



- This is a cut on two contracted formulae.
- First, the cut interacts with the blue (right) contraction. This duplicates the left branch (the red cut) and creates a new series of contractions below the two new cuts.
- Second, the top cut interacts with its red (left) contraction. This duplicates the right branch and creates a new series of contractions below the two new cuts.
- The cut at the bottom is now in the original situation, with a cut on two contracted formulae.
- It follows that naive cut-reduction is not strongly normalizing.

We cannot have a non-trivial semantics of classical proof and:

- involutive negation  $\overline{\overline{A}} \cong A$
- conjunction and disjunction are products and coproducts
- disjunction acts as implication  $A \supset B = \overline{A} \lor B$



• Intuitionistic natural deduction proves  $A \vdash (A \supset \bot) \supset \bot$ , and extends to classical natural deduction by including a rule  $\bot E$ ,

 $\begin{bmatrix} \overline{A} \end{bmatrix}^{\alpha} \\ \vdots \\ \underline{\bot} \\ \overline{A} \ \bot^{E,\alpha}$ 

which proves  $(A \supset \bot) \supset \bot \vdash A$  (double-negation elimination). However, the two proofs do not form an isomorphism: composing them does not necessarily give an identity.

Classical natural deduction is computational, and double-negation elimination relates to the call/cc construct (call–with–current–continuation); see Parigot's  $\lambda\mu$ -calculus and subsequent work.

- Omitting some of the equations for products and coproducts (while keeping the necessary contraction and weakening rules) can be sufficient to prevent the semantics from becoming trivial. However, equating proofs under cut-elimination automatically gives products and coproducts, so this is no longer possible.
- Additive linear logic has involutive negation, products, and coproducts, but its disjunction does not have an axiom (only the separate meta-connective ⊢ does), and so cannot take the role of an implication. While it is possible to build a classical model this way, again equating proofs under cut-elimination is ruled out.





