Maps in Shape Collections

Descriptor and Subspace Learning

- Feature selection for shape matching
- Extraction the most stable correspondences from a collection of mappings

Networks of Maps

- Cycle consistency constraint
- Latent spaces
- Application to co-segmentation

Metrics and Shape Differences

- A functional representation of intrinsic distortions introduced for analysis purposes
- Potential application to geometry synthesis

Part I

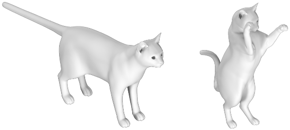
Descriptor and Subspace Learning

- Feature selection for shape matching
- Extraction the most stable correspondences from a collection of mappings

Functional Map Approximation

Functional map approximation [Ovsjanikov et al., 2012]:

$$\mathbf{C}_i^{\star} = \arg\min_{\mathbf{C}} \|\mathbf{C}\mathbf{A}_0 - \mathbf{A}_i\|_F^2 + \alpha\|\mathbf{C}\boldsymbol{\Delta}_0 - \boldsymbol{\Delta}_i\mathbf{C}\|_F^2$$



$\mathcal{M}$ $\mathcal{N}$

- $\mathbf{A}_i$ functions on $\mathcal{N}_i$
- $\boldsymbol{\Delta}_i$ Laplacian on $\mathcal{N}_i$

Functional Map Approximation

Functional map approximation [Ovsjanikov et al., 2012]:

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- **Probe functions**
  Any functions stable by nearly-isometric deformation
  In practice: HKS [Sun et al., 2009], WKS [Aubry et al., 2011], Curvatures...

▶ Non-unique solution

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- **Probe functions**
  Any functions stable by nearly-isometric deformation
  In practice: HKS [Sun et al., 2009], WKS [Aubry et al., 2011], Curvatures...

- **Regularization:** Assume nearly isometric deformations
  Commutativity of $\mathbf{C}$ with the Laplace-Beltrami operator:

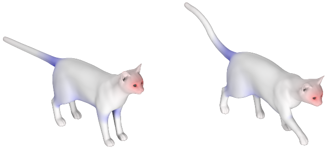
$$\mathbf{C}\boldsymbol{\Delta}_0 = \boldsymbol{\Delta}_i\mathbf{C}$$

▶ It can be difficult to a obtain good approximation

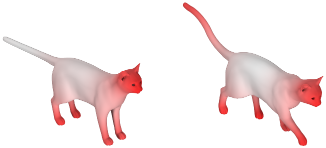
Main Challenges

$$\mathbf{C}_i^\star = \arg\min_{\mathbf{C}} \|\mathbf{C}\mathbf{A}_0 - \mathbf{A}_i\|_F^2 + \alpha\|\mathbf{C}\boldsymbol{\Delta}_0 - \boldsymbol{\Delta}_i\mathbf{C}\|_F^2$$

- The probe functions can be inconsistent



(a) Smoothed Gaussian curvature.



(b) Logarithm of the absolute value of Gaussian Curvature.

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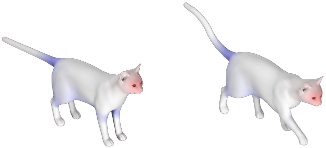

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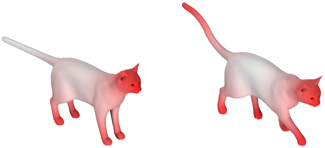


(b) Logarithm of the absolute value of Gaussian Curvature.

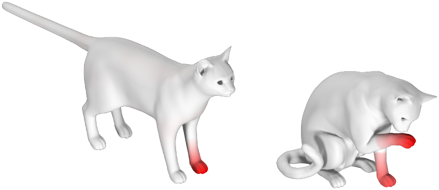
Weight the probe functions [Corman et al., 2014]:

$$\mathbf{C}_i^\star(\mathbf{D}) = \arg\min_{\mathbf{C}} \|\mathbf{C}\mathbf{A}_0\mathbf{D} - \mathbf{A}_i\mathbf{D}\|_F^2 + \alpha\|\mathbf{C}\boldsymbol{\Delta}_0 - \boldsymbol{\Delta}_i\mathbf{C}\|_F^2$$

Main Challenges

$$\mathbf{C}_i^\star = \arg\min_{\mathbf{C}} \|\mathbf{C}\mathbf{A}_0 - \mathbf{A}_i\|_F^2 + \alpha\|\mathbf{C}\boldsymbol{\Delta}_0 - \boldsymbol{\Delta}_i\mathbf{C}\|_F^2$$

- The approximation is not reliable on the entire functional space



$\mathbf{f}$ $\mathbf{C}^\star\mathbf{f}$

Main Challenges

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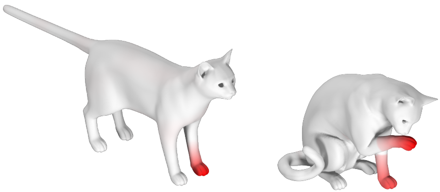
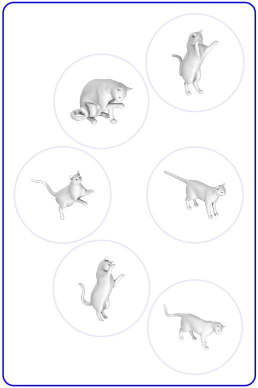


Learn the functional subspace $S_p \subset L^2(\mathcal{M})$ of dimension p such that:

$$\mathbf{C}_T\mathbf{f} \approx \mathbf{C}^\star\mathbf{f}, \quad \forall \mathbf{f} \in S_p$$

Feature Selection

Training Set



Feature Selection

$$\mathbf{D}^{\star} \in \underset{\mathbf{D}}{\arg\min} \sum_{i=1}^{N} \|\mathbf{C}_i^{\star}(\mathbf{D}) - \mathbf{C}_i\| \quad ; \quad \mathbf{Y}_p \in \underset{\mathbf{Y}^{\top}\mathbf{Y}=\mathbf{I}_p}{\arg\min} \sum_{i=1}^{N} \|(\mathbf{C}_i^{\star}(\mathbf{D}^{\star}) - \mathbf{C}_i)\mathbf{Y}\|_F^2$$

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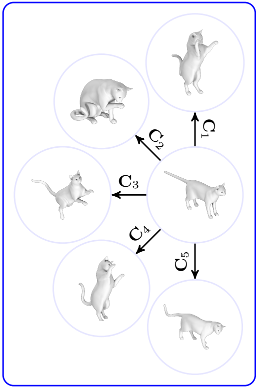

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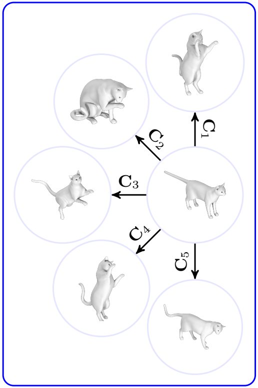
Training Set $\Longrightarrow$

$\mathbf{D}^\star$: optimal weights
$\mathbf{Y}_p$: basis of S_p



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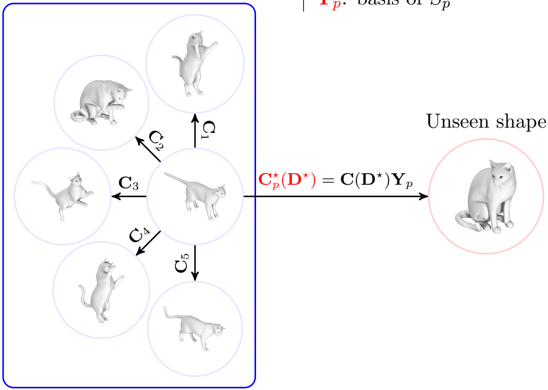
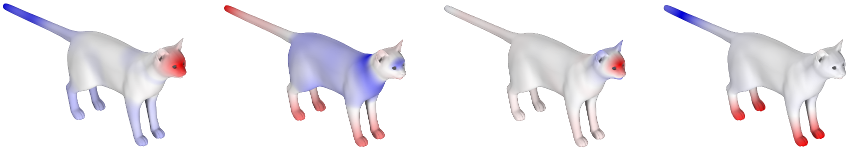


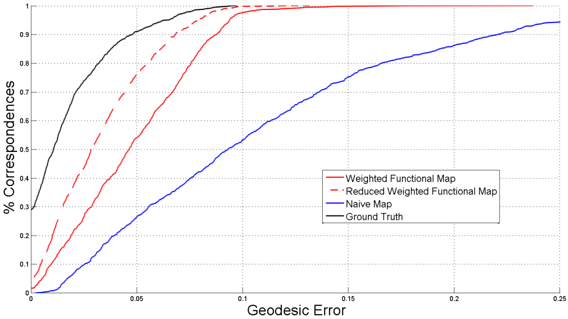
Stable function subspace

Reduced basis extraction:



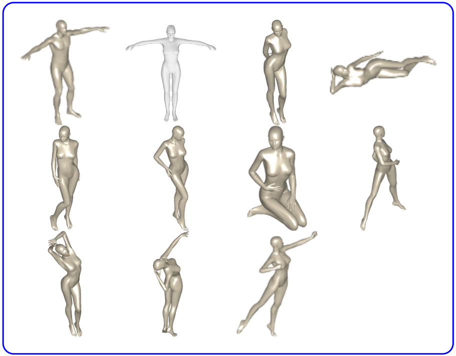
$\mathbf{y}_1$ $\mathbf{y}_2$ $\mathbf{y}_3$ $\mathbf{y}_4$

Correspondences:

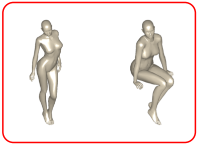


Non-Isometric matching

Training Set

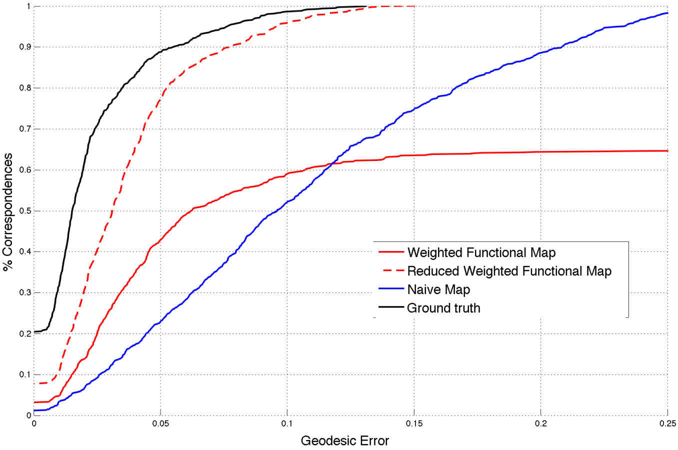


Unseen Poses

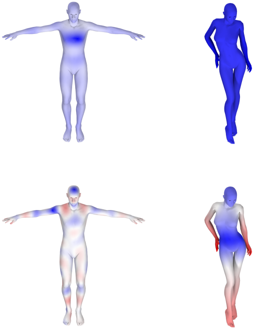


- 100 basis functions
- 310 probe functions
- Training set: 10 shapes of women + 1 reference shape of man
- 50 functions in the reduced basis

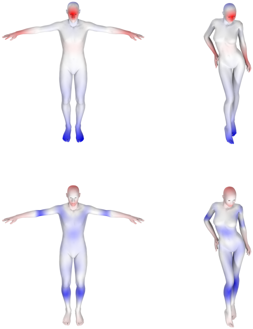
Results: Non Isometric matching



Conclusion



Naive Map



Learned Map

- The functional maps quality can be improved by weighting the probe functions
- Learning makes the functional maps more stable with respect to large deformations

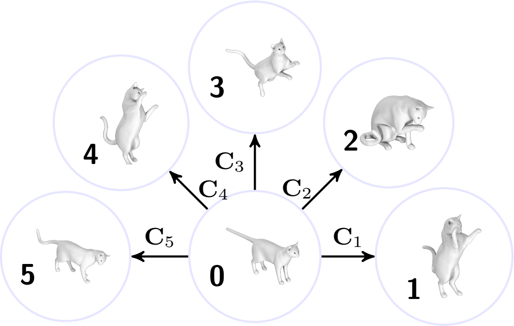
Part II

Network of Maps

A non-supervised regularization for shape matching

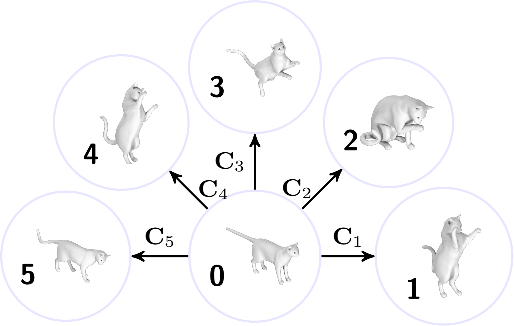
- Cycle consistency constraint
- Latent spaces

Graph of Maps



- Compact description the entire network by composition (e.g. $\mathbf{C}_{45} = \mathbf{C}_{05}\mathbf{C}_{40}$)

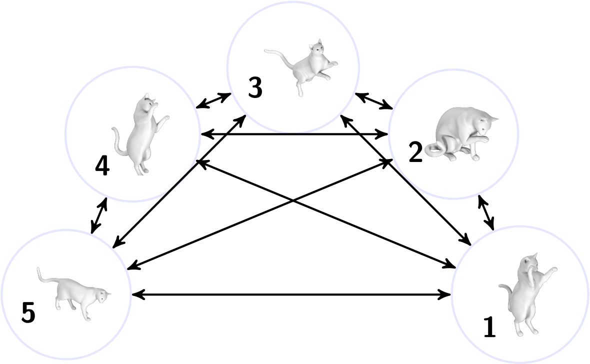
Graph of Maps



- Compact description the entire network by composition (e.g. $\mathbf{C}_{45} = \mathbf{C}_{05}\mathbf{C}_{40}$)

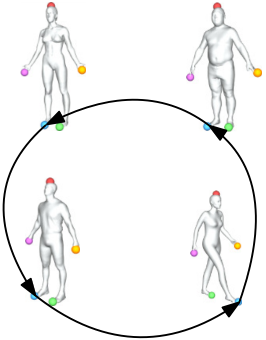
- Suppose a star graph structure
- The results depends on the reference shape

Graph of Maps



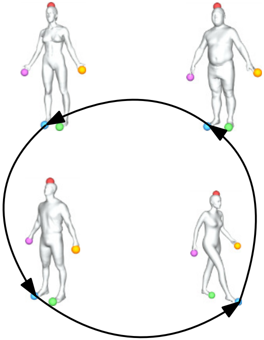
How to use general graph structure?
How to impose coherence and consistency?
How a shzpe collection help solving shape matching problem?

Cycle Consistency Constraint



Consistent Path

Cycle Consistency Constraint



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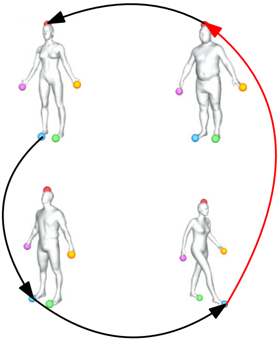

Inconsistent Path

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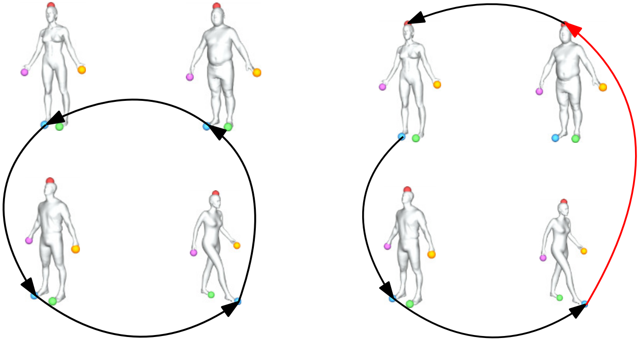

Consistent Path　　　　Inconsistent Path

- Strong regularization
- Allows detection and correction of errors
- Characterized by: $\mathbf{C}_{ij} = \mathbf{C}_{kj}\mathbf{C}_{ik}$

Cycle Consistency and Low Rank Matrix

- Can be difficult to enforce in an optimization problem:

$$\mathbf{C}_{ij} = \mathbf{C}_{kj}\mathbf{C}_{ik}$$

- Equivalent to a low rank or semi-definiteness condition on a big mapping matrix [Huang et al., 2014]

$$\mathbf{C} := \begin{pmatrix} \mathbf{C}_{11} & \cdots & \mathbf{C}_{N1} \\ \vdots & \ddots & \vdots \\ \mathbf{C}_{1N} & \cdots & \mathbf{C}_{NN} \end{pmatrix} = \begin{pmatrix} \mathbf{Y}_1^+ \\ \vdots \\ \mathbf{Y}_N^+ \end{pmatrix} \begin{pmatrix} \mathbf{Y}_1 & \cdots & \mathbf{Y}_N \end{pmatrix} \succeq 0$$

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  - $\mathbf{C}$ is semi-definite
  - Rank of $\mathbf{C}$ is very low compared to the number of shapes

Computation of a Functional Map Network

Given descriptors on each shape, we can compute the functional map network:

$$\mathbf{C}^{\star} = \min_{\mathbf{C}} \sum_{(i,j)\in\mathcal{G}} \|\mathbf{C}_{ij}\mathbf{A}_i - \mathbf{A}_j\|_{2,1} + \mathrm{Reg}(\mathbf{C}_{ij}) + \lambda\|\mathbf{C}\|_{\star}$$

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- Convex optimization problem solved with ADMM

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- Nuclear norm $\|\mathbf{X}\|_\star = \sum_i \sigma_i(\mathbf{X})$ is the convex regularization of the rank
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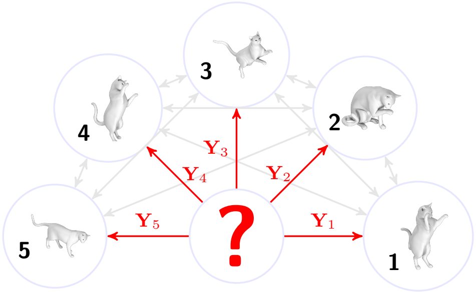
Unlike separate computation of the functional map this setting:

- Removes descriptors outliers
- Enforces coherence between in the network

Latent Spaces



$$\begin{pmatrix} \mathbf{C}_{11} & \cdots & \mathbf{C}_{N1} \\ \vdots & \ddots & \vdots \\ \mathbf{C}_{1N} & \cdots & \mathbf{C}_{NN} \end{pmatrix} = \begin{pmatrix} \mathbf{Y}_1^+ \\ \vdots \\ \mathbf{Y}_N^+ \end{pmatrix} \begin{pmatrix} \mathbf{Y}_1 & \cdots & \mathbf{Y}_N \end{pmatrix}$$

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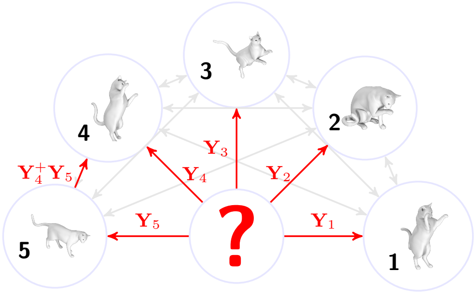

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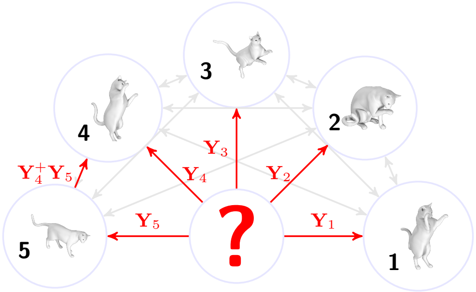

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- The Y_i can be understood as functional maps to an abstract surface called “latent space”

Orthogonal Basis Synchronization

Cycle consistency as hard constraint:

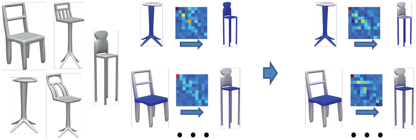
$$\min_{\mathbf{Y}_1,\ldots,\mathbf{Y}_N} \sum_{(i,j)\in\mathcal{G}} \|\mathbf{C}_{ij} - \mathbf{Y}_j^{+}\mathbf{Y}_i\|_F^2 \text{ s.t. } \mathbf{Y}_i^{\top}\mathbf{Y}_i = \mathbf{I}$$

Given a map network $\mathbf{C}_{ij}$, $(i,j) \in \mathcal{G}$ (with possible inconsistencies and missing edges), performing the factorization can be used to:

- Regularize and clean up functional maps
- Extract shared structure
- Find the most representative reference abstract shape
- Efficient storage of large network

Application to Cosegmentation [Huang et al., 2014]

Input: Shape collection and local descriptors
Output: Consistent segmentation



- Joint map optimization

$$\mathbf{C}^\star = \min_{\mathbf{C}} \sum_{(i,j)\in\mathcal{G}} \|\mathbf{C}_{ij}\mathbf{A}_i - \mathbf{A}_j\|_{2,1} + \lambda\|\mathbf{C}\|_\star$$

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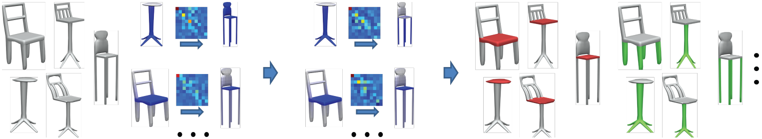

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- Orthogonal basis synchronization

$$\min_{\mathbf{Y}_1,\ldots,\mathbf{Y}_N} \sum_{(i,j)\in\mathcal{G}} \|\mathbf{C}^{\star}_{ij} - \mathbf{Y}_j^{+}\mathbf{Y}_i\|_F^2 \text{ s.t. } \mathbf{Y}_i^{\top}\mathbf{Y}_i = \mathbf{I}$$

Part III

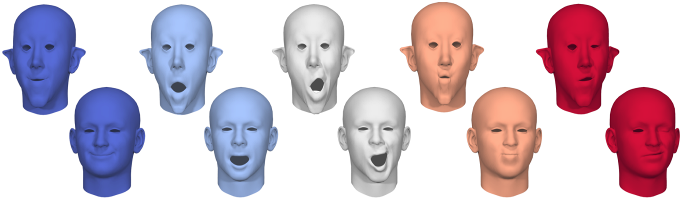
Shape Difference Operators

A functional representation of intrinsic distortions

- Introduced for analysis purposes
- Potential application to geometry synthesis

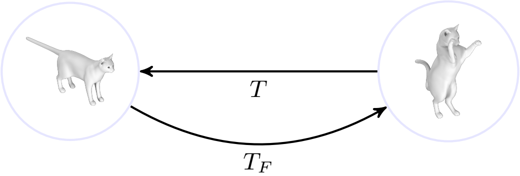
Shape Differences Overview [Rustamov et al., 2013]

- Fully characterize the distortion using two linear functional operators
- Can compute areas of maximal distortion through eigendecomposition
- Can compare distortions of different pairs of shapes



[Rustamov et al., 2013]

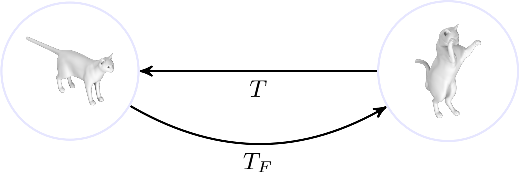
Area-based Shape Difference



Area-based shape difference: $D_A : L^2(\mathcal{M}) \to L^2(\mathcal{M})$

$$\int_{\mathcal{M}} f\, D_A(g)\, \mathrm{d}\mu = \int_{\mathcal{N}} T_F(f)\, T_F(g)\, \mathrm{d}\mu$$

Area-based Shape Difference



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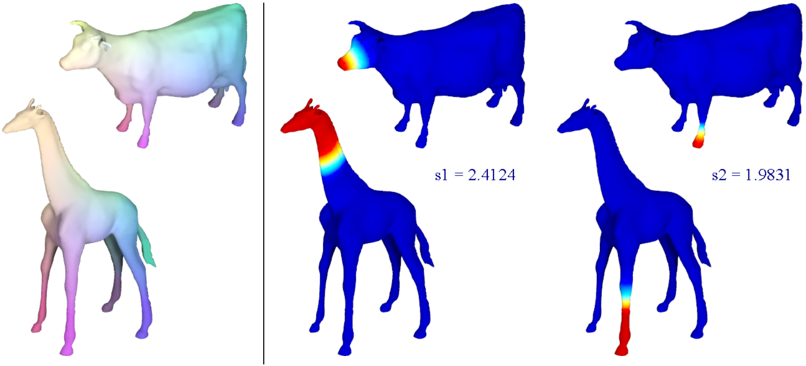
$$\int_{\mathcal{M}} f \, D_A(g) \, \mathrm{d}\mu = \int_{\mathcal{N}} T_F(f) \, T_F(g) \, \mathrm{d}\mu$$

$$D_A(f)(p) = \frac{\mathrm{Area}\left(T^{-1}(p)\right)}{\mathrm{Area}(p)} f(p)$$

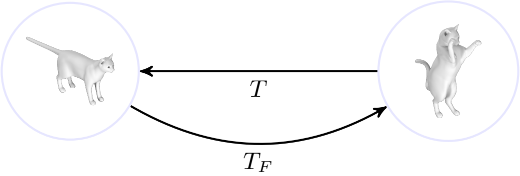
- $D_A(f) = f$ if and only if T area preserving map

Most Distorted Areas



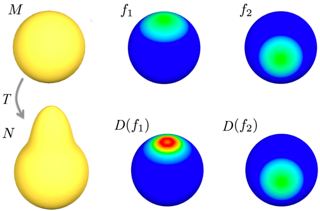
[Rustamov et al., 2013]

Conformal Shape Difference



Conformal shape difference: $D_C : H_0^1(\mathcal{M}) \to H_0^1(\mathcal{M})$

$$\int_{\mathcal{M}} \langle \nabla f, \nabla D_C(g) \rangle \, \mathrm{d}\mu = \int_{\mathcal{N}} \langle \nabla T_F(f), \nabla T_F(g) \rangle \, \mathrm{d}\mu$$



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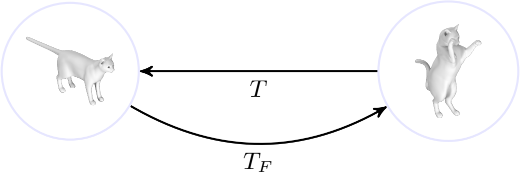
Conformal Shape Difference



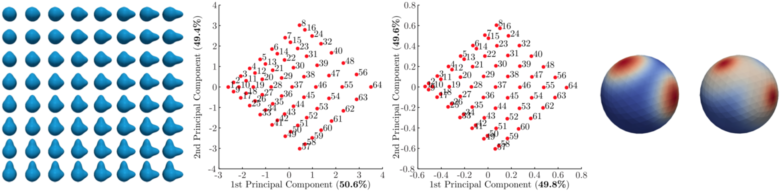
Conformal shape difference: $D_C : H_0^1(\mathcal{M}) \to H_0^1(\mathcal{M})$

$$\int_{\mathcal{M}} \langle \nabla f, \nabla D_C(g) \rangle \, \mathrm{d}\mu = \int_{\mathcal{N}} \langle \nabla T_F(f), \nabla T_F(g) \rangle \, \mathrm{d}\mu$$

- $D_C(f) = f$ if and only if T conformal map

Low-Dimension Embeddings

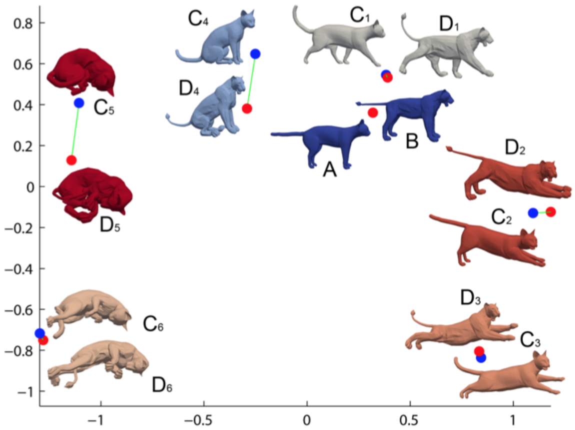
- D_A, D_C fully encode the metric



[Rustamov et al., 2013]

Shape Search

Find a shape D_i, such that the difference between shapes B and D_i is as-close-as possible to the difference between A and C_i.



[Rustamov et al., 2013]

Shape Differences for Synthesis?

- Shape difference operators for analysis:
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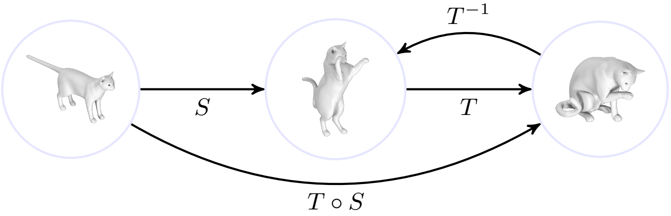
  - Comparison of deformations

Shape Differences for Synthesis?

- Shape difference operators for analysis:
  - Meaningful low-dimensional embedding
  - Visualization of most distorted areas
  - Comparison of deformations
- Shape difference operators are easily created:
  - Deformation manipulation
  - Deformation transfer
  - Shape interpolation
  - Intrinsic symmetrization

How much information is contained in the shape difference operators?

Shape Differences Algebra



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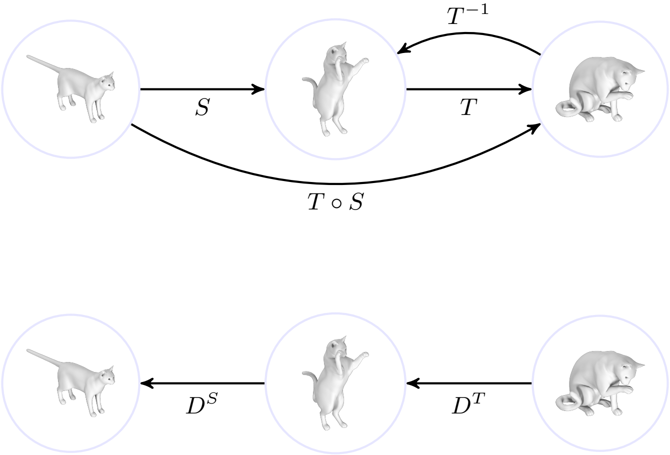

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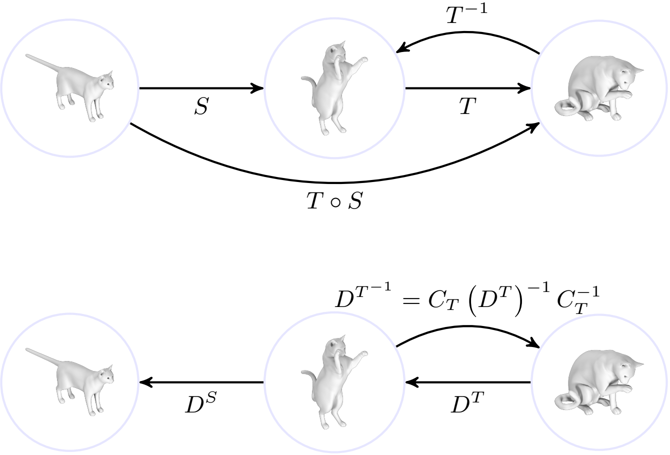

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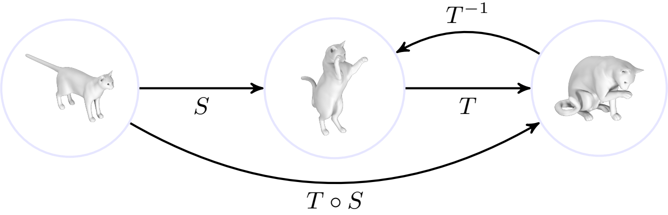
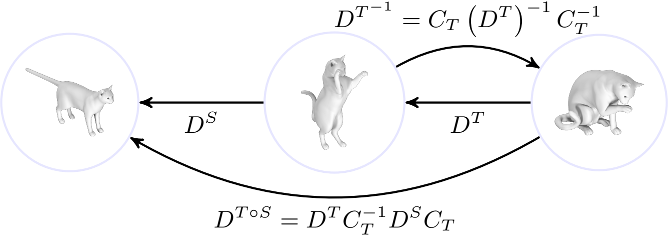
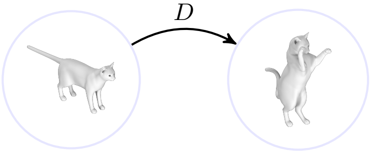




Intrinsic Deformation Transfer

- Deformation on M described by a shape difference $D : L^2(\mathcal{M}) \to L^2(\mathcal{M})$ can be transported to another shape using a functional map:

$$T_F D T_F^{-1} : L^2(\mathcal{N}) \to L^2(\mathcal{N})$$



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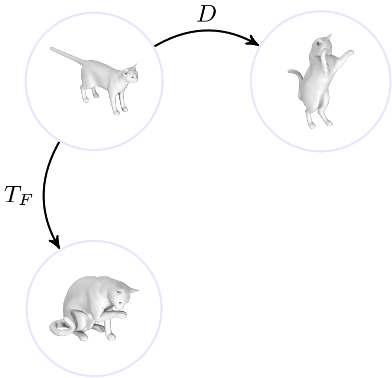

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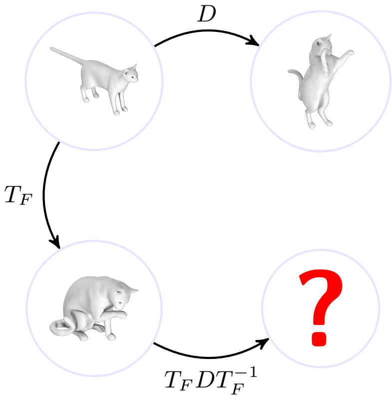
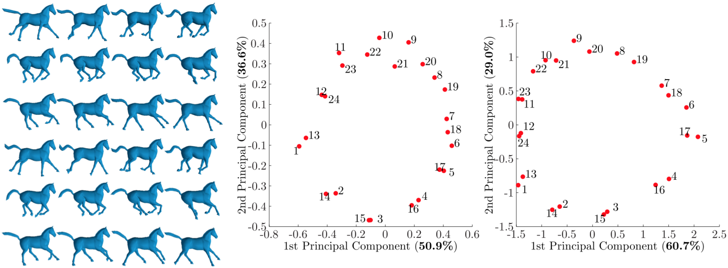


Shape Interpolation

► Use the low-dimension embedding to produce non-linear shape interpolation



[Rustamov et al., 2013]

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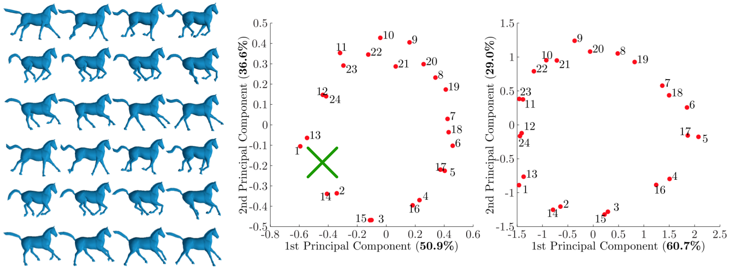

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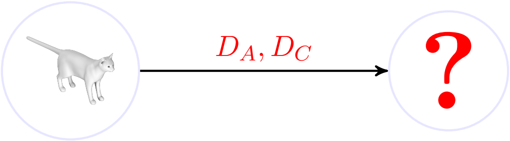
Main Challenge for Synthesis

- Recovering geometry from operators:



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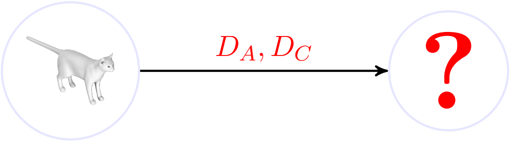

"Shape differences fully encode the metric"
What does it mean for the discrete geometry?

Shape Difference on Triangle Meshes

Assumptions:

- Triangle meshes with same connectivity
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Theorem

Assume that the mesh M is manifold without boundary. Then, for almost all choices of areas μ, the map $\ell^2 \mapsto D_C(\mu, \ell^2)$ uniquely determines ℓ, which is recoverable via a linear solve.

Recovering Intrinsic Geometry

[Boscaini et al., 2015]

- Solve a non-linear optimization problem:

$$\ell^\star = \arg\min_{\ell} \|D_A(\ell) - \bar{D}_A\|_F^2 + \|D_C(\ell) - \bar{D}_C\|_F^2$$

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[Corman et al., 2016]

- Two convex optimization problems:

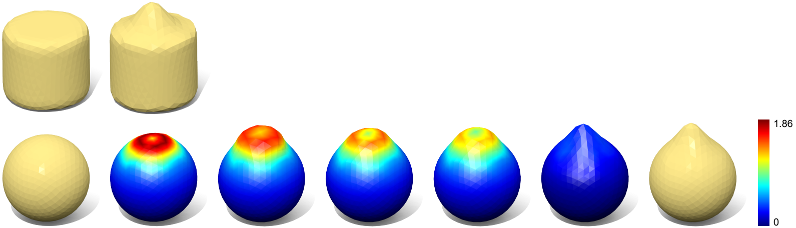
1. Find the triangle areas μ:

$$\mu^\star = \arg\min_{\mu} \|D_A(\mu) - \bar{D}_A\|_F^2$$
$$\text{s.t. } \mu > 0$$

2. Given the areas, find the squared edge lengths ℓ^2:

$$\min_{\ell^2} \|D_C(\mu^\star, \ell^2) - \bar{D}_C\|_F^2$$
$$\text{s.t. } \ell_i < \ell_j + \ell_k \,;\ \text{Area}(\ell_i^2, \ell_j^2, \ell_k^2) \geq \mu_{ijk}$$

Shape Analogy



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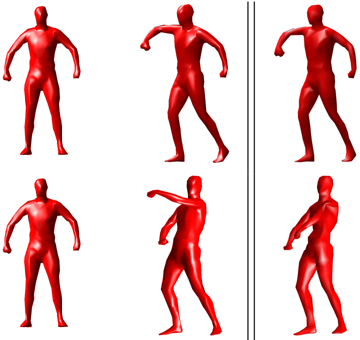
Intrinsic Shape Difference Operators

- Intrinsic information only, in general not enough to recover geometry



Source Target Intrinsic

Intrinsic Shape Difference Operators

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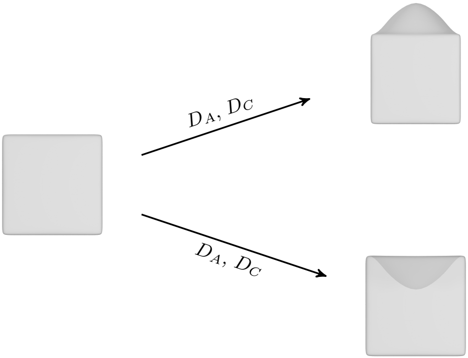
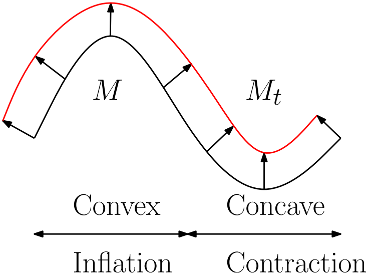


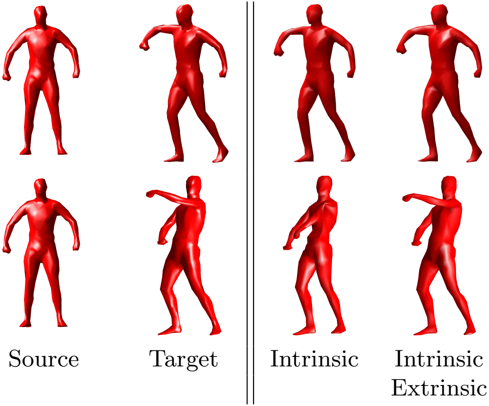
Encoding Curvature using Normal Flow



- Evolution of the area linked to Mean Curvature
- The second fundamental form can be recovered given the metric tensors at time 0 and at time $t > 0$

Geometry From Operators

- Mesh embedding uniquely defined by four operators

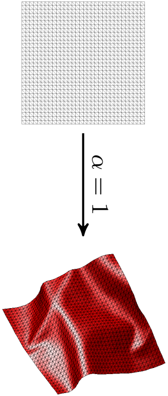


[Corman et al., 2016]

Shape Interpolation

- Linear interpolation in shape differences space:

$$D_{\alpha} = (1-\alpha)I + \alpha D$$



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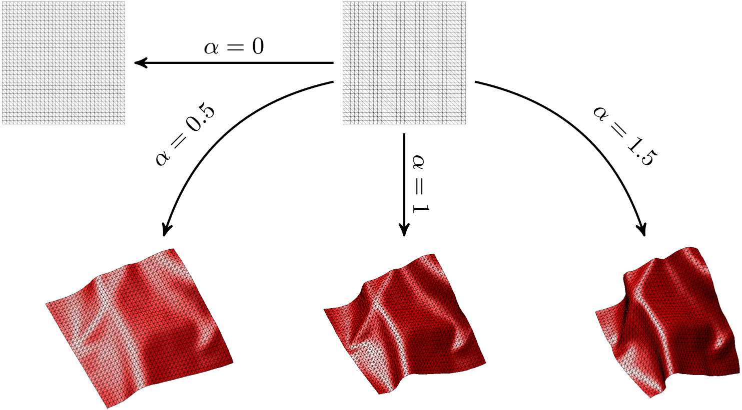

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Limitations:

- Need to solve an isometric embedding problem
- Impractical for large meshes
- Solver that is oblivious of the initial mesh embedding

Conclusion

- Descriptor learning for shape matching [Corman et al., 2014]
- Computation of map collection with cycle consistency constraint [Huang et al., 2014]
- Shape collection visualization with shape differences [Rustamov et al., 2013]
- Shape editing [Boscaini et al., 2015, Corman et al., 2016]

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