Constraint satisfaction problems
and
related logic

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Keywords

(descriptive) complexity, finite model theory, constraint satisfaction problems (CSP), homomorphism problems, monotone monadic syntactic NP without inequalities (MMSNP), Heyting algebra, dichotomy.

Abstract

Constraint satisfaction problems (CSP) can be modelled in terms of the existence of a homomorphism between finite structures. Feder and Vardi have attempted to characterise logically the class CSP. They introduced a fragment of existential second order logic, the logic MMSNP. They proved that MMSNP is computationally equivalent to CSP and that CSP is strictly included in MMSNP.
In this work, we introduce a new class of combinatorial problems, the so-called forbidden patterns problems (FP), that correspond exactly to the problems definable by a sentence of MMSNP. We introduce some novel algebraic tools that allow us to characterise exactly those problems that are in FP but not in CSP. Furthermore, given a representation for a problem in FP, we are able to construct a normal form, and according to a very simple criterion, we can decide whether this problem is in CSP or not. If it is then we can construct a template for this problem (that is give this problem as a problem in CSP) and, otherwise, for any given candidate for the role of template, we can build a counter-example.
This result can be seen as a generalisation of a recent and independent work by Tardif and Nešetřil. We compare our proof with their proof. Their work yields a better characterisation in the restricted case of monochrome forbidden patterns problems. However, our construction of the template (whenever possible) is superior to theirs. Their proof relies heavily on a correspondence between gap pairs and duality pairs in the category of σ-structures. We simplify and generalise their proof to Heyting algebras . We also show that the class of forbidden patterns problems can be related to such an algebra.
We showed also that FP behaves essentially like CSP with respect to complexity: we gave complete forbidden patterns problems for the class NL,P and NP, that are not in CSP. We briefly review restrictions that can lead to the tractability of forbidden patterns problems.
Finally, we investigate homomorphism problems for unary algebras: we prove a dichotomy in the case of a single unary function symbol and show that the class of problems with two unary function symbols is computationally equivalent to CSP.