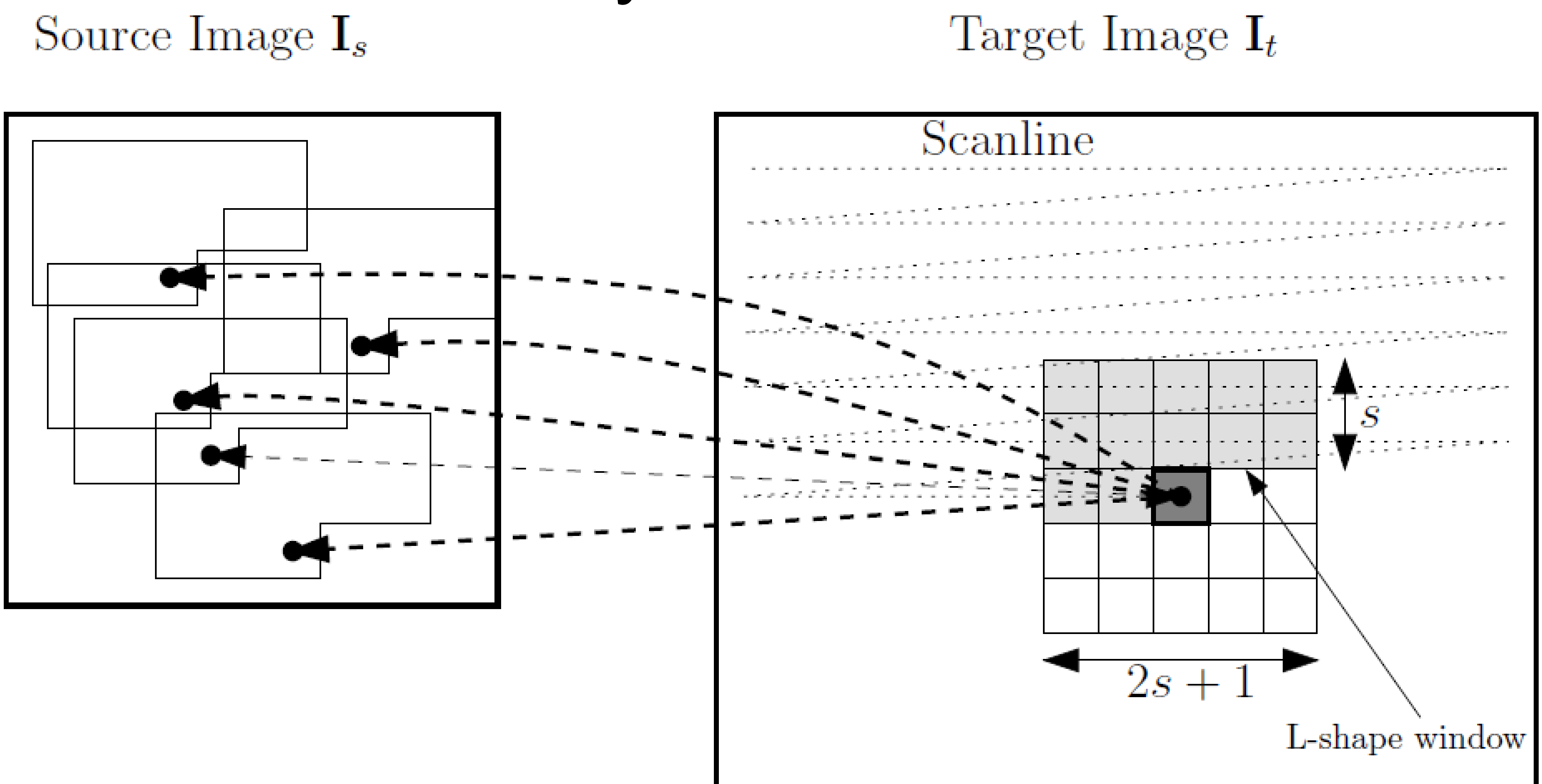
http://graphics.cs.cmu.edu/people/efros/research/EfrosLeung.html

``Texture Synthesis by Non-parametric Sampling''

Alexei A. Efros and Thomas K. Leung

IEEE International Conference on Computer Vision (ICCV'99),

Stochastic texture synthesis

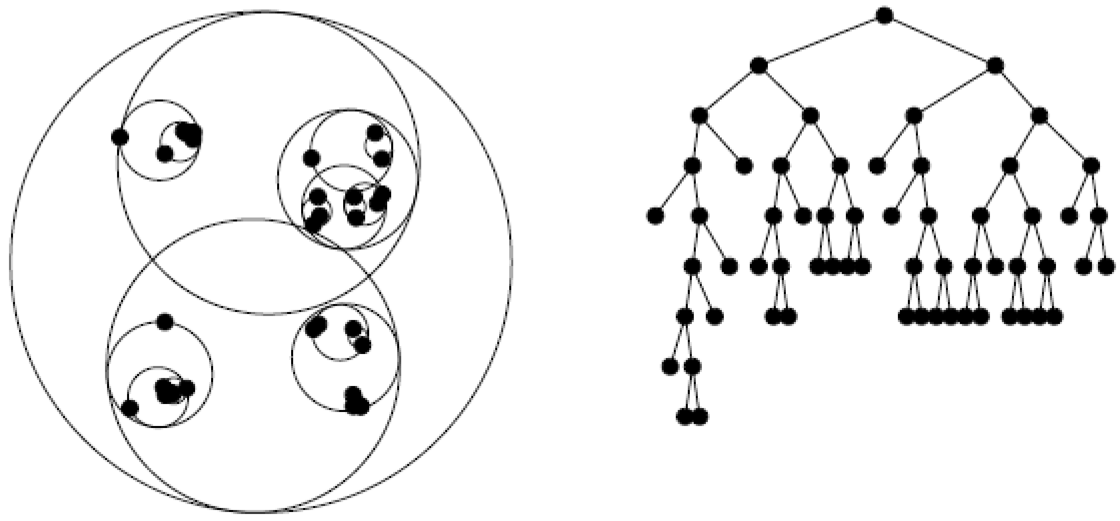


$$\mathrm{SSD}(x_s, y_s; x_t, y_t) = \sum_{l=-s}^{s} \sum_{c=-s}^{s} \mathrm{LShape}(l, c) \left(\mathbf{I}_s[x_s + c, y_s + l] - \mathbf{I}_t[x_t + c, y_t + l]\right)^2$$

$$(x_s, y_s) = \mathrm{argmin}_{(x,y)\in \mathbf{I}_s} \mathrm{SSD}(x, y; x_t, y_t).$$

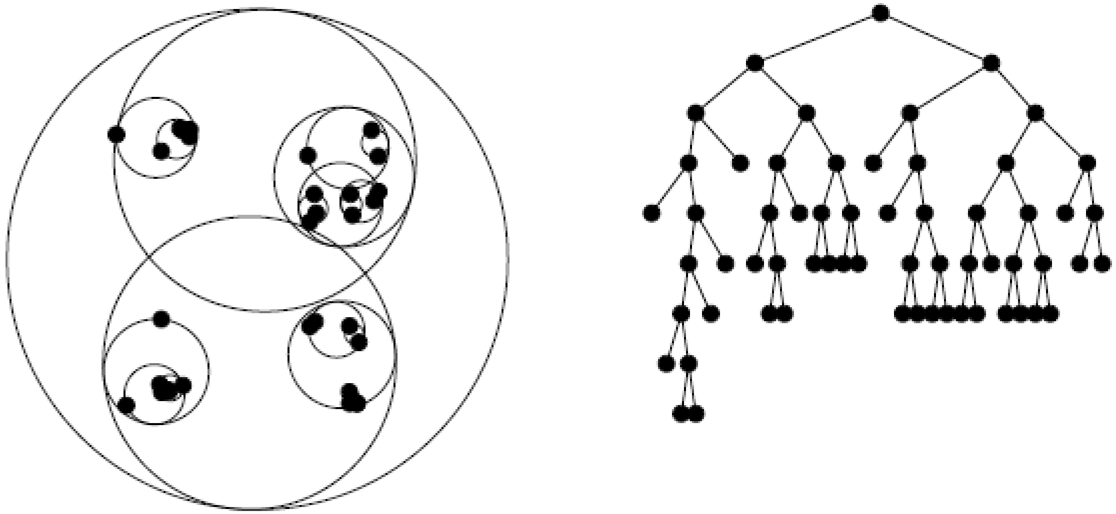
Fast nearest neighbor queries in high dimensions

Ball tree data structures for nearest neighbor search



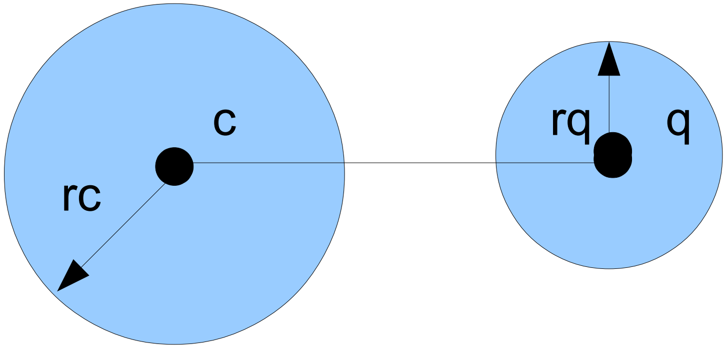
Compute a k-means on S with k=2
Split S into S1 and S2 according to the two centroids
Perform recursion on S1, and S2 until |S1|<n0 and |S2|<n0

Nearest neighbor queries using ball trees



Pruning some of the nodes:
Let NN(q) denote the current best nearest neighbor of q

if ||q-c||-rq> rc then PRUNE (do not explore the subtree)



At leaves, perform the naive linear search, and potentially update NN(q)

Careful seeding for k-means: Perform just a careful initialization!!!

Interpolate between the two methods:

Let $D(x)$ be the distance between x and the nearest cluster center. Sample proportionally to $(D(x))^\alpha = D^\alpha(x)$

Original Lloyd's: $\alpha = 0$

Furthest Point: $\alpha = \infty$

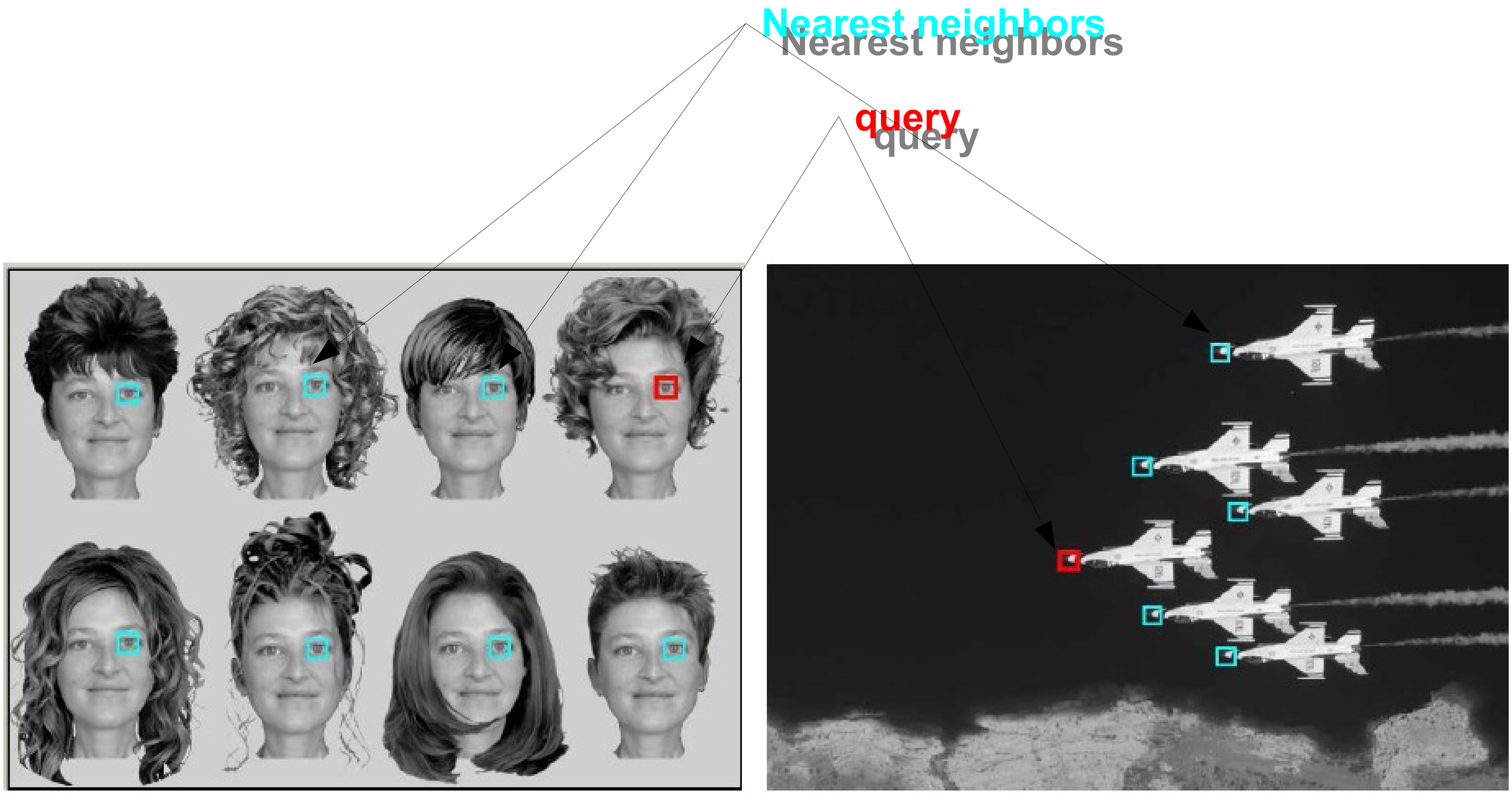
k-means++: $\alpha = 2$

1a. Choose an initial center c_1 uniformly at random from $\mathcal{X}$.

1b. Choose the next center c_i, selecting $c_i = x' \in \mathcal{X}$ with probability $\frac{D(x')^2}{\sum_{x \in \mathcal{X}} D(x)^2}$.

1c. Repeat Step 1b until we have chosen a total of k centers.

Theorem: k-means++ is $\Theta(\log k)$ approximate in expectation

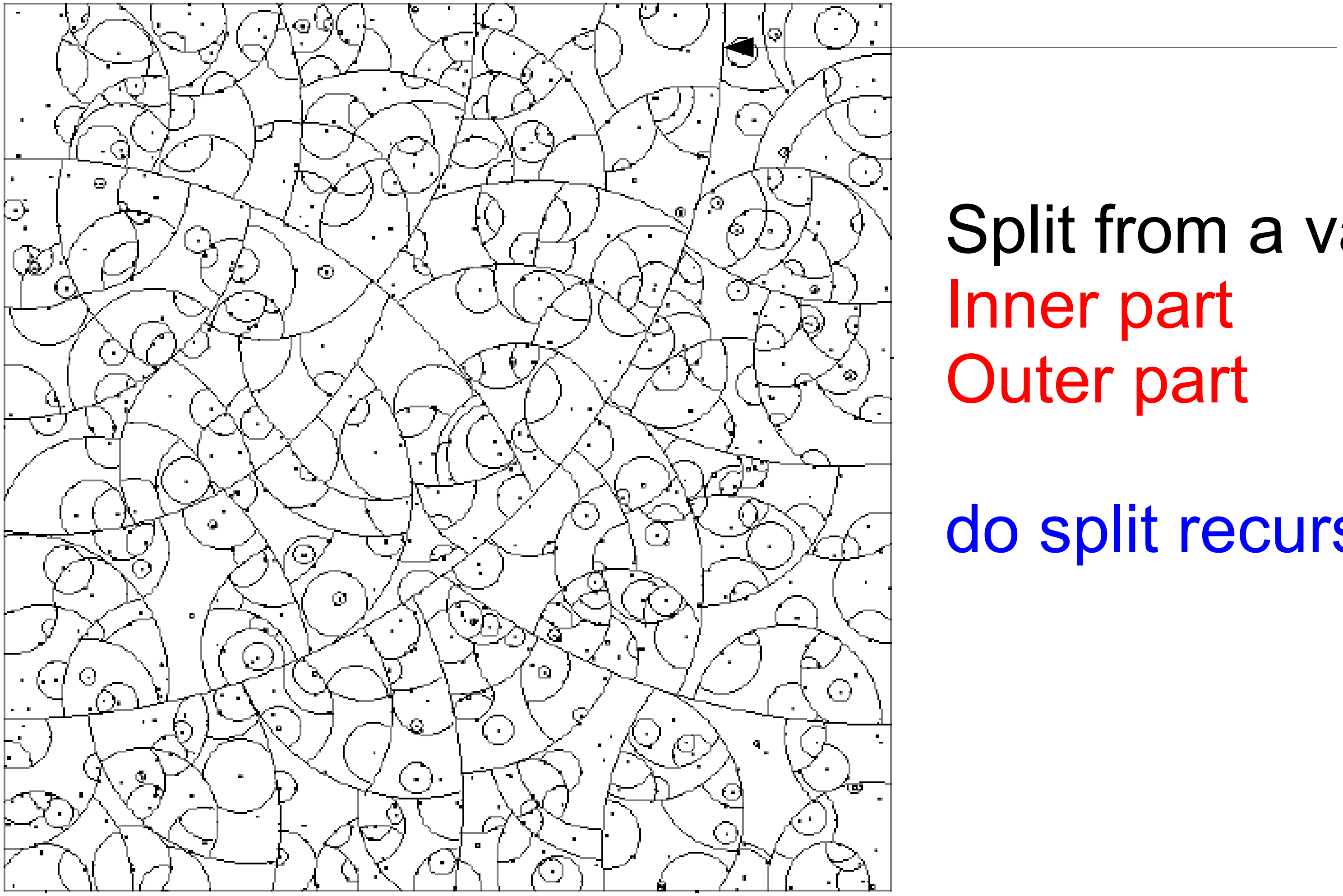


21x21 patches.
21x21 patches.

Vp-trees worked best (=fastest) for image patches.... [ECCV'08]

Vantage point trees (or vp-trees)

Partition the data according to a vantage point and a distance threshold
Relative distances are thus used.



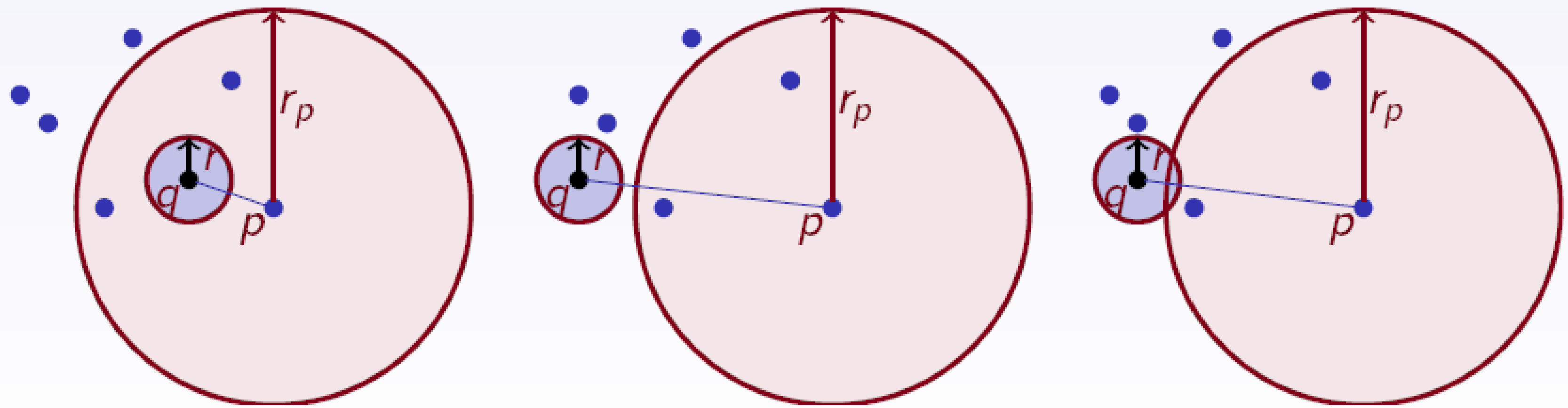
Split from a vantage point:
Inner part
Outer part

do split recursion

Vantage point trees: Pruning condition

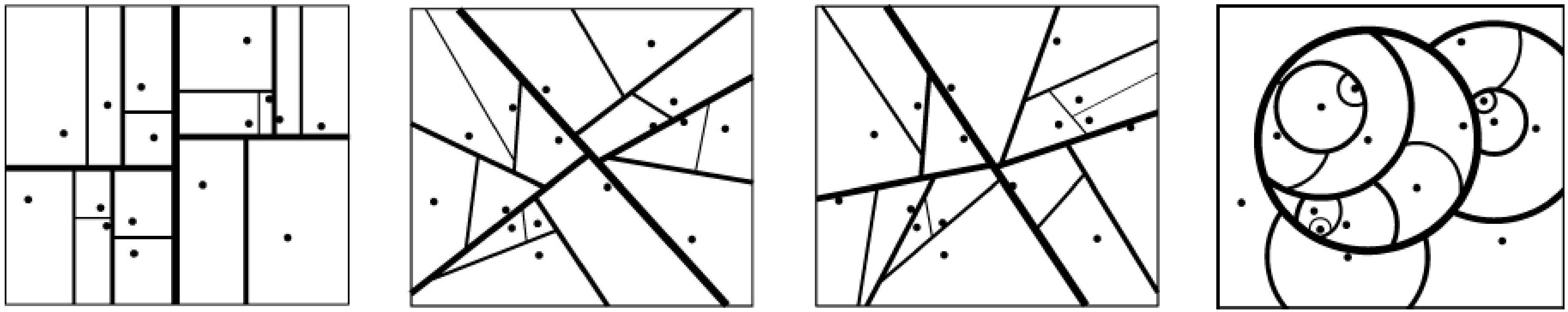
If $d(q,p) > r_p + r$ prune the inner branch
If $d(q,p) < r_p - r$ prune the outer branch

For $r_p - r \leq d(q,p) \leq r_p + r$ we have to inspect both branches

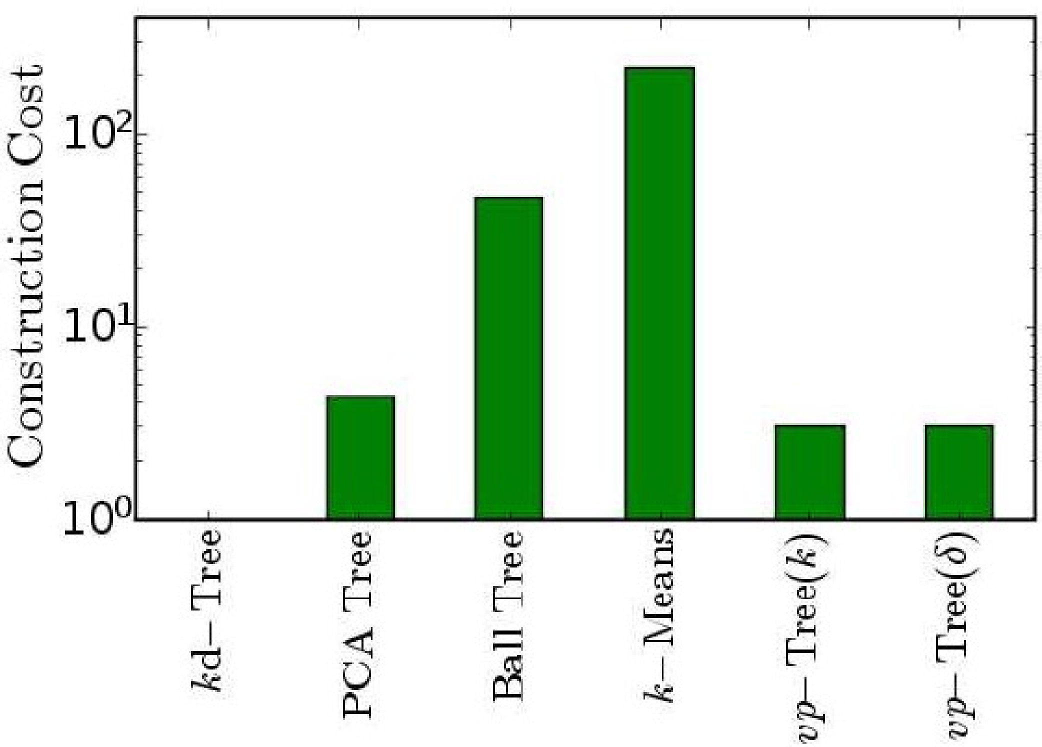


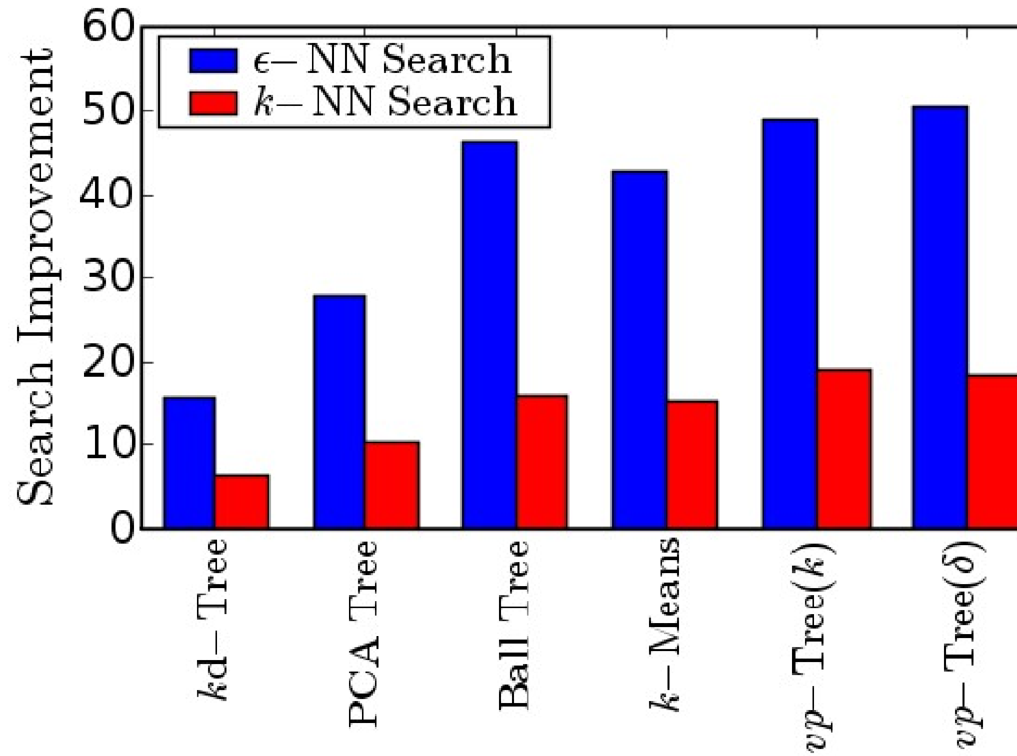
Prune outer **Prune inner** **Cannot prune**

Many ways to partition the point sets



(a) kd-Tree (b) PCA Tree (c) Ball Tree (d) vp-Tree

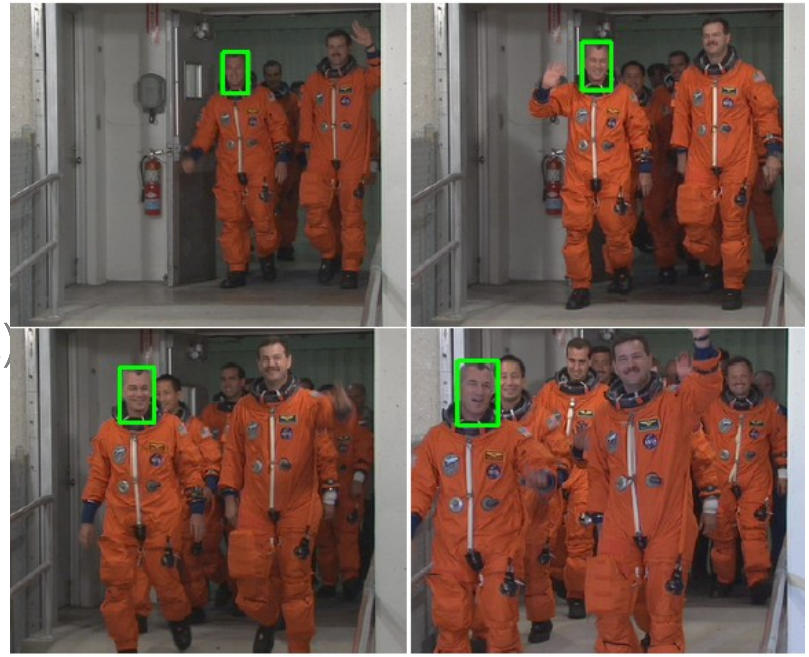


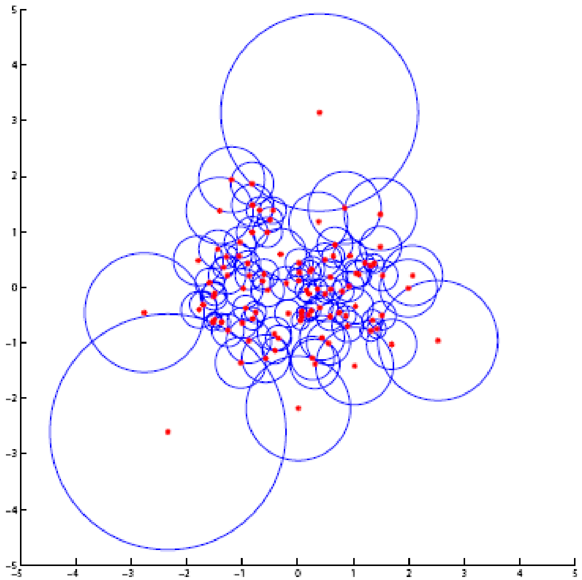


GPGPU: General Purpose GPU

GPU cores
GPU cores

Tracking
Tracking
ROIs
ROIs
(region of interest)
(region of interest)





Descripteur	Dimension	ANN-C++	BF-CUDA	Gain
C	3	1m 33s	53s	1.8
CP	5	2m 05s	1m 05s	1.9
CG	5	2m 35s	1m 07s	2.3
CGP	7	4m 27s	1m 19s	3.3
C_3	11	6m 40s	1m 17s	5.2
C_3P	13	5m 43s	1m 12s	4.8

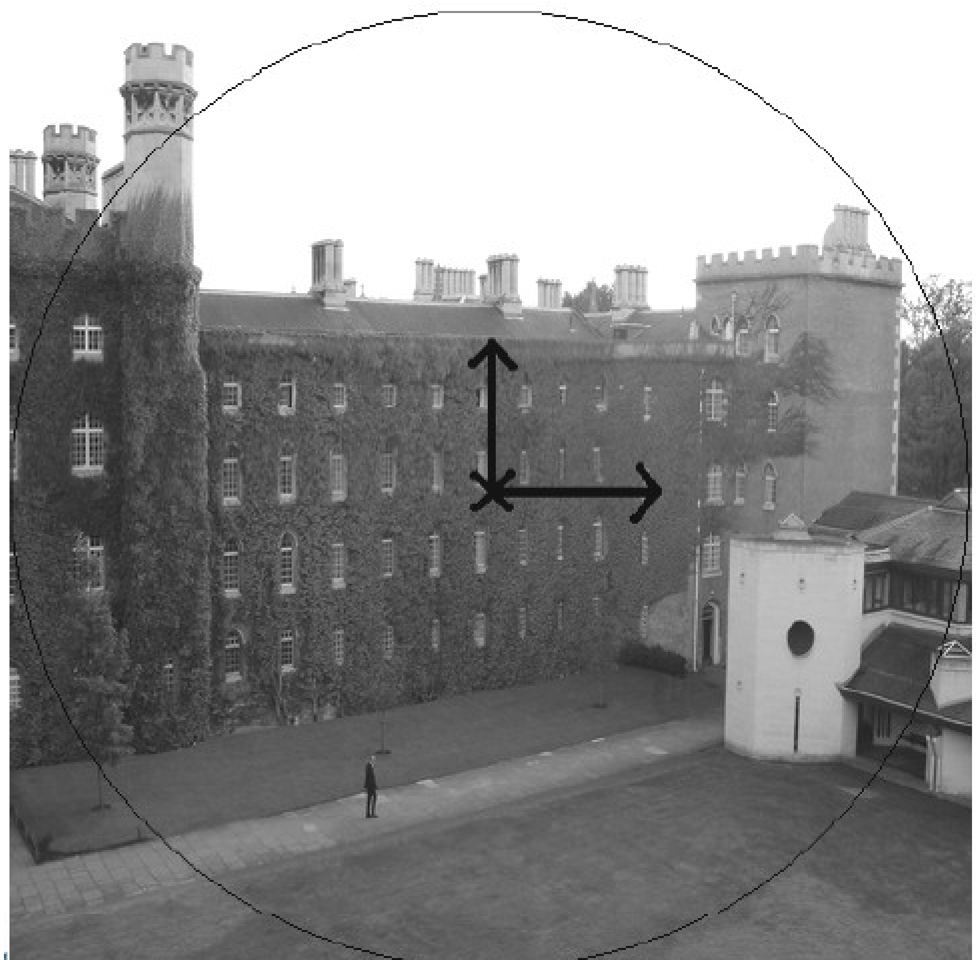
Distance au k=2 plus proche voisin

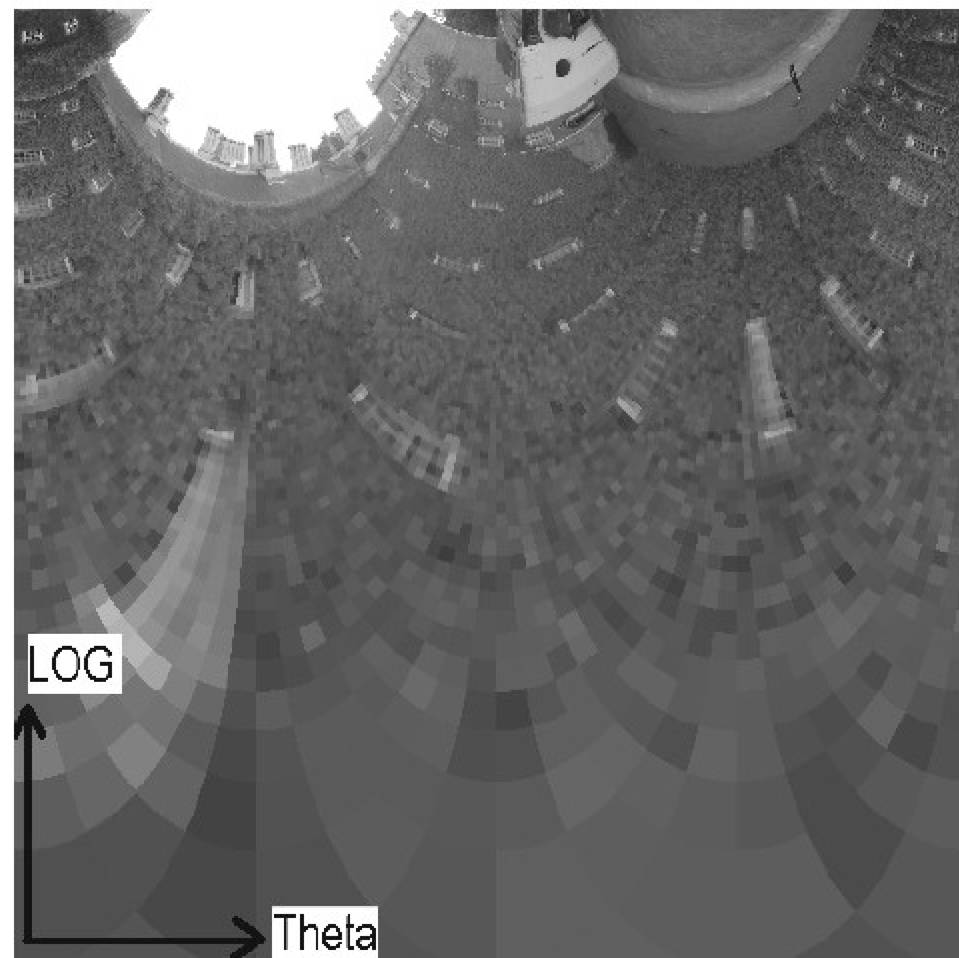
C: couleur (YUV)
C3: 3x3 neigh in Y
G: gradient
P: position
k=3

Log-Polar coordinates

$$\rho = \log \sqrt{x^2 + y^2},$$
$$\theta = \arctan \frac{y}{x}.$$

$$\rho = \log \sqrt{(x - x_o)^2 + (y - y_o)^2},$$
$$\theta = \arctan \frac{y - y_o}{x - x_o}.$$



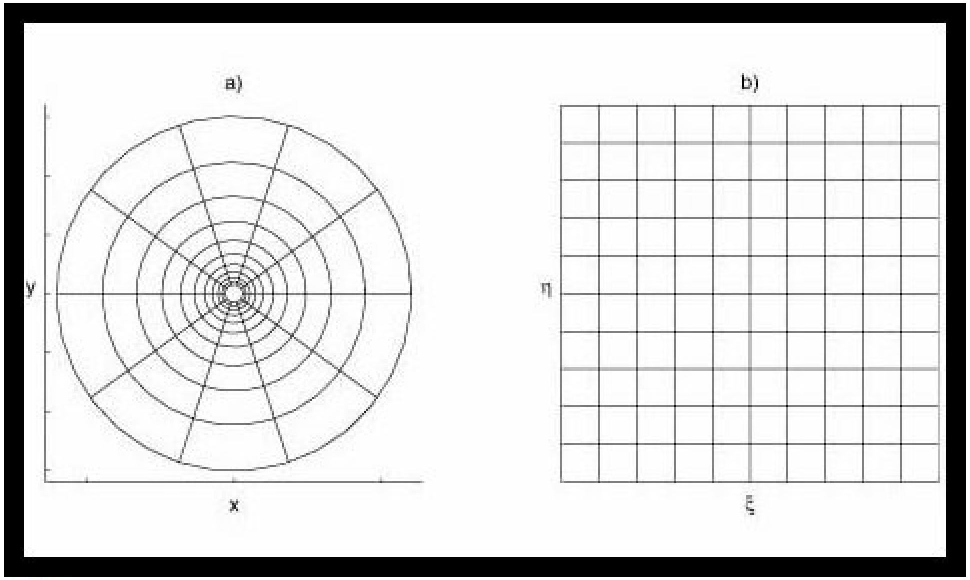


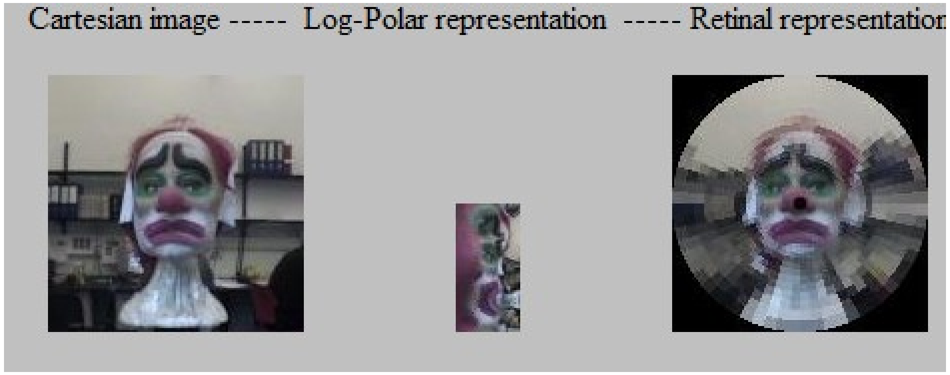
Log-Polar coordinates

Scales and rotations become translations

$$\mathbf{S}_s\mathbf{x} = (sx, sy) \longrightarrow (\log s + \rho(\mathbf{x}), \theta(\mathbf{x})),$$

$$\mathbf{R}_\phi\mathbf{x} = (x\cos\phi + y\sin\phi, y\cos\phi - x\sin\phi) \longrightarrow (\rho(\mathbf{x}), \phi + \theta(\mathbf{x})).$$



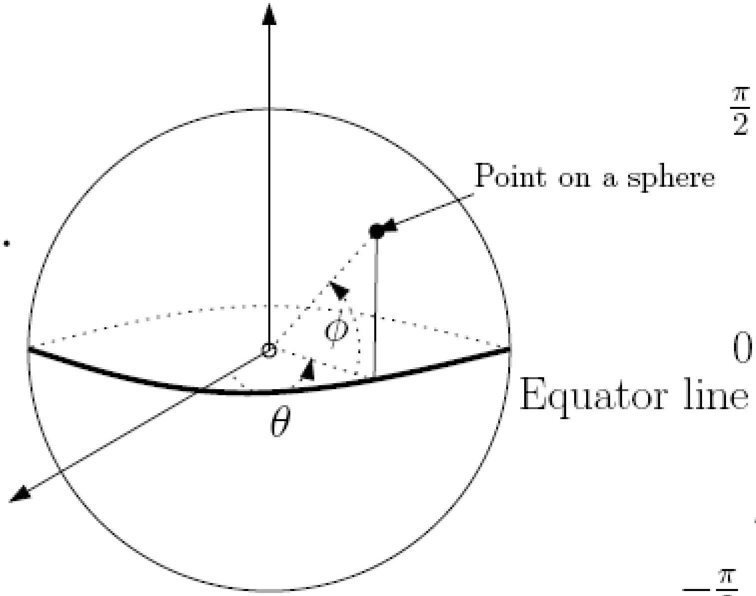


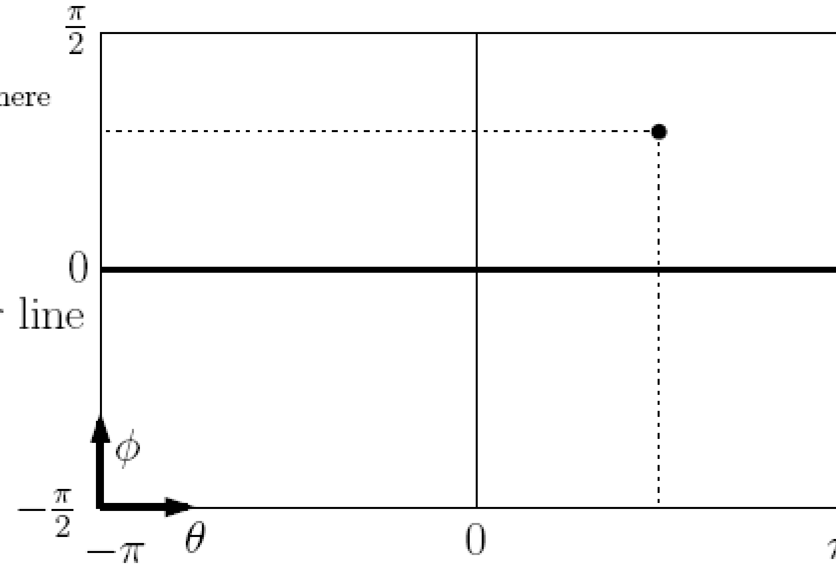
Data reduction for retinal images...

Spherical coordinates

$$\theta = \arctan\frac{x}{z} \quad \text{and} \quad \phi = \arctan\frac{y}{\sqrt{x^2+z^2}}.$$

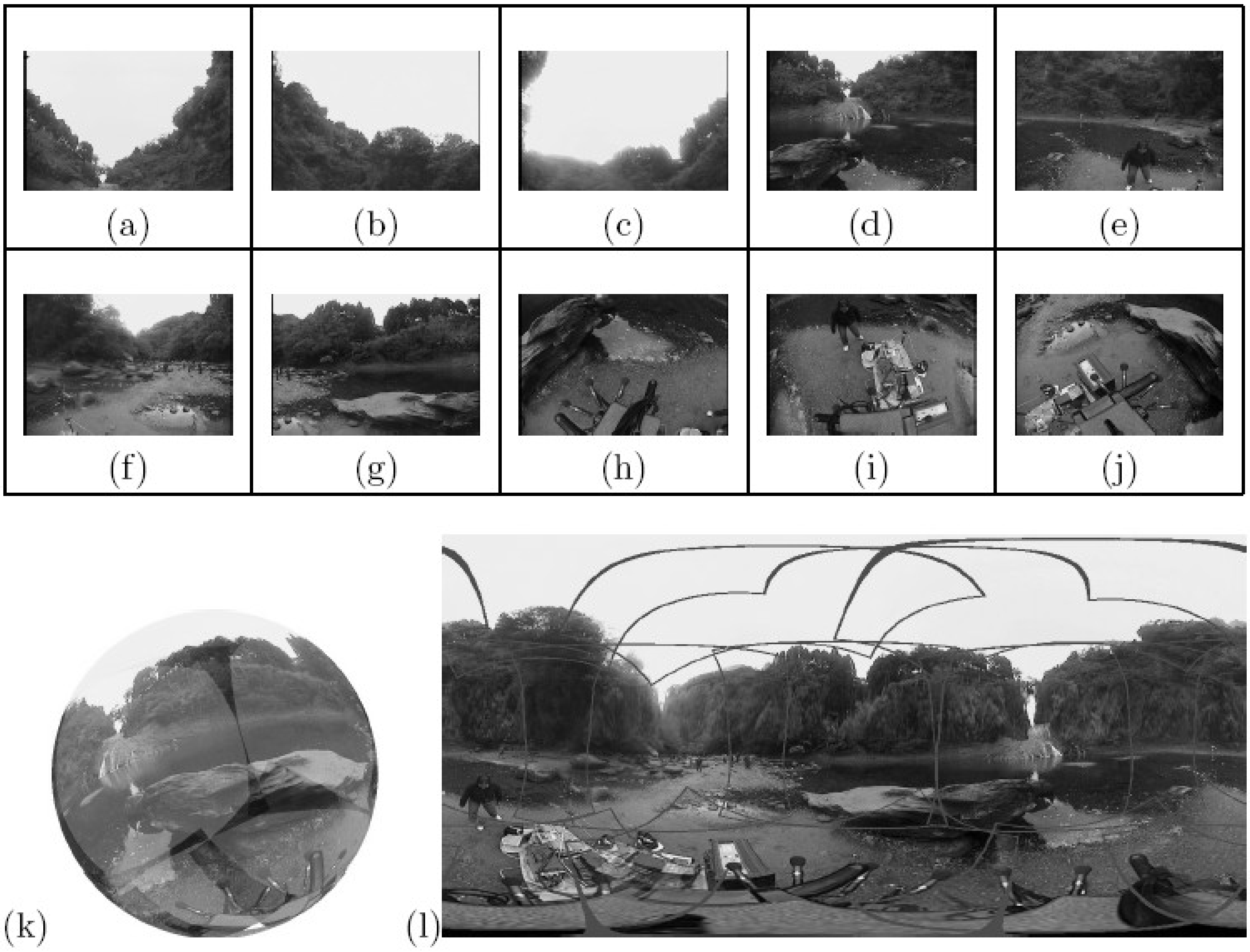
$$\mathbf{r} = \begin{bmatrix} \cos\phi\sin\theta \\ \sin\phi \\ \cos\phi\cos\theta \end{bmatrix}.$$



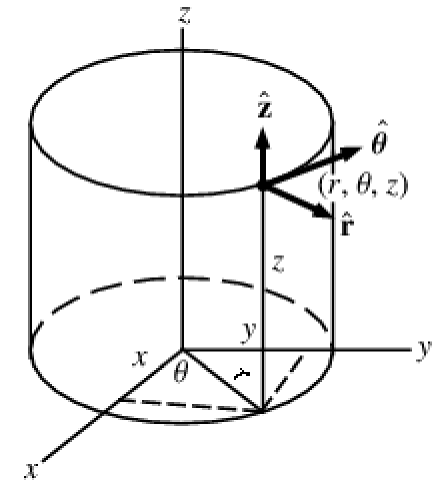


Spherical coordinates

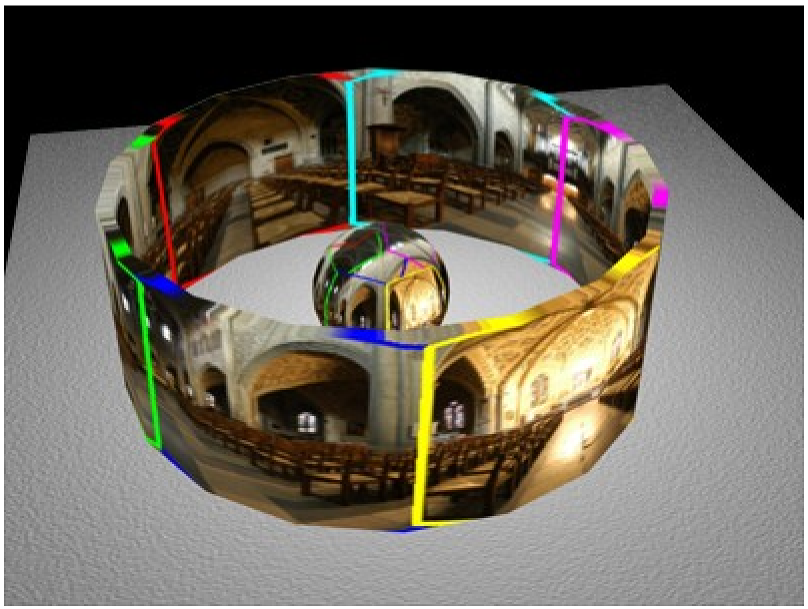
High Resolution Full Spherical Videos
Frank Nielsen, ITCC'02

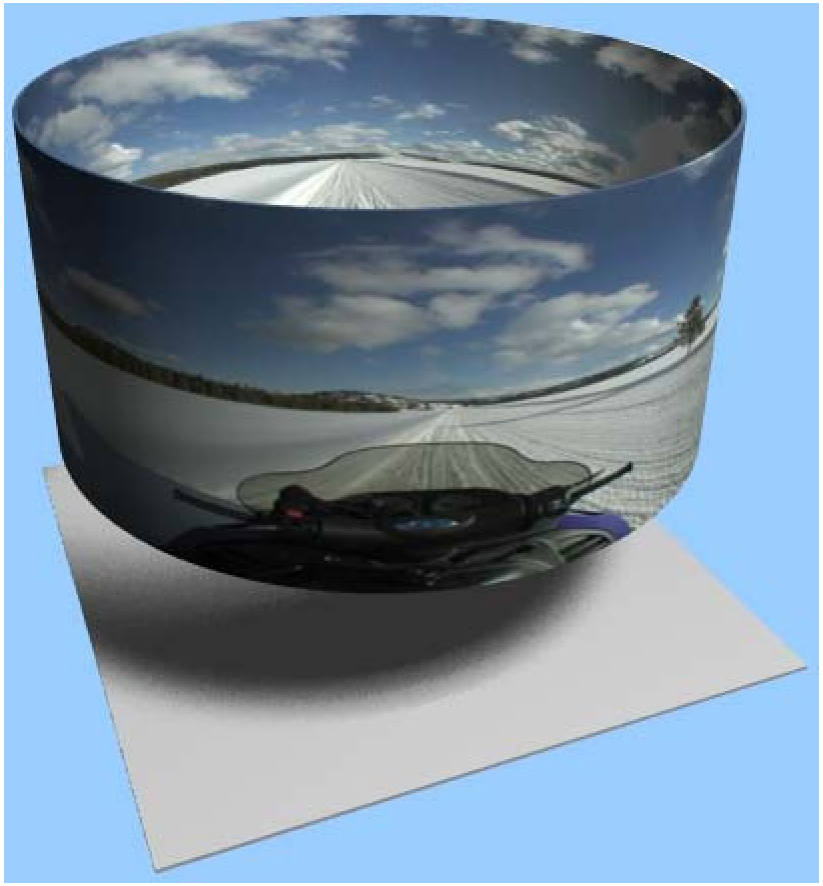


Cylindrical coordinates



$$\theta = \arctan \frac{x}{z}, \qquad s = \frac{y}{\sqrt{x^2 + z^2}}.$$

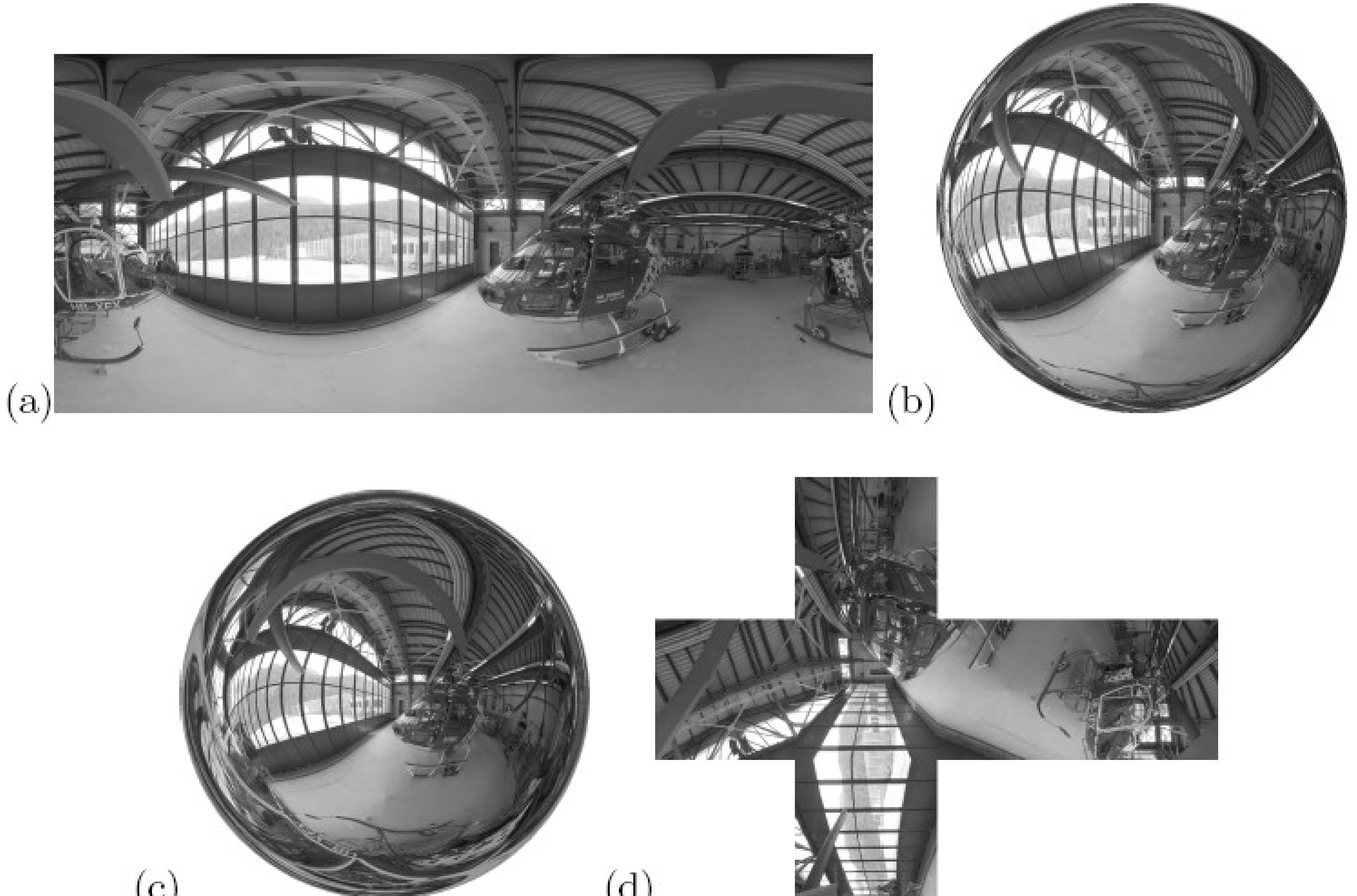




Panoramic image stitching:

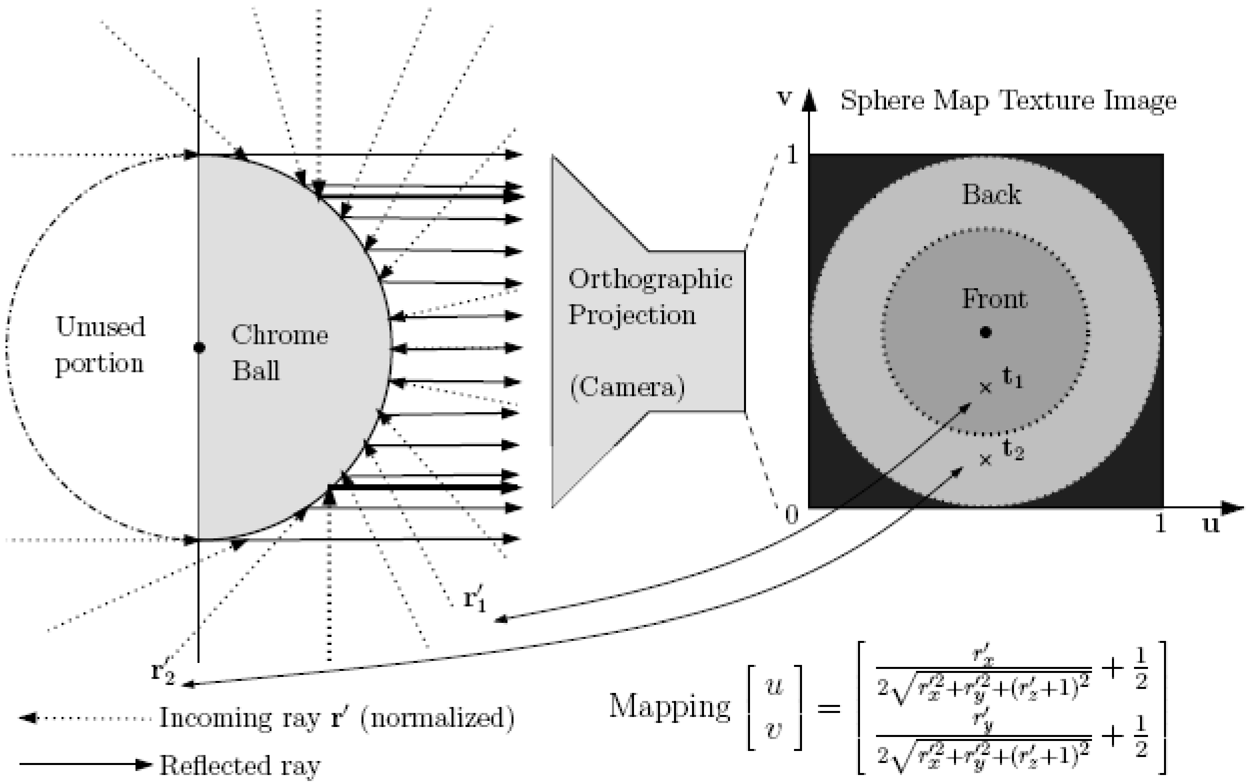
Align by a translation into the cylindrical coordinate map...

Environment maps



Equirectangular, mirror ball, cubic, etc.

Environment maps: Mirror ball

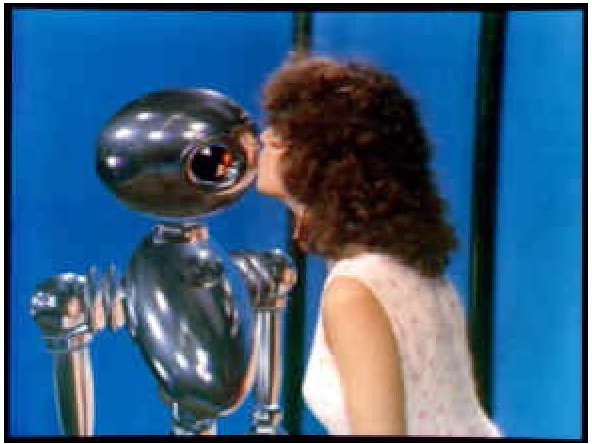


$$\text{Mapping} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{r'_x}{2\sqrt{r'^2_x + r'^2_y + (r'_z+1)^2}} + \frac{1}{2} \\ \frac{r'_y}{2\sqrt{r'^2_x + r'^2_y + (r'_z+1)^2}} + \frac{1}{2} \end{bmatrix}$$

Environment maps for reflections

Blinn, 1976

1982



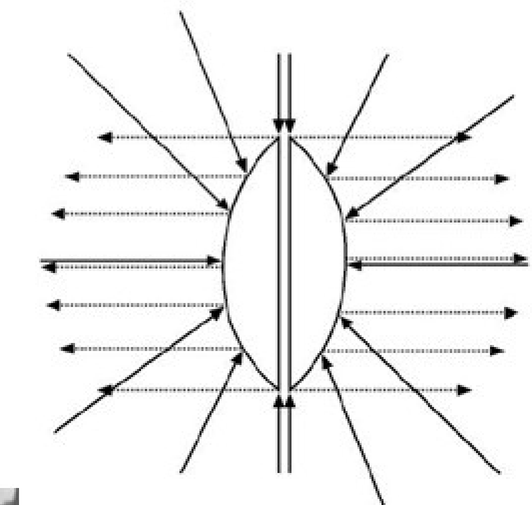
Interface, 1985
Lance Williams

http://www.debevec.org/ReflectionMapping/

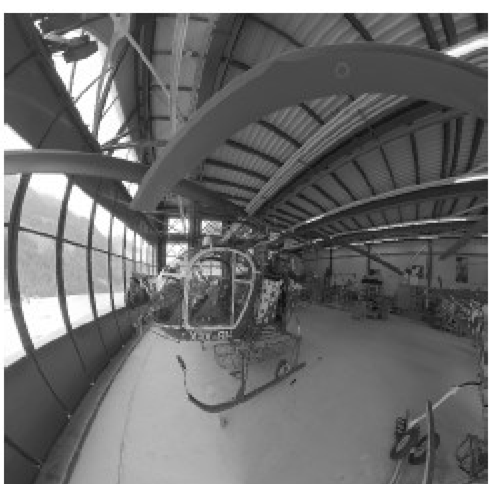
Environment maps for reflections

Abyss, Terminator 2 (1991)

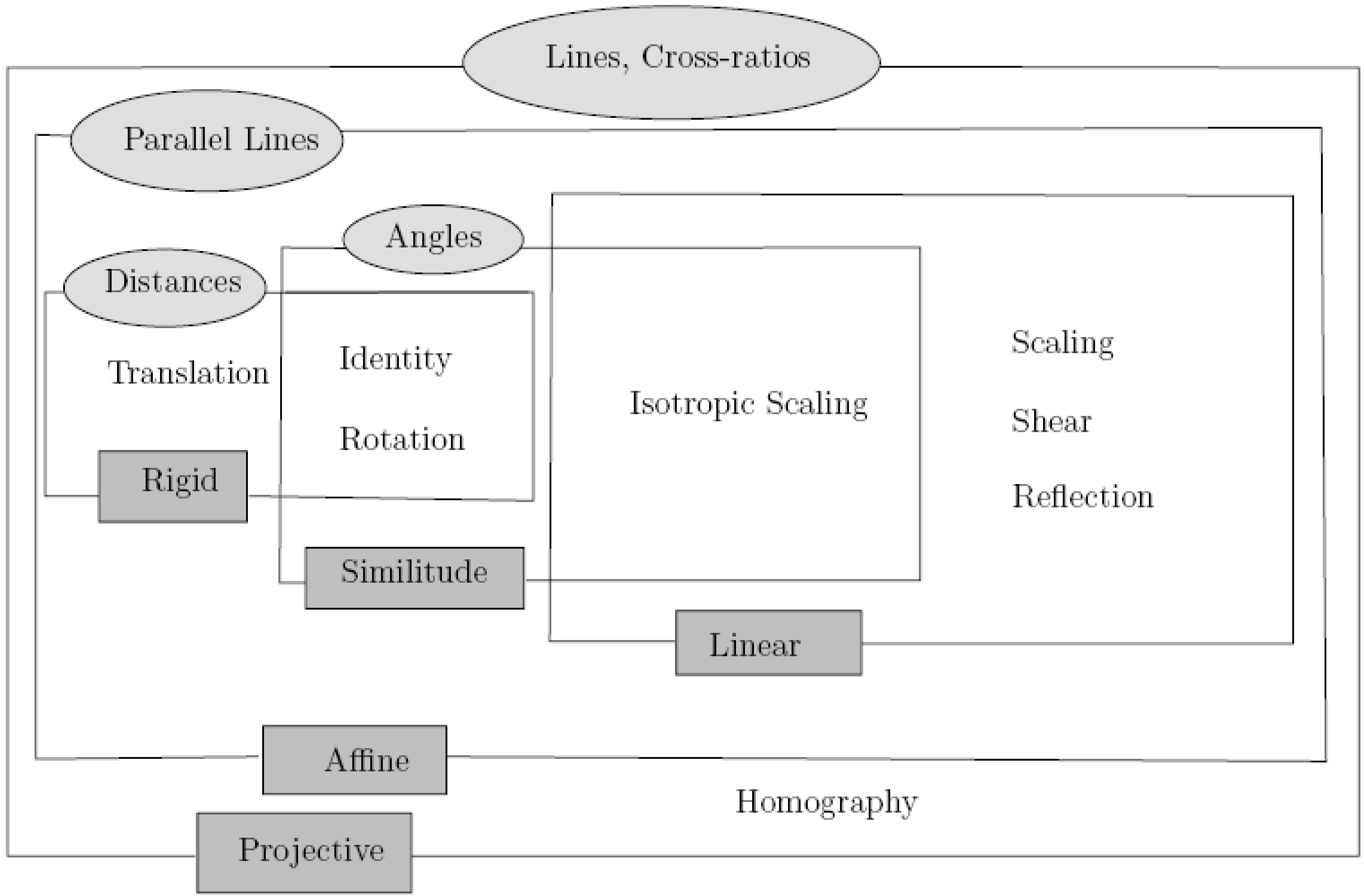
Best environment map for real-time graphics?



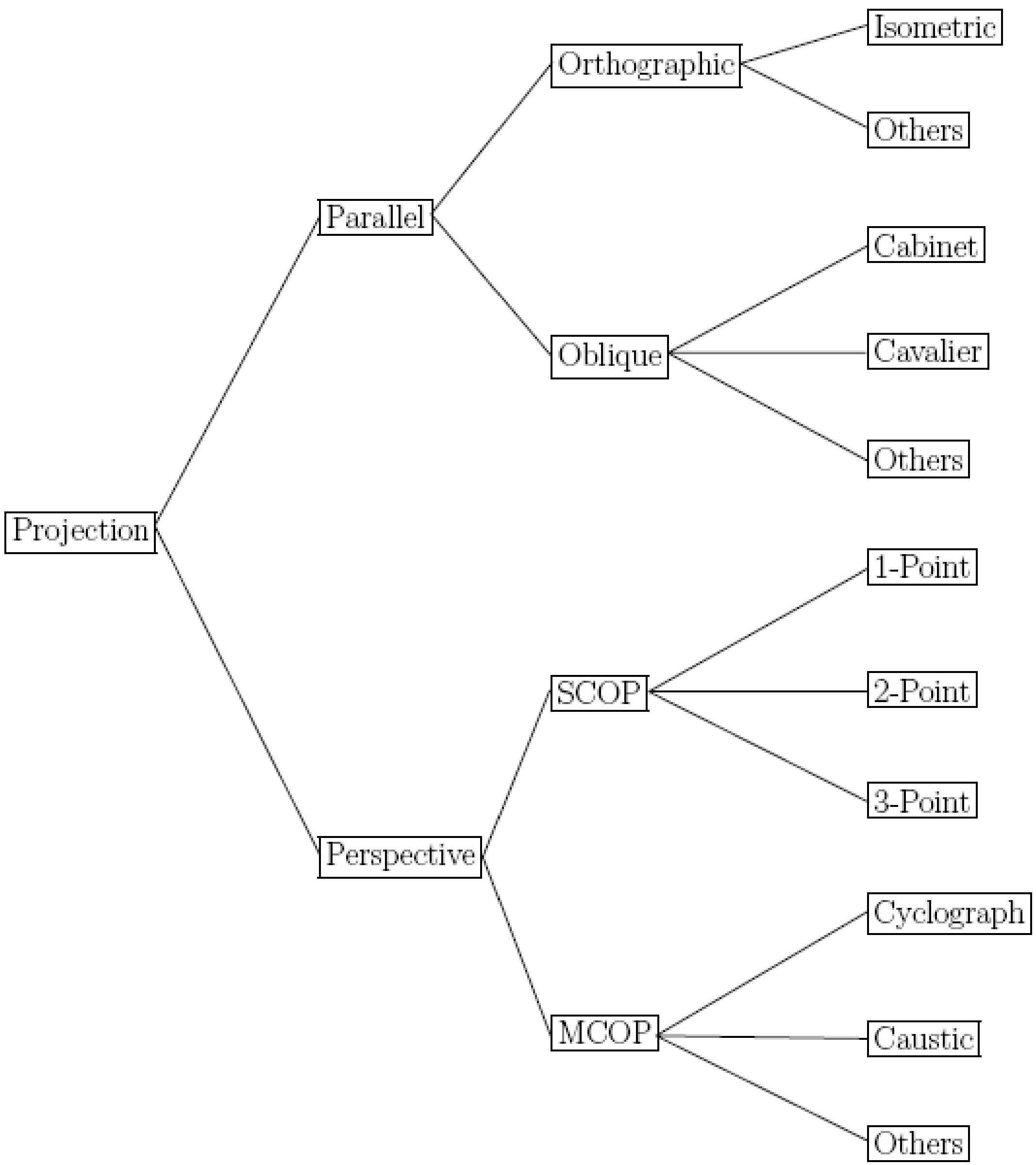
Dual paraboloid (2 images front/back only)



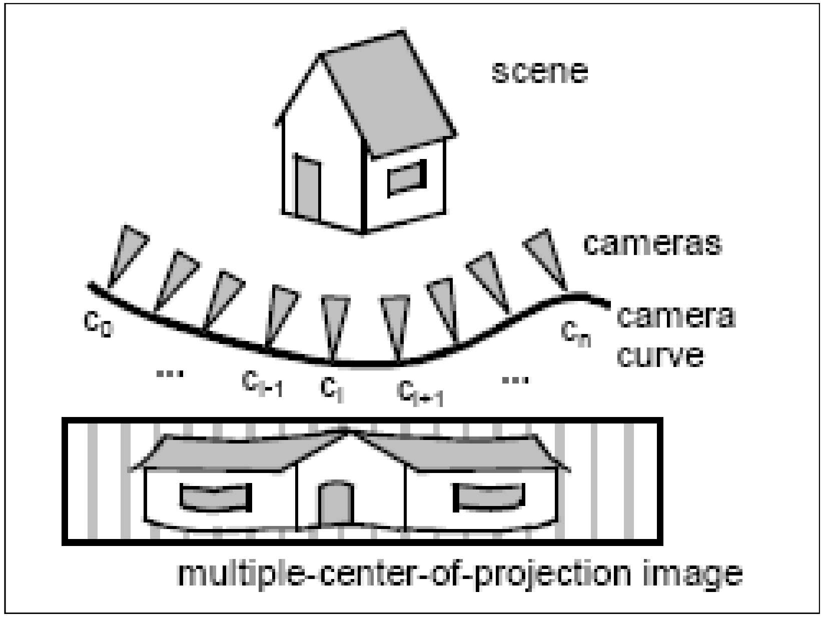
Transformations and their invariants

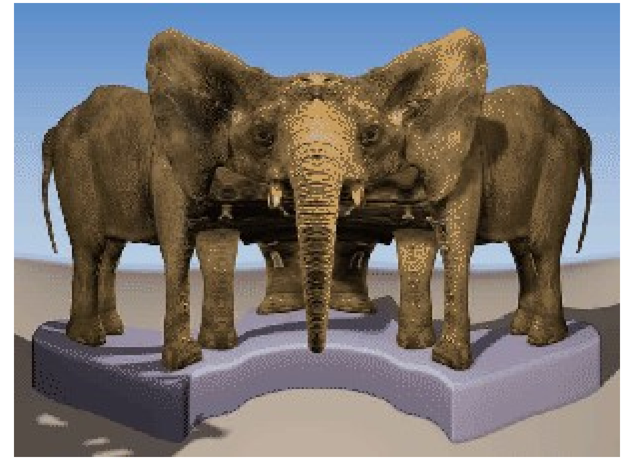


Taxonomy of projections:

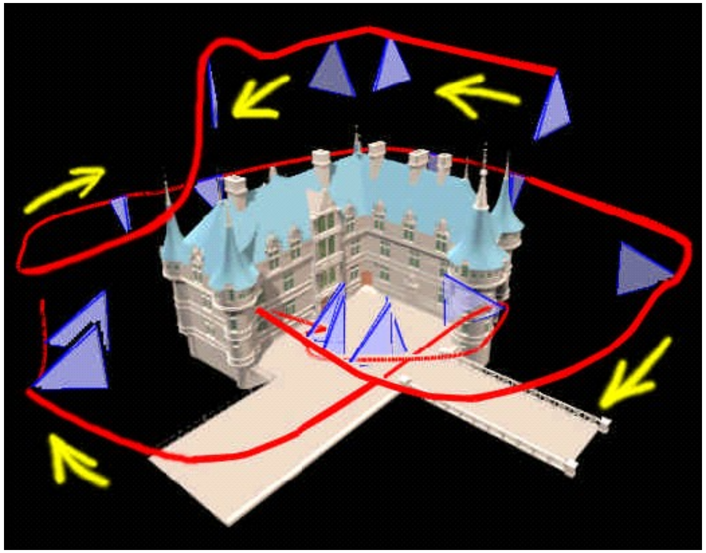


Multiple centers of projections (MCOP)

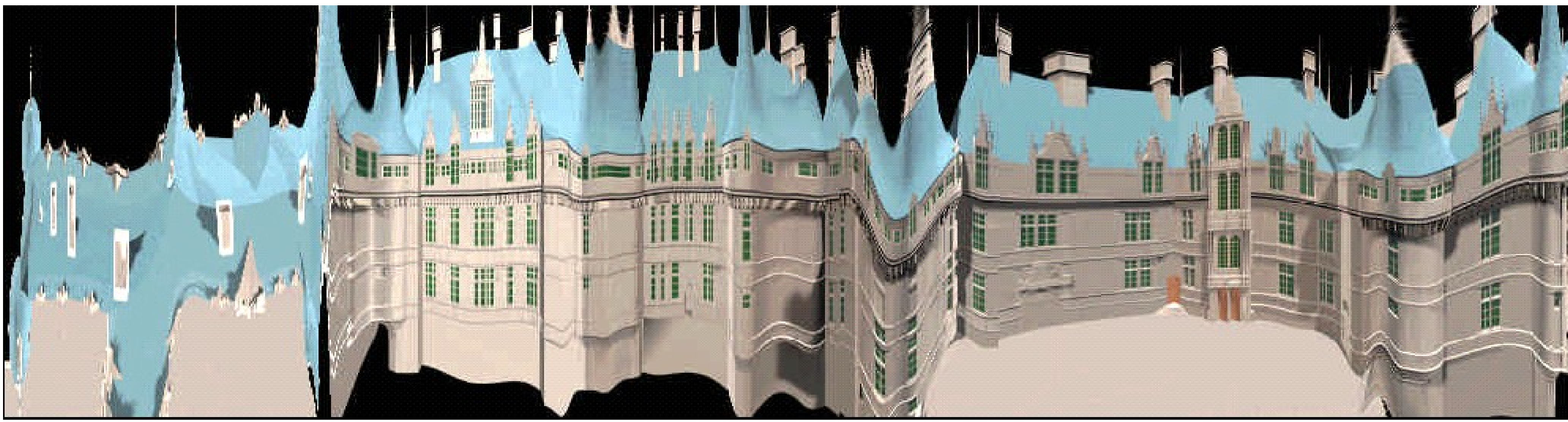


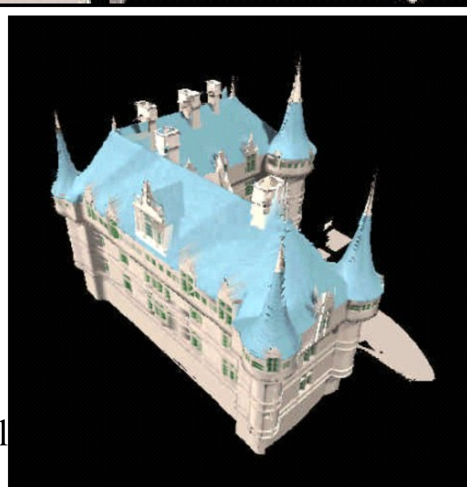


Reconstruction from a single MCOP images
Generalizes epipolar geometry
Resolution dependent

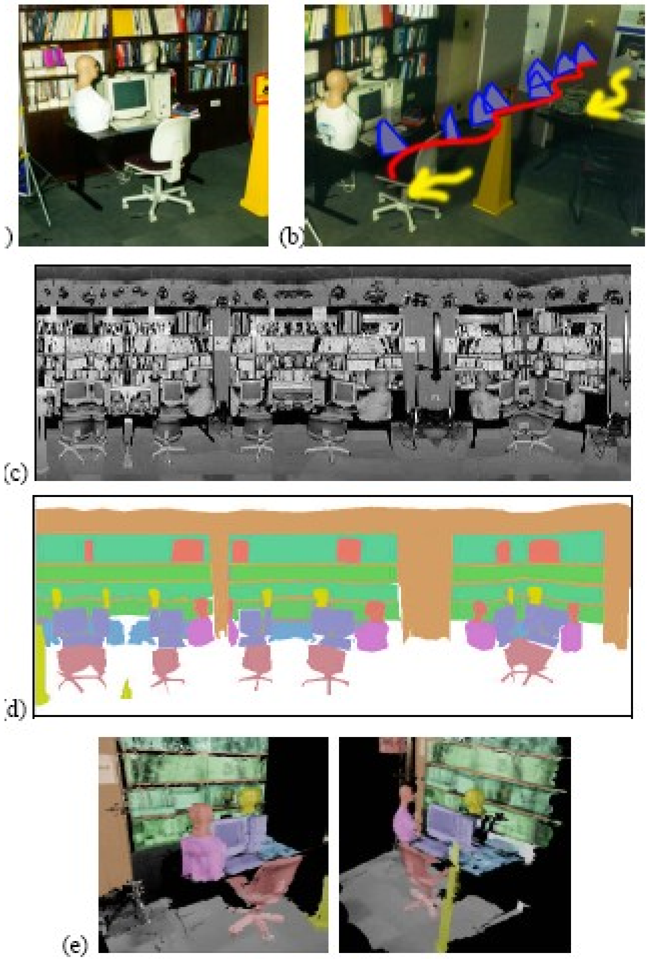


Acquisition Example



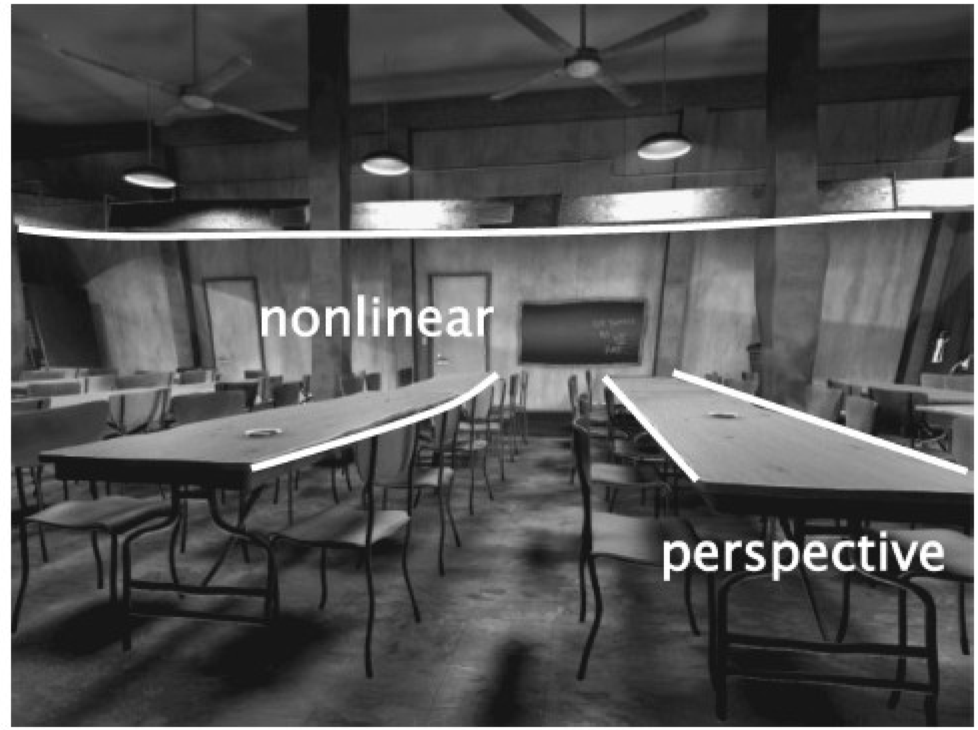


Multiple centers of projections (MCOP)



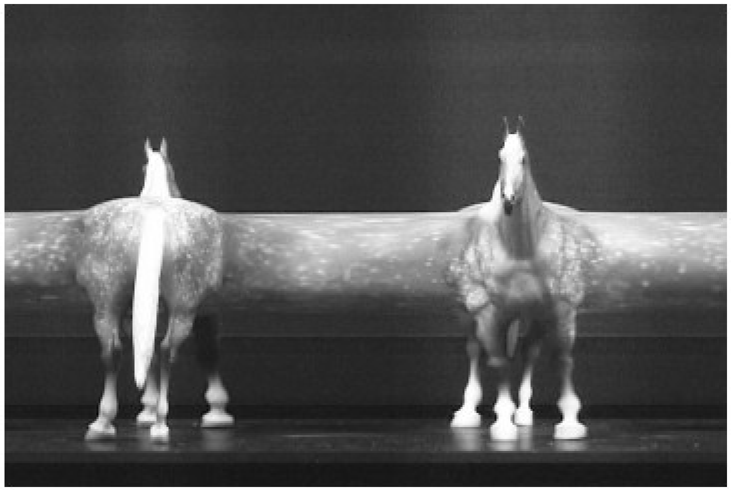
Difficult to obtain in practice...
Localization is difficult

Multiple centers of projections (MCOP)

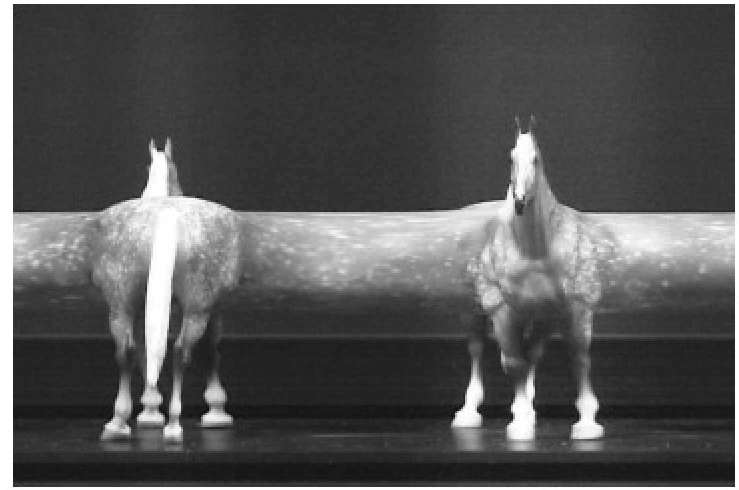


The art of depiction...

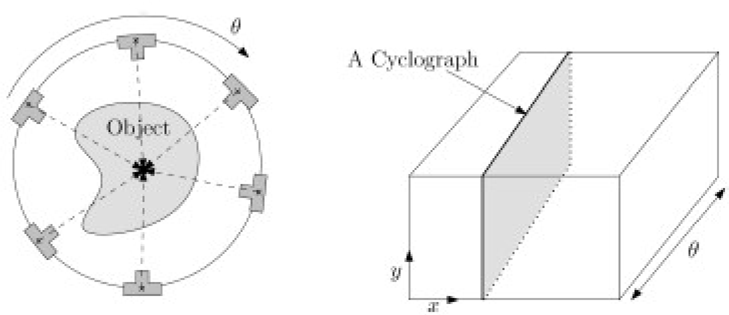
Stereo cyclographs...



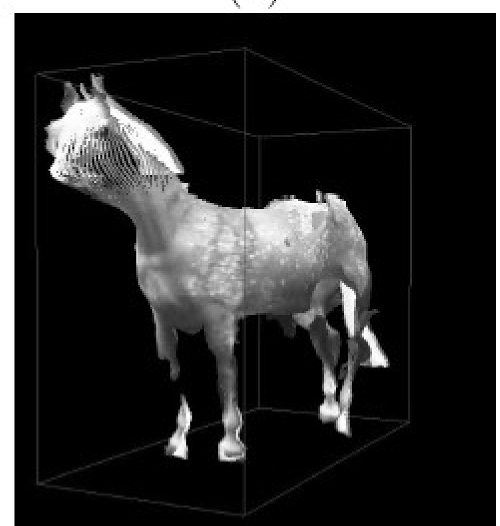
(a)



(b)

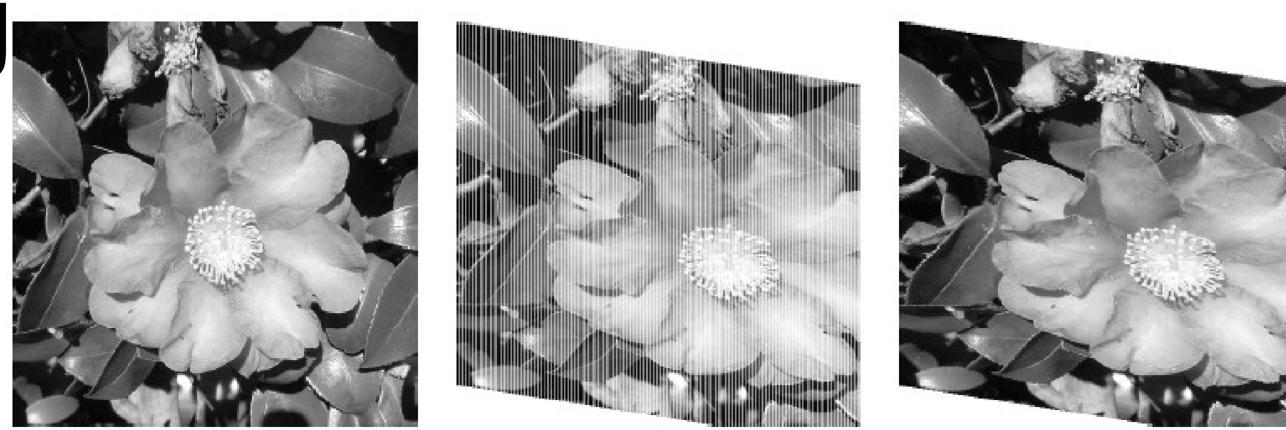


(c)



(d)

Image backward vs forward mapping
Image warping



ForwardMapping($\mathbf{I}_s, f$)

◁ Create a warped image $\mathbf{I}_d$ by forward mapping ▷
◁ f: warping function ▷
Initialize an empty image $\mathbf{I}_d$
◁ for all image lines ▷
for $y \leftarrow 1$ **to** h_s
  do ◁ for all column pixels ▷
    for $x \leftarrow 1$ **to** w_s
      do ◁ Compute the source-to-destination mapping ▷
        $(u, v) \leftarrow f(x, y)$
        ◁ Round coordinates to integers ▷
        ◁ (no interpolation required) ▷
        $(u_r, v_r) \leftarrow (\lfloor u \rceil, \lfloor v \rceil)$
        ◁ Should check index bounds ▷
        $\mathbf{I}_d[u_r, v_r] = \mathbf{I}_s[x, y]$

BackwardMapping($\mathbf{I}_s, f$)

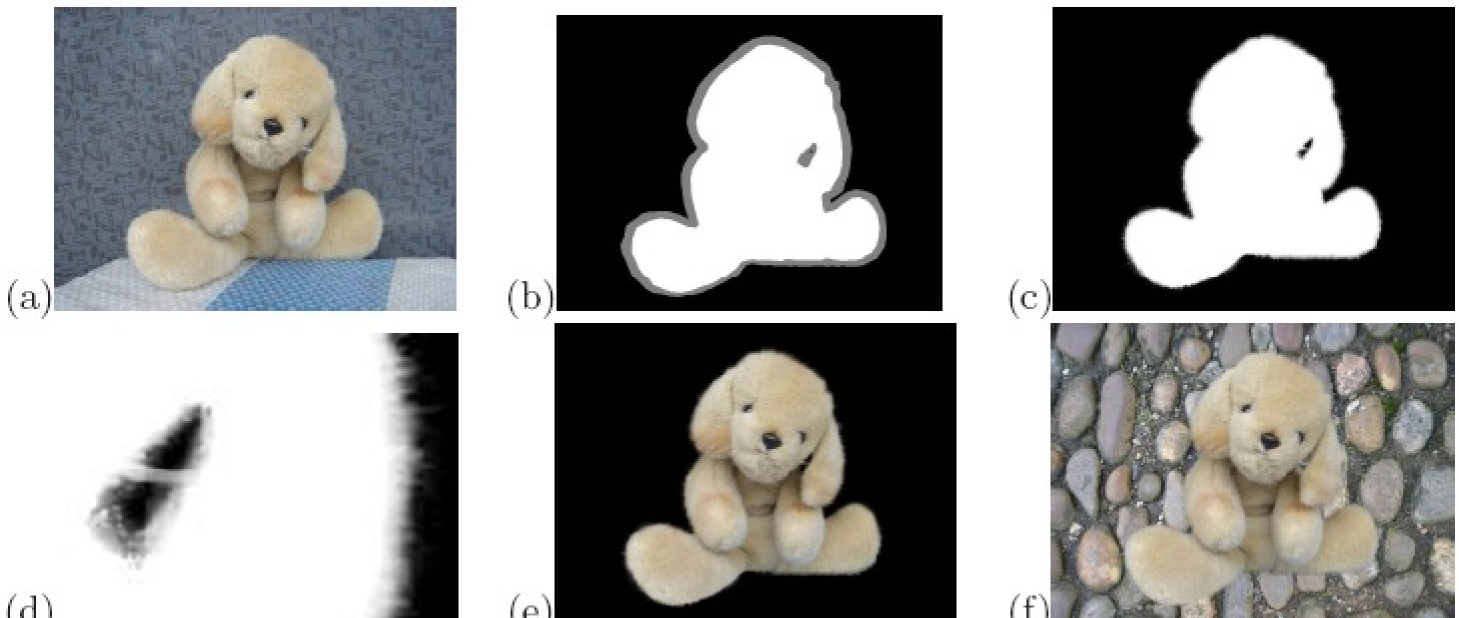
◁ Destination image $\mathbf{I}_d$ of dimension $w_d \times h_d$ ▷
for $v \leftarrow 1$ **to** h_d
  do for $u \leftarrow 1$ **to** w_d
    do $(x, y) = g(u, v)$
      ◁ Backward mapping requires resampling ▷
      $\mathbf{I}_d[u, v] = $ Resample($\mathbf{I}_s, x, y$)

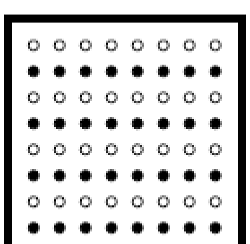
Resampling
Interpolation

Image Blending: Alpha channel

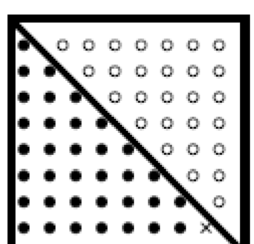
$$\mathbf{I}[i,j] = \boldsymbol{\alpha}[i,j]\mathbf{F}[i,j] + (1-\boldsymbol{\alpha}[i,j])\mathbf{B}[i,j],$$

$$\mathbf{I} = \alpha\mathbf{F} + (1-\alpha)\mathbf{B},$$

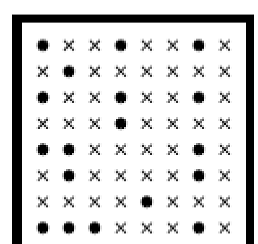




Full semitransparent coverage ($\alpha = \frac{1}{2}$)



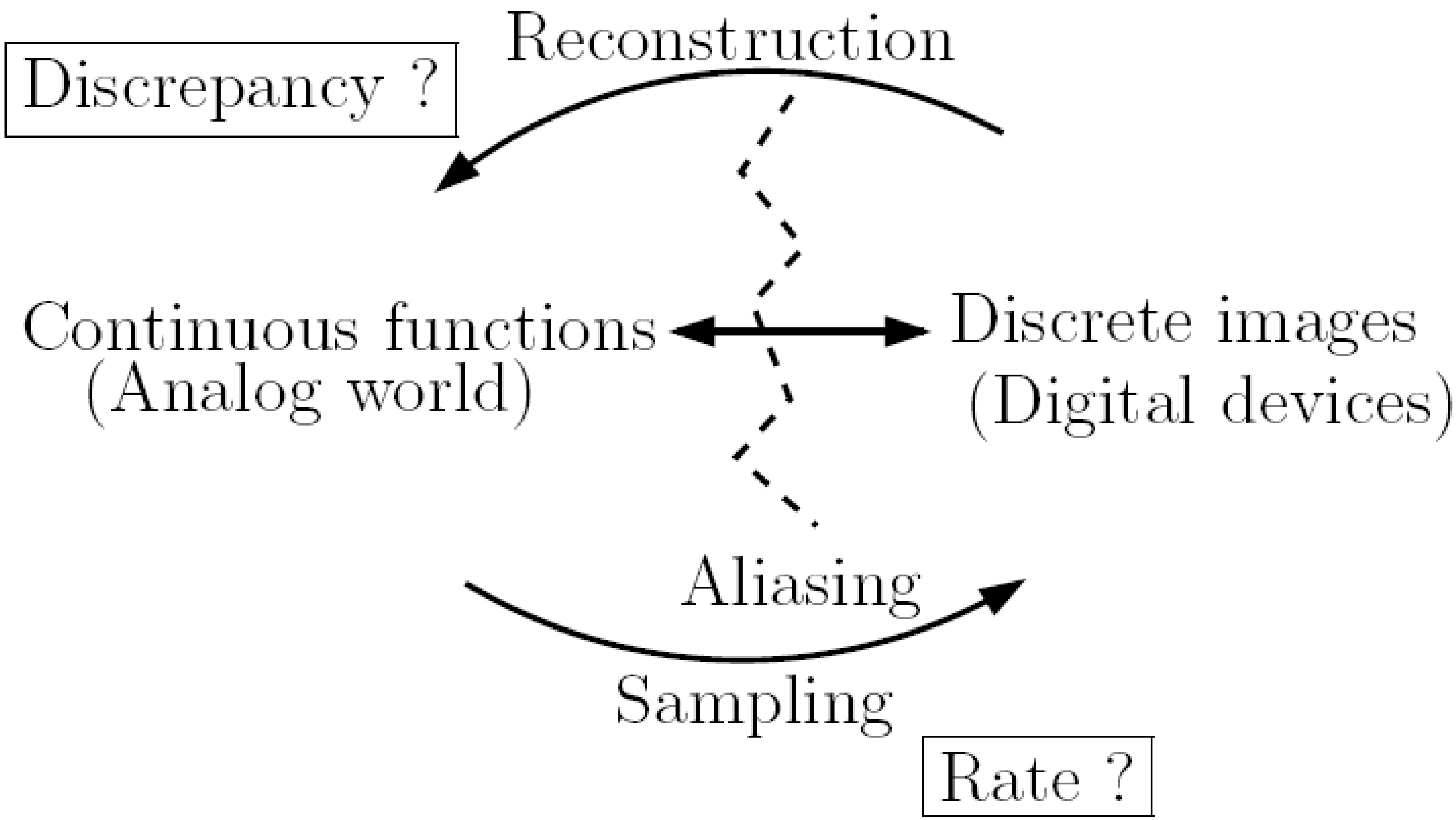
Partial opaque coverage ($\alpha = \frac{1}{2}$)



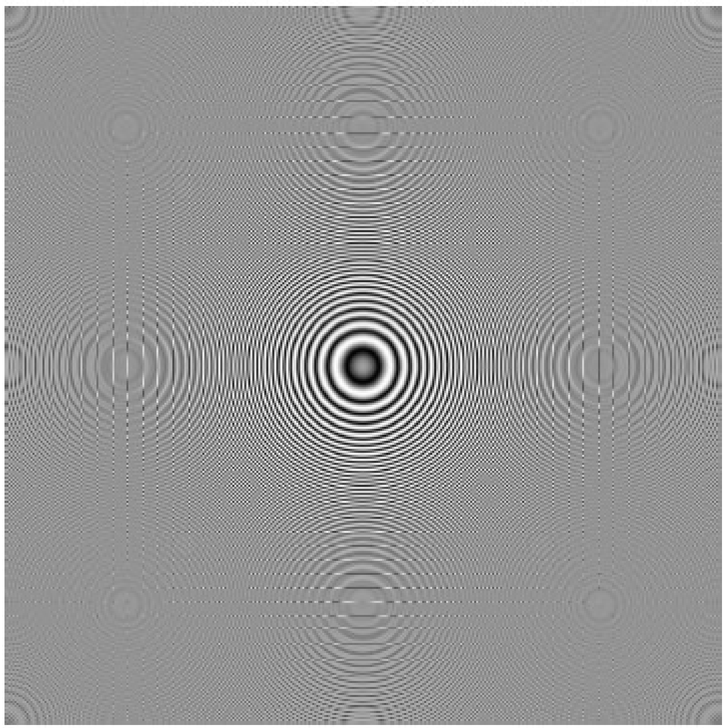
Blending two colors • ×

Interpretation at the microscopic level...

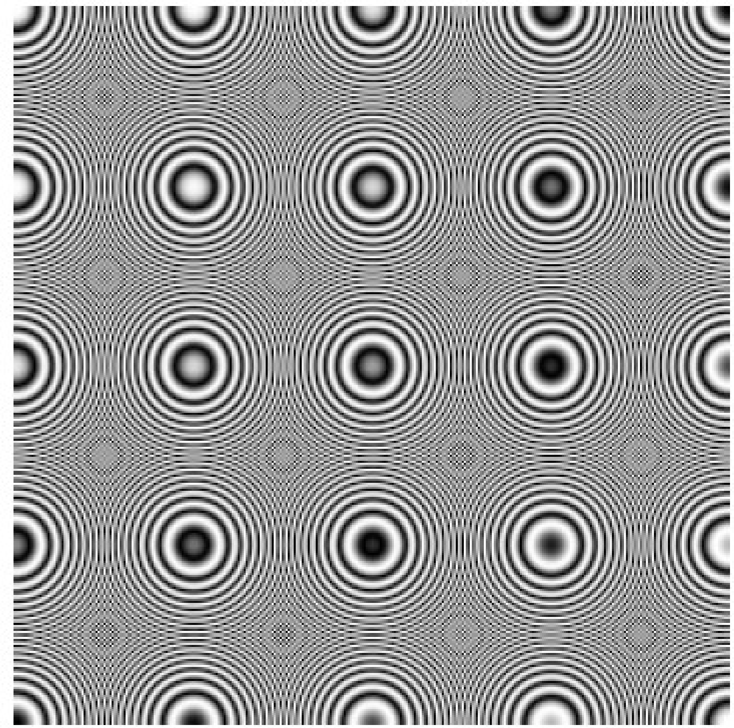
Image sampling/reconstruction



Zone plate: Aliasing/ringing effect



1024x1024



256x256
downsampled
using bilinear interpolation

Continuous versus discrete convolutions

$$(f \otimes g)(x) = \int_{t=-\infty}^{\infty} f(t)g(x-t)dt = \int_{t=-\infty}^{\infty} g(t)f(x-t)dt = (g \otimes f)(x).$$

$$\mathbf{C}[i,j] = \mathbf{A} \otimes \mathbf{B} = \sum_k \sum_l \mathbf{A}[k,l]\mathbf{B}[i-k, j-l].$$

$$\mathbf{G} = \frac{1}{273} \begin{bmatrix} 1 & 4 & 7 & 4 & 1 \\ 4 & 16 & 26 & 16 & 4 \\ 7 & 26 & 41 & 26 & 7 \\ 4 & 16 & 26 & 16 & 4 \\ 1 & 4 & 7 & 4 & 1 \end{bmatrix}$$

Fourier analysis

$$f(x+T)=f(x)$$

Fourier discovered that ***all periodic signals*** can be represented as a sum (eventually infinite) of sinusoidal waves: sin(·) functions, the basis functions.

Let *f*(·) denote the continuous function in the spatial domain and *F*(·) denote the dual complex function, also called ***spectral function***.

Euler formula (period 2pi)

$$\exp(ix)=\cos x+i\sin x$$

$$\exp(ix)=\cos x+i\sin x=\cos(x+2\pi)+i\sin(x+2\pi)=\exp(i(x+2\pi))$$

Fourier analysis: Duality spatial/spectral domain

Spatial domain

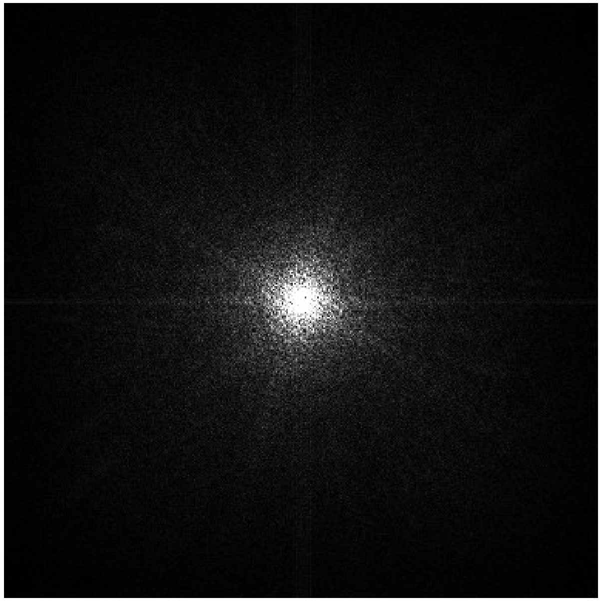
$$f(x, y) = \int_{u=-\infty}^{\infty} \int_{v=-\infty}^{\infty} F(u, v) \exp\left(i 2\pi (ux + vy)\right) du dv.$$

Spectral domain

$$F(u, v) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} f(x, y) \exp\left(-i 2\pi (ux + vy)\right) dx dy.$$

$$F(u, v) = A(u, v) + iB(u, v)$$

Fourier analysis: Phase/amplitude



Frequency magnitudes

$$F(u,v) = A(u,v) + iB(u,v)$$

Polar coordinates:

$$F(u,v) = |F(u,v)| \exp(i\phi(u,v))$$

$$P(u,v) = A^2(u,v) + B^2(u,v)$$

Power spectrum

$$|F(u,v)|$$

Amplitude

$$\phi(u,v) = \arctan \frac{B(u,v)}{A(u,v)}$$

Phase

Fourier analysis: Convolution theorem

Convolution in spatial domain is a multiplication in Fourier domain

$$\mathcal{F}(f \otimes g) = \sqrt{2\pi}(\mathcal{F}f) \times (\mathcal{F}g) = \sqrt{2\pi} F \times G$$

Convolution in frequency domain is a multiplication in spatial domain

$$F \otimes G = \sqrt{2\pi}\mathcal{F}(f \times g)$$

Fourier analysis: Discrete transformations

$$f_j = \frac{1}{n}\sum_{k=0}^{n-1} x_k \exp(-2\pi i \frac{jk}{n}),\ \forall\, 0 \leq j \leq n-1$$

$$x_k = \sum_{j=0}^{n-1} f_j \exp(2\pi i \frac{jk}{n})$$

1D

$$F(u,v) = \frac{1}{wh}\sum_{x=0}^{w-1}\sum_{y=0}^{h-1} f(x,y) \exp\left(-2\pi i \left(\frac{xu}{w} + \frac{yv}{h}\right)\right)$$

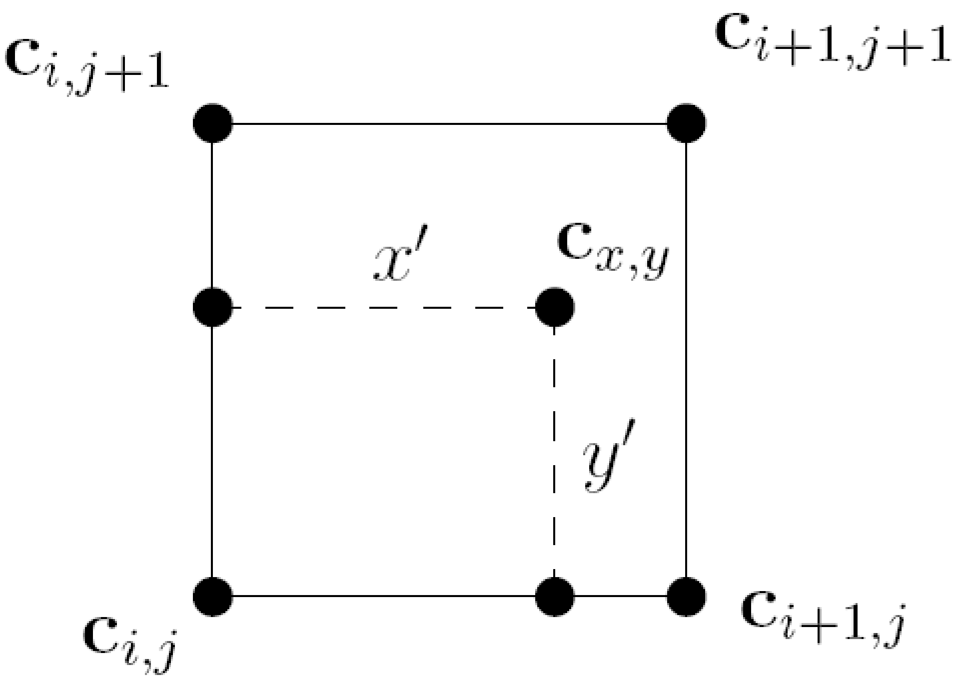
$$f(x,y) = \sum_{u=0}^{w-1}\sum_{v=0}^{h-1} F(u,v) \exp\left(2\pi i \left(\frac{xu}{w} + \frac{yv}{h}\right)\right)$$

2D

Interpolation/reconstruction filters

Bilinear interpolation

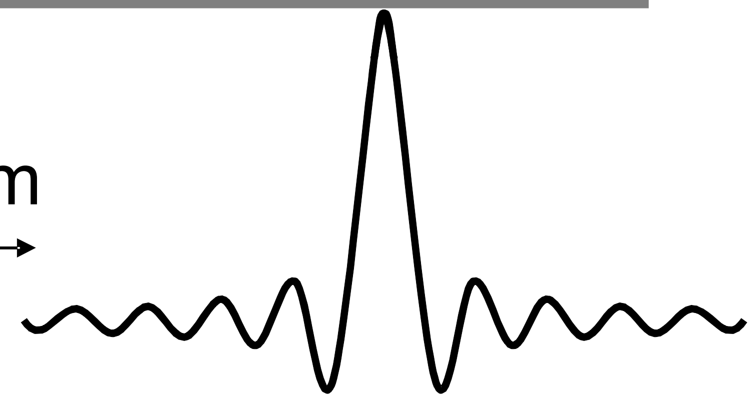
$$\mathbf{c}_{x,y} = (1-x')y'\mathbf{c}_{i,j+1} + x'y'\mathbf{c}_{i+1,j+1} + (1-x')(1-y')\mathbf{c}_{i,j} + x'(1-y')\mathbf{c}_{i+1,j}$$



Interpolation/reconstruction filters

Sinc (Lanczos) is ideal low-pass filter (infinite support)

Fourier transform



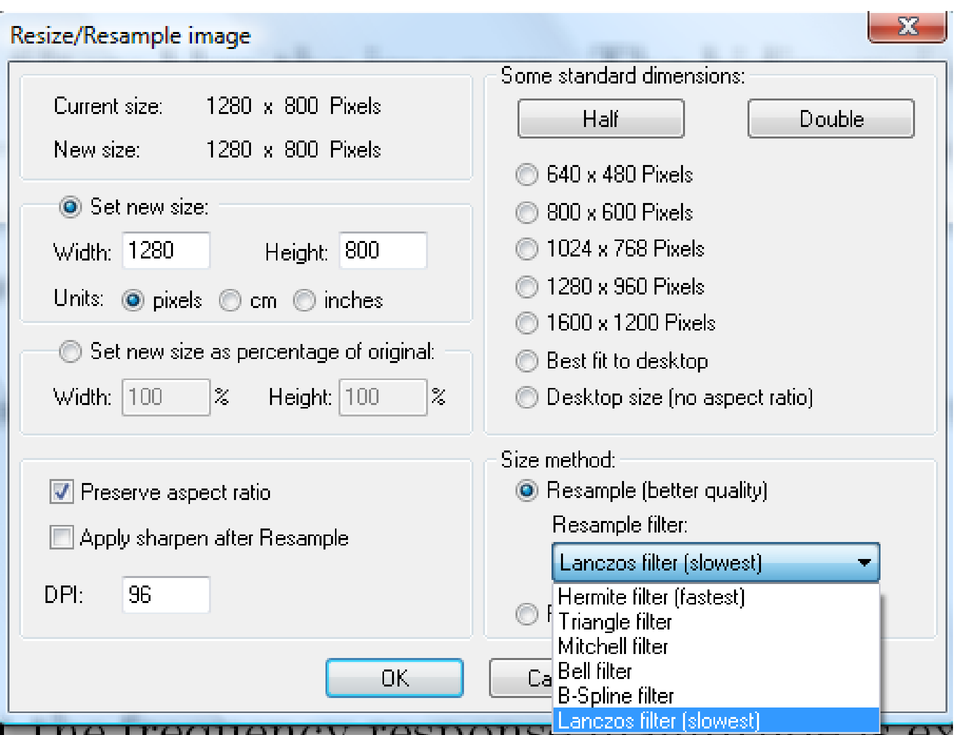
Ideal low-pass filter

Fourier domain

The sinc function

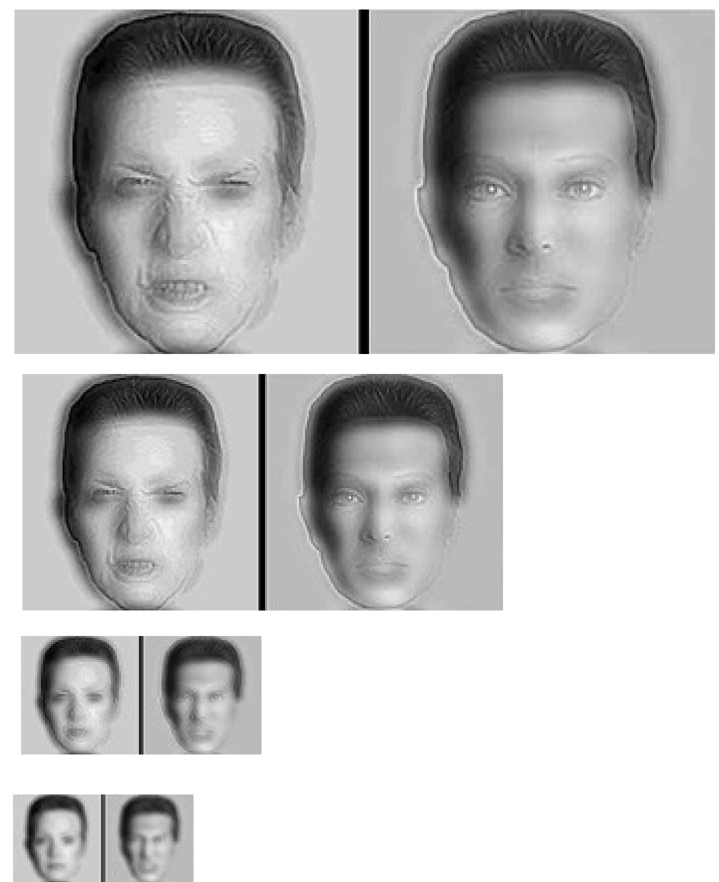
$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

Spatial domain



Windowed sinc

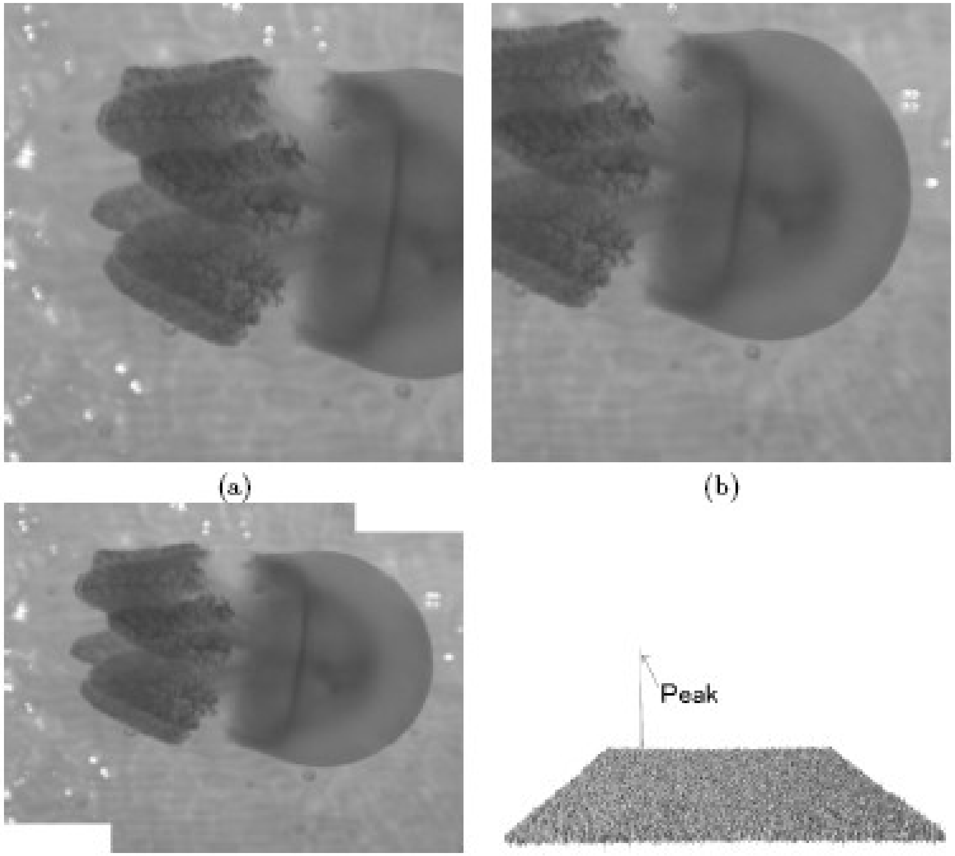
Mr Angry
Mrs Calm



Low/high frequency perception

Phase correlation

Stitch by 2D translation two images



$$f_2(x, y) = f_1(x + x_t, y + y_t)$$

$$F_2(u, v) = F_1(u, v) \exp(-2\pi i(ux_t + vy_t))$$

$$\underbrace{\frac{F_1(u, v)F_2^*(u, v)}{|F_1(u, v)F_2^*(u, v)|}}_{\text{Cross-power spectrum}} = \exp(2\pi i(ux_t + vy_t))$$

FFT can be computed in O(nlog n) time

F* is the conjugate function

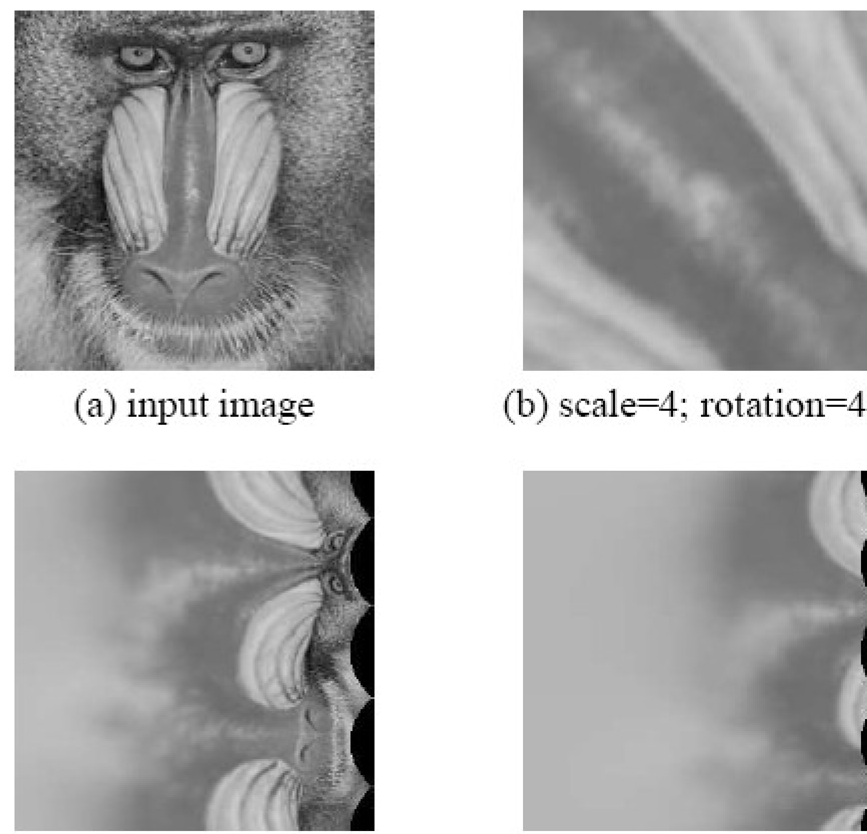
Phase correlation

$$\underbrace{\frac{F_1(u,v)F_2^*(u,v)}{|F_1(u,v)F_2^*(u,v)|}}_{\text{Cross-power spectrum}} = \exp(2\pi i(ux_t + vy_t))$$

Algorithme: Calculer le cross-power spectrum des deux images

Calculer la transformation inverse FFT, et chercher le sommet dans l'image spatiale

Extend to rotation and scale using the log-polar transform



(a) input image

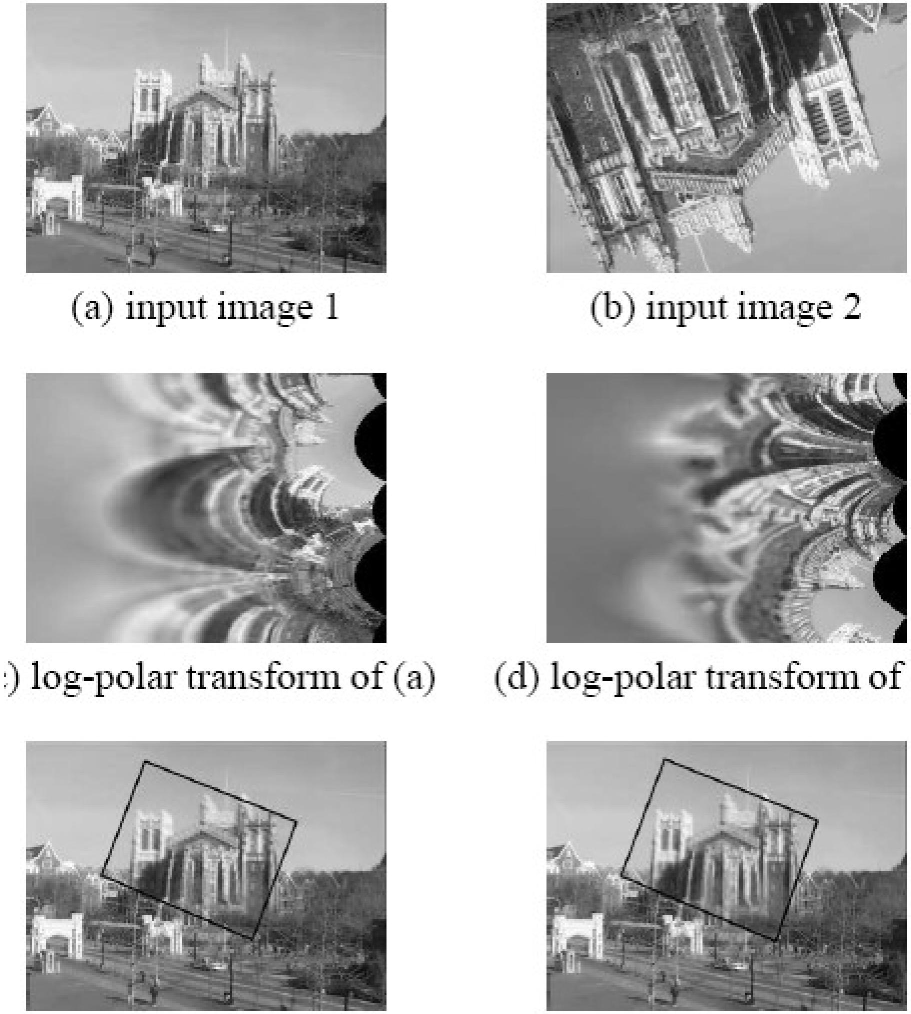
(b) scale=4; rotation=45°

(c) log-polar transform of (a)

(d) log-polar transform of (b)

Phase correlation

Extend up to affine transformations



(a) input image 1 (b) input image 2

:) log-polar transform of (a) (d) log-polar transform of (b)

(e) log-polar registration (f) log-polar/affine registration

ROBUST IMAGE REGISTRATION USING LOG-POLAR TRANSFORM, ICIP 2000

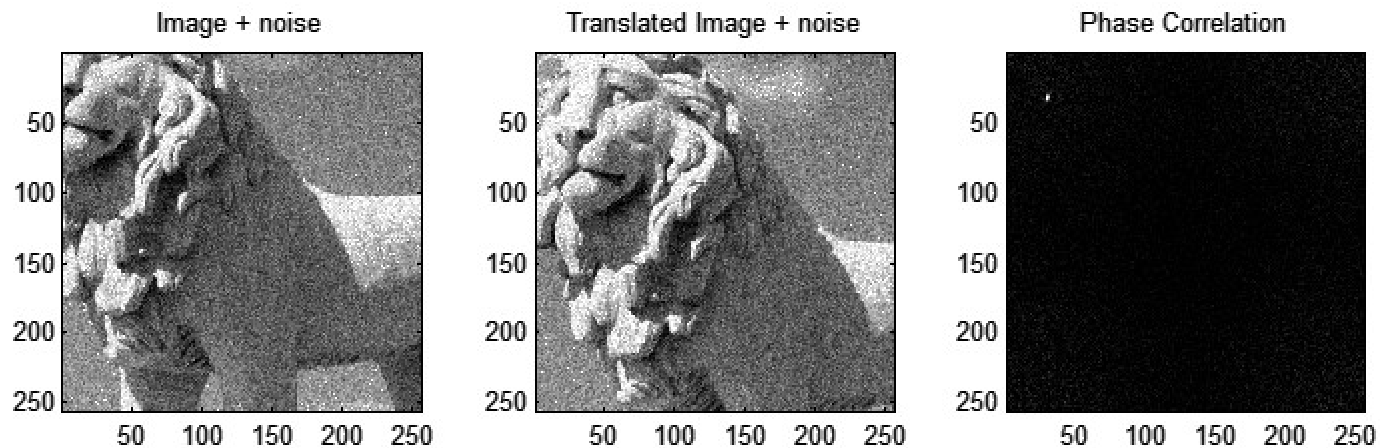
C.D. Kuglin and D.C. Hines. The phase correlation image aligment method. *Proc. Int. Conf. on Cybernetics and Society*, pages 163–165, 1975.

E. De Castro and C. Morandi. Registration of translated and rotated images using finite fourier transforms.*IEEE Trans. Pattern Analysis and Machine Intelligence*, (3):700–703, September 1987.

Phase correlation: Detecting the peak

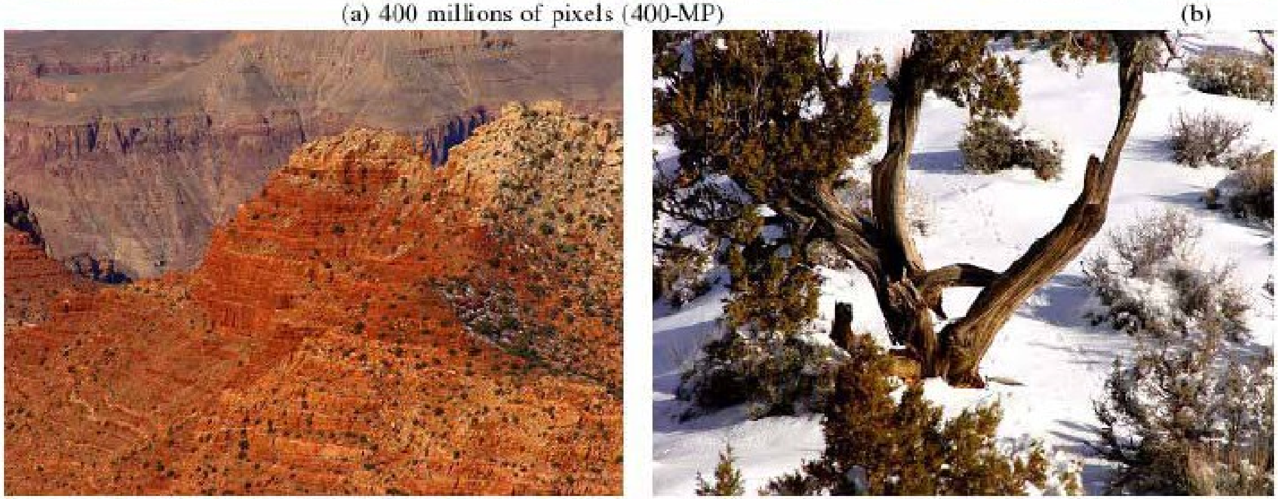
ClairVoyance: A Fast and Robust Precision Mosaicing System for Gigapixel Images

Sub-pixel accuracy if we fit a quadratic function



(a) 400 millions of pixels (400-MP)

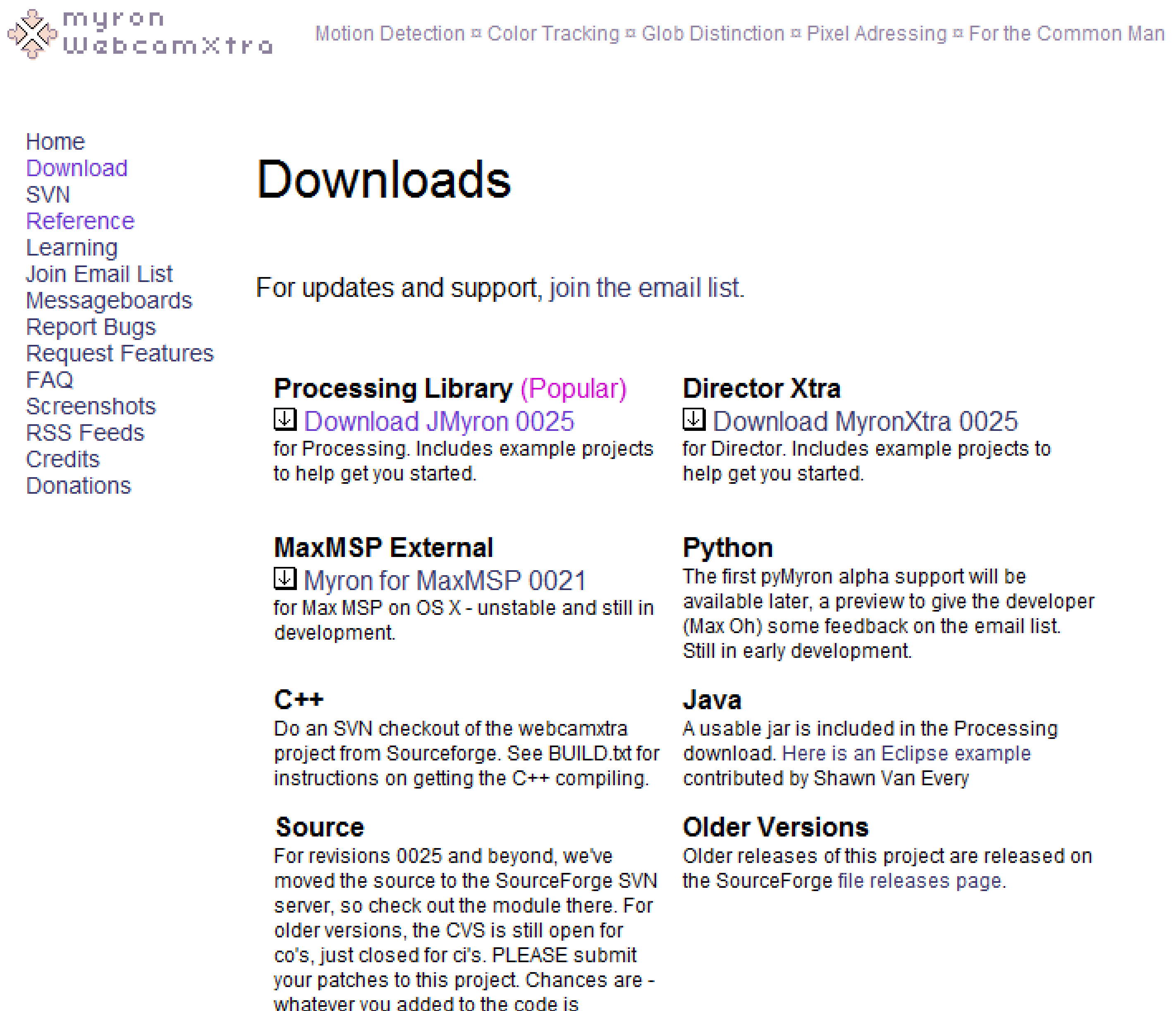
(b)



Clairvoyance system
IEEE IECON 2006

Veuillez commencer vos projets sous Processing et autres Java APIs

Utilisez JMyron sous Processing pour capturer la webcam
http://webcamxtra.sourceforge.net/


myron WebcamXtra
Motion Detection ¤ Color Tracking ¤ Glob Distinction ¤ Pixel Adressing ¤ For the Common Man

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Processing Library (Popular)
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for Processing. Includes example projects to help get you started.

Director Xtra
Download MyronXtra 0025
for Director. Includes example projects to help get you started.

MaxMSP External
Myron for MaxMSP 0021
for Max MSP on OS X - unstable and still in development.

Python
The first pyMyron alpha support will be available later, a preview to give the developer (Max Oh) some feedback on the email list. Still in early development.

C++
Do an SVN checkout of the webcamxtra project from Sourceforge. See BUILD.txt for instructions on getting the C++ compiling.

Java
A usable jar is included in the Processing download. Here is an Eclipse example contributed by Shawn Van Every

Source
For revisions 0025 and beyond, we've moved the source to the SourceForge SVN server, so check out the module there. For older versions, the CVS is still open for co's, just closed for ci's. PLEASE submit your patches to this project. Chances are - whatever you added to the code is

Older Versions
Older releases of this project are released on the SourceForge file releases page.