DIRECTED ALGEBRAIC TOPOLOGY

AND

CONCURRENCY

Emmanuel Haucourt

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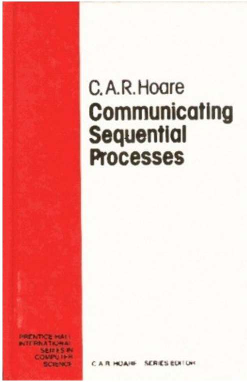
MPRI : Concurrency (2.3.1)
– Lecture 1 –

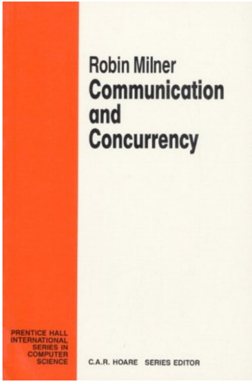
2024 – 2025

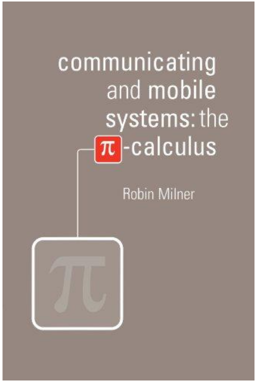
A QUICK OVERVIEW

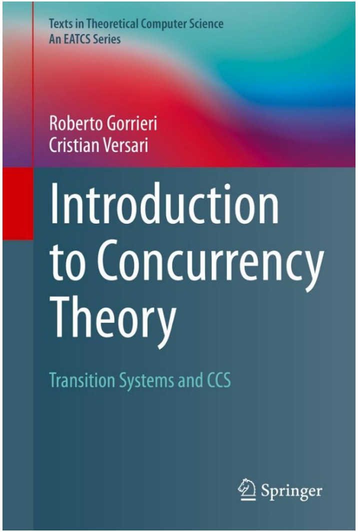
OF

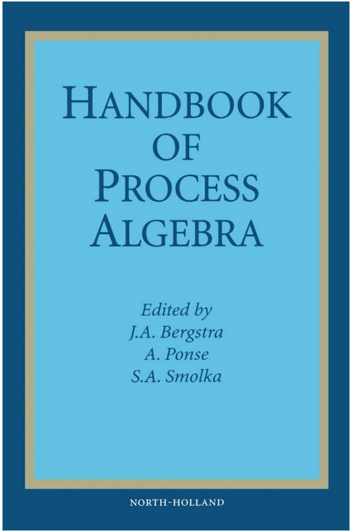
CONCURRENCY THEORY


C.A.R.Hoare
Communicating Sequential Processes


Robin Milner
Communication and Concurrency
PRENTICE HALL INTERNATIONAL SERIES IN COMPUTER SCIENCE
C.A.R. HOARE SERIES EDITOR


communicating and mobile systems: the π-calculus
Robin Milner


Texts in Theoretical Computer Science
An EATCS Series
Roberto Gorrieri
Cristian Versari
Introduction to Concurrency Theory
Transition Systems and CCS
Springer

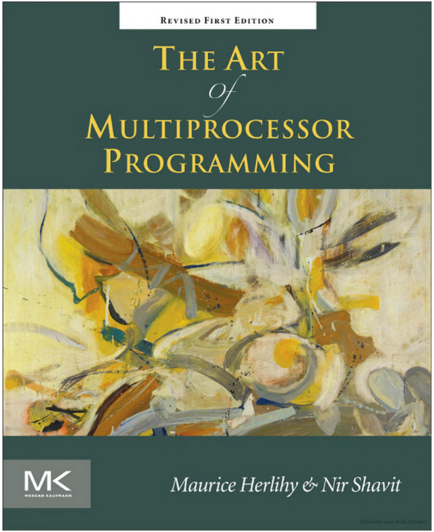

Handbook of Process Algebra
Edited by
J.A. Bergstra
A. Ponse
S.A. Smolka
NORTH-HOLLAND

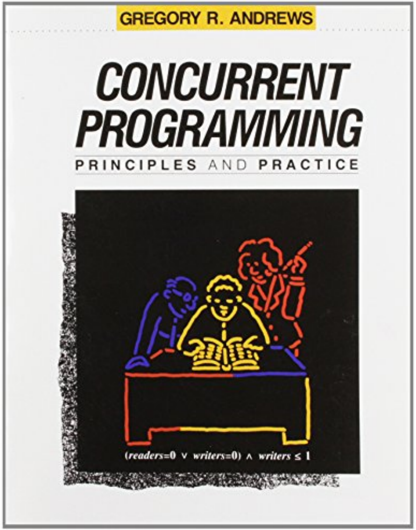
Handbook of Logic in Computer Science

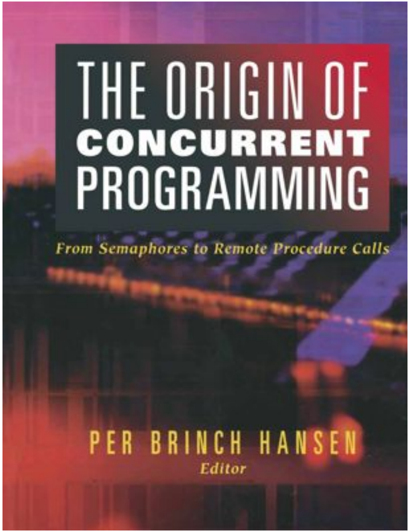
VOLUME 4

S. ABRAMSKY, DOV M. GABBAY,
and T. S. E. MAIBAUM

OXFORD SCIENCE PUBLICATIONS


Revised First Edition
The Art of Multiprocessor Programming
MK
Morgan Kaufmann
Maurice Herlihy & Nir Shavit


GREGORY R. ANDREWS
CONCURRENT PROGRAMMING
PRINCIPLES AND PRACTICE
(readers=0 ∨ writers=0) ∧ writers ≤ 1


THE ORIGIN OF
CONCURRENT
PROGRAMMING
From Semaphores to Remote Procedure Calls
PER BRINCH HANSEN
Editor

PARALLEL AUTOMATA META LANGUAGE

Syntax

Paradigm

Cooperating sequential processes, E. W. Dijkstra, 1965.
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- `proc` <identifier> = <basic block>
- `init` <multiset of identifiers>
  e.g. `init a 2b 3c`

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v	content of $v \in \mathcal{V}$	$x \in \mathbb{R}$	constant
$\wedge$	minimum	$\vee$	maximum
$+$	addition	$-$	substraction
$*$	multiplication	$/$	division
$\leqslant$	less or equal	$\geqslant$	greater of equal
$<$	strictly less	$>$	strictly greater
$=$	equal	$\neq$	not equal
$\neg$	complement	$\%$	modulo
$\perp$	bottom		

nullary	unary
$\perp$, $x \in \mathbb{R}$, $v \in \mathcal{V}$	$\neg$
binary	
$\wedge, \vee, +, -, *, /, <, >, \leqslant, \geqslant, =, \neq, \%$	

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- `(`*L*`)` enclose a list of instructions between parenthesis to make it a single instruction

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- If all the expressions evaluate to zero, then L_{n+1} is triggered.

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Due to the branchings, basic blocks are actually trees.

Control Flow Graphs

Control flow graphs and flowcharts

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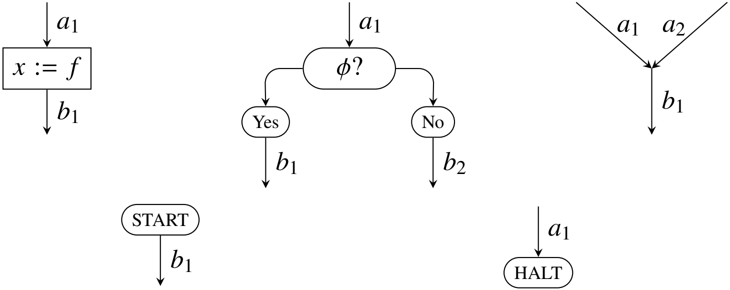
Generators



The Hasse-Syracuse algorithm in PAML

```
var x = 7
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proc p = ()+[x=1]+J(q)

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```

```
init p
```

Building the control flow graph

of the Hasse-Syracuse algorithm

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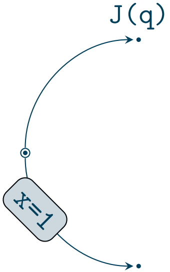
entry point of the

basic block of p



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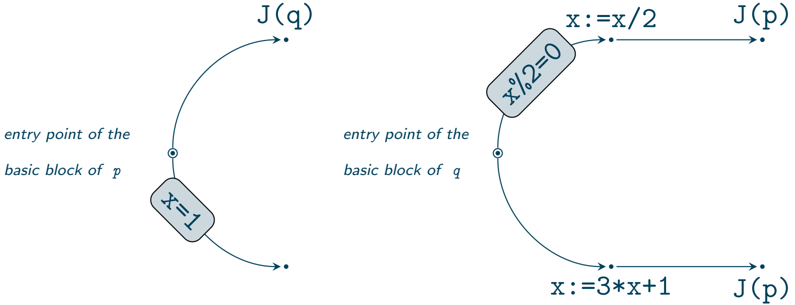

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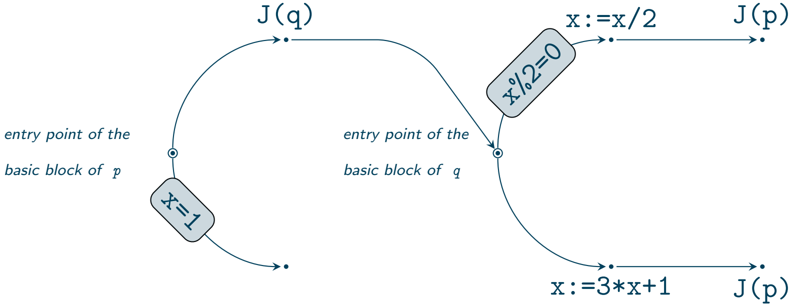

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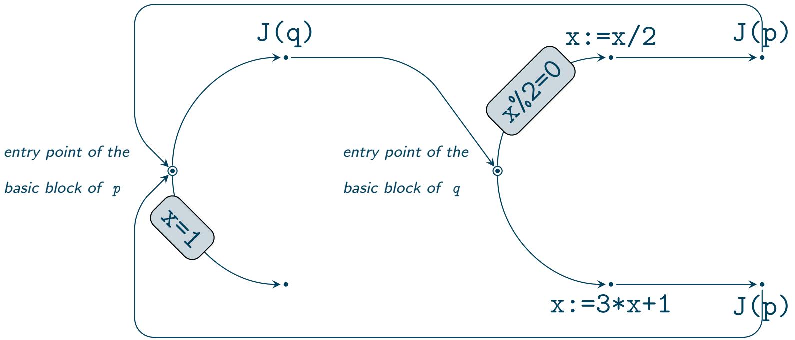

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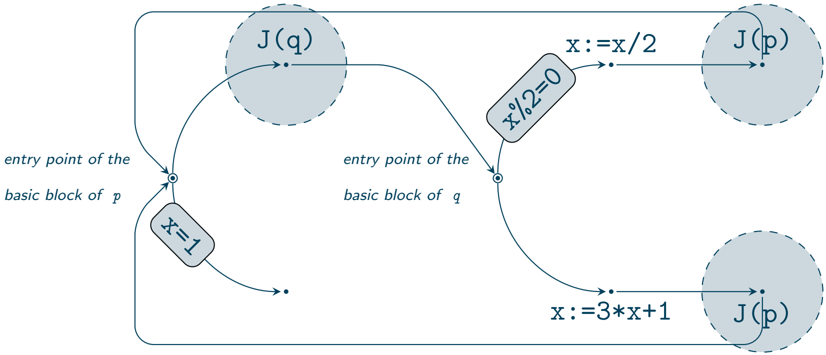

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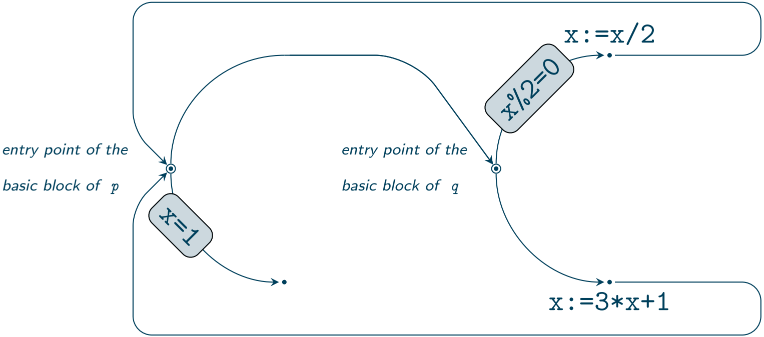

Reducing the Control Flow Graph

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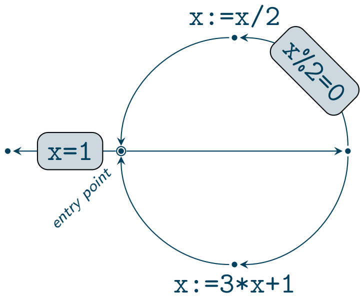

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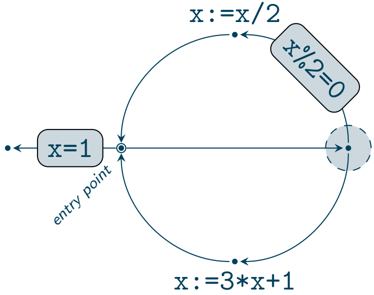

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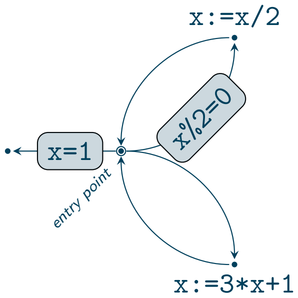

An execution trace on a control flow graph

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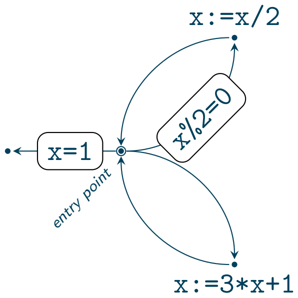

An execution trace on a control flow graph

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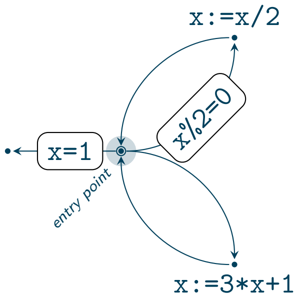
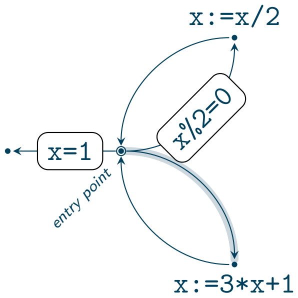


the current value of x is 7

An execution trace on a control flow graph

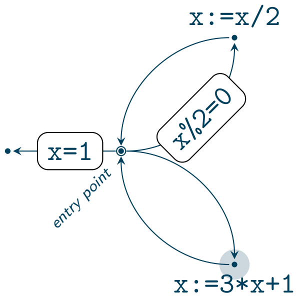
of the Hasse-Syracuse algorithm



the current value of x is 7

An execution trace on a control flow graph

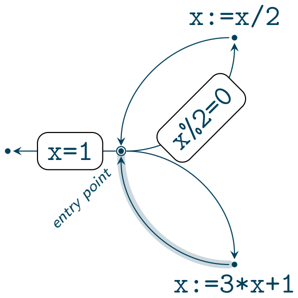
of the Hasse-Syracuse algorithm



the current value of `x` is 22

An execution trace on a control flow graph

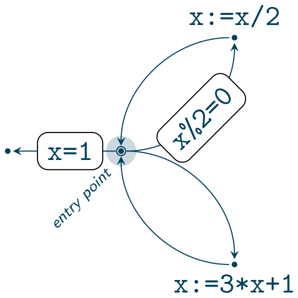
of the Hasse-Syracuse algorithm



the current value of x is 22

An execution trace on a control flow graph

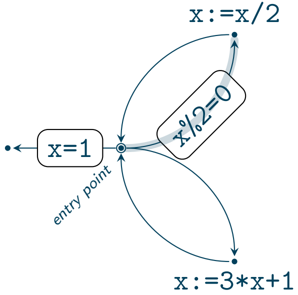
of the Hasse-Syracuse algorithm



the current value of x is 22

An execution trace on a control flow graph

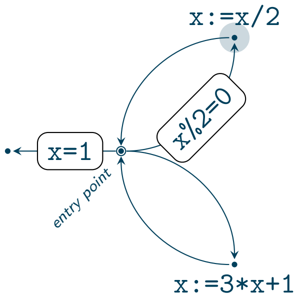
of the Hasse-Syracuse algorithm



the current value of `x` is 22

An execution trace on a control flow graph

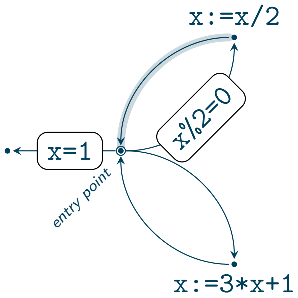
of the Hasse-Syracuse algorithm



the current value of x is 11

An execution trace on a control flow graph

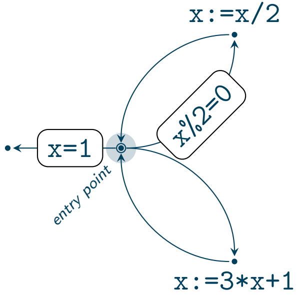
of the Hasse-Syracuse algorithm



the current value of x is 11

An execution trace on a control flow graph

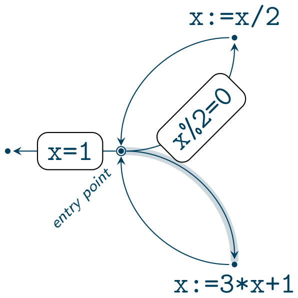
of the Hasse-Syracuse algorithm



the current value of x is 11

An execution trace on a control flow graph

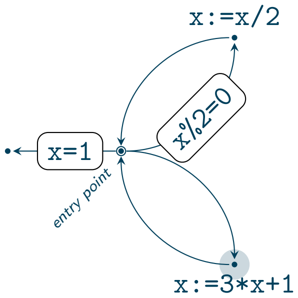
of the Hasse-Syracuse algorithm



the current value of `x` is 11

An execution trace on a control flow graph

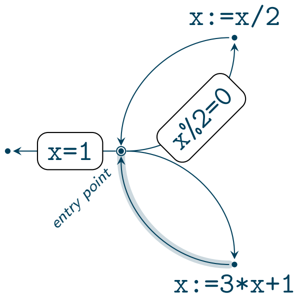
of the Hasse-Syracuse algorithm



the current value of x is 34

An execution trace on a control flow graph

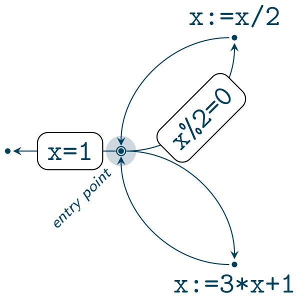
of the Hasse-Syracuse algorithm



the current value of `x` is 34

An execution trace on a control flow graph

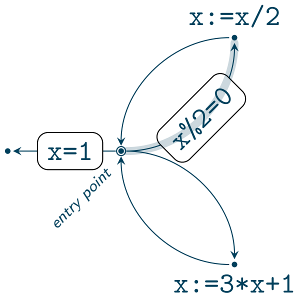
of the Hasse-Syracuse algorithm



the current value of `x` is 34

An execution trace on a control flow graph

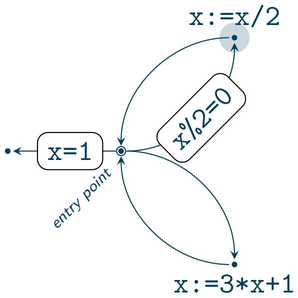
of the Hasse-Syracuse algorithm



the current value of x is 34

An execution trace on a control flow graph

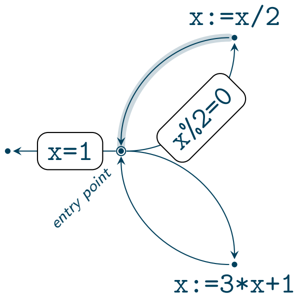
of the Hasse-Syracuse algorithm



the current value of x is 17

An execution trace on a control flow graph

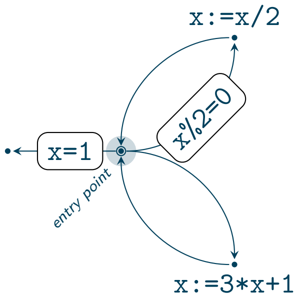
of the Hasse-Syracuse algorithm



the current value of x is 17

An execution trace on a control flow graph

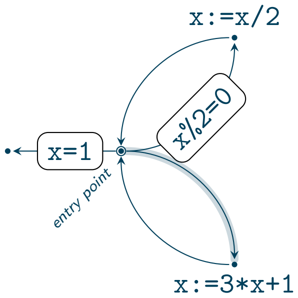
of the Hasse-Syracuse algorithm



the current value of `x` is 17

An execution trace on a control flow graph

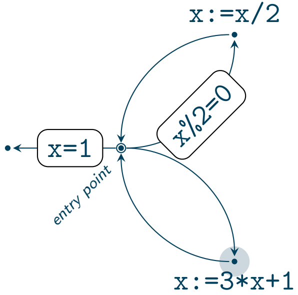
of the Hasse-Syracuse algorithm



the current value of x is 17

An execution trace on a control flow graph

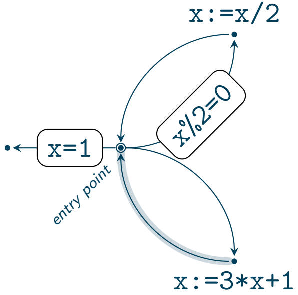
of the Hasse-Syracuse algorithm



the current value of x is 52

An execution trace on a control flow graph

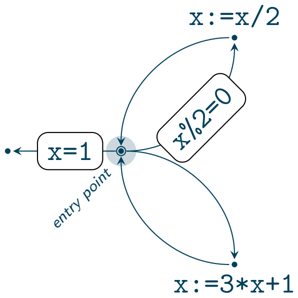
of the Hasse-Syracuse algorithm



the current value of x is 52

An execution trace on a control flow graph

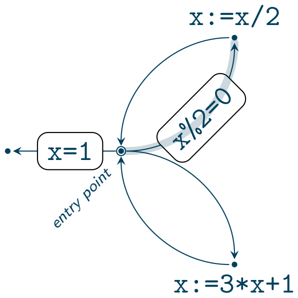
of the Hasse-Syracuse algorithm



the current value of `x` is 52

An execution trace on a control flow graph

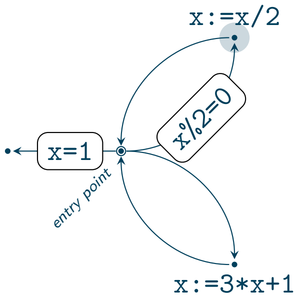
of the Hasse-Syracuse algorithm



the current value of `x` is 52

An execution trace on a control flow graph

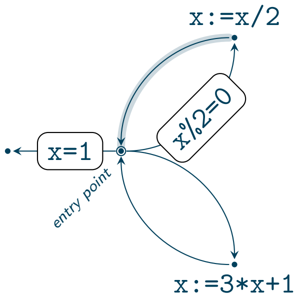
of the Hasse-Syracuse algorithm



the current value of x is 26

An execution trace on a control flow graph

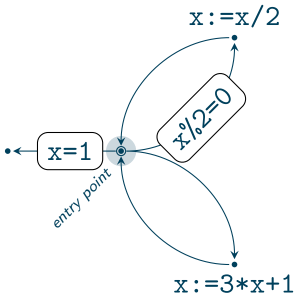
of the Hasse-Syracuse algorithm



the current value of x is 26

An execution trace on a control flow graph

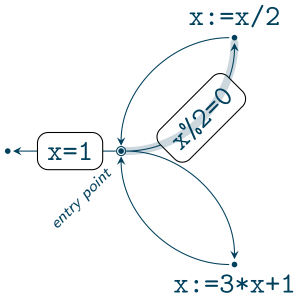
of the Hasse-Syracuse algorithm



the current value of `x` is 26

An execution trace on a control flow graph

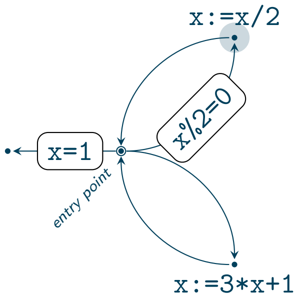
of the Hasse-Syracuse algorithm



the current value of x is 26

An execution trace on a control flow graph

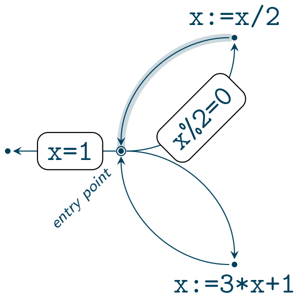
of the Hasse-Syracuse algorithm



the current value of x is 13

An execution trace on a control flow graph

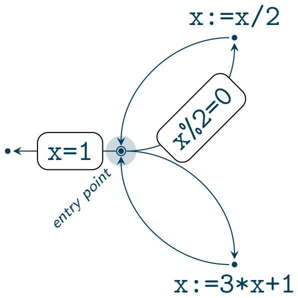
of the Hasse-Syracuse algorithm



the current value of x is 13

An execution trace on a control flow graph

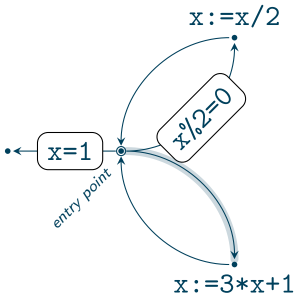
of the Hasse-Syracuse algorithm



the current value of x is 13

An execution trace on a control flow graph

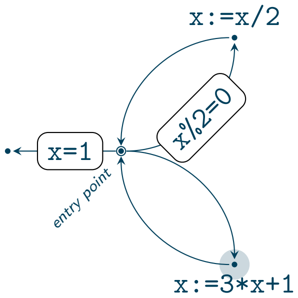
of the Hasse-Syracuse algorithm



the current value of x is 13

An execution trace on a control flow graph

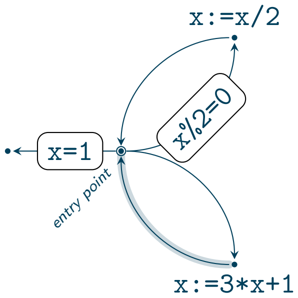
of the Hasse-Syracuse algorithm



the current value of x is 40

An execution trace on a control flow graph

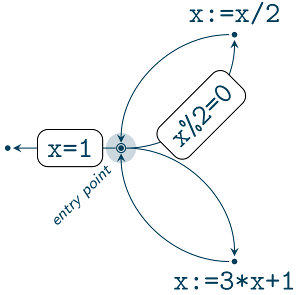
of the Hasse-Syracuse algorithm



the current value of `x` is 40

An execution trace on a control flow graph

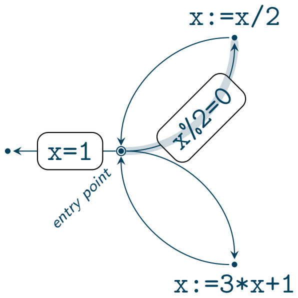
of the Hasse-Syracuse algorithm



the current value of `x` is 40

An execution trace on a control flow graph

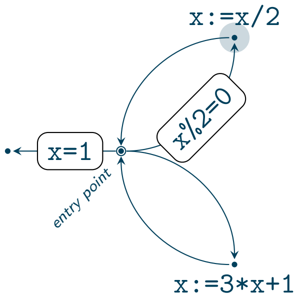
of the Hasse-Syracuse algorithm



the current value of `x` is 40

An execution trace on a control flow graph

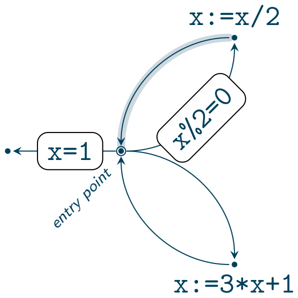
of the Hasse-Syracuse algorithm



the current value of `x` is 20

An execution trace on a control flow graph

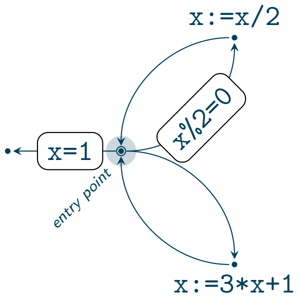
of the Hasse-Syracuse algorithm



the current value of x is 20

An execution trace on a control flow graph

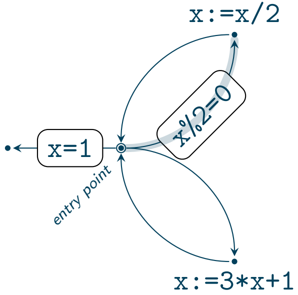
of the Hasse-Syracuse algorithm



the current value of x is 20

An execution trace on a control flow graph

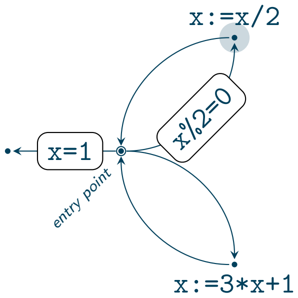
of the Hasse-Syracuse algorithm



the current value of x is 20

An execution trace on a control flow graph

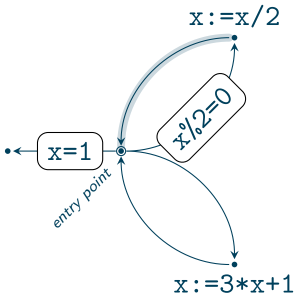
of the Hasse-Syracuse algorithm



the current value of x is 10

An execution trace on a control flow graph

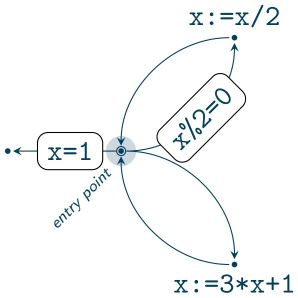
of the Hasse-Syracuse algorithm



the current value of `x` is 10

An execution trace on a control flow graph

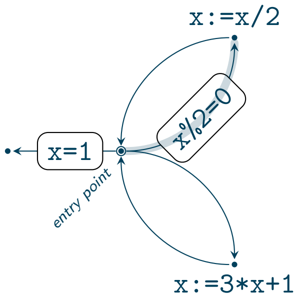
of the Hasse-Syracuse algorithm



the current value of `x` is 10

An execution trace on a control flow graph

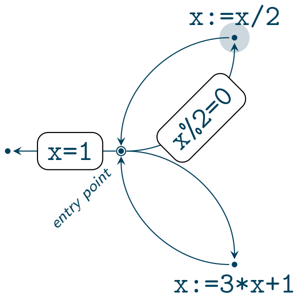
of the Hasse-Syracuse algorithm



the current value of x is 10

An execution trace on a control flow graph

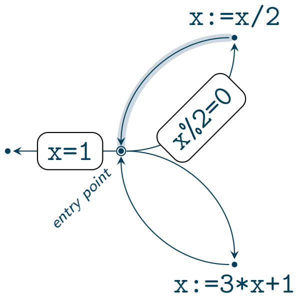
of the Hasse-Syracuse algorithm



the current value of `x` is 5

An execution trace on a control flow graph

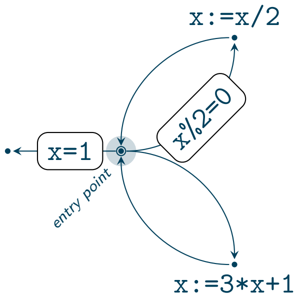
of the Hasse-Syracuse algorithm



the current value of `x` is 5

An execution trace on a control flow graph

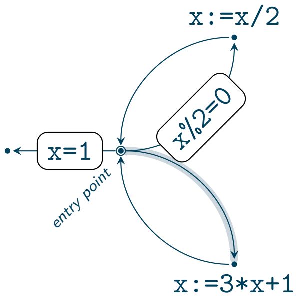
of the Hasse-Syracuse algorithm



the current value of `x` is 5

An execution trace on a control flow graph

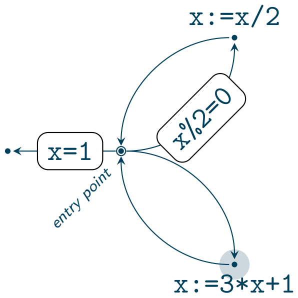
of the Hasse-Syracuse algorithm



the current value of `x` is 5

An execution trace on a control flow graph

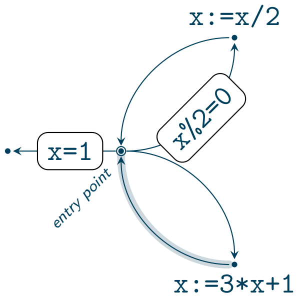
of the Hasse-Syracuse algorithm



the current value of `x` is 16

An execution trace on a control flow graph

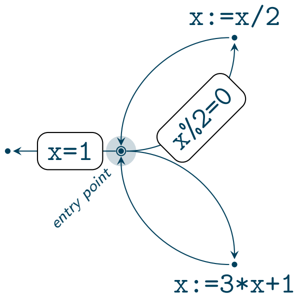
of the Hasse-Syracuse algorithm



the current value of `x` is 16

An execution trace on a control flow graph

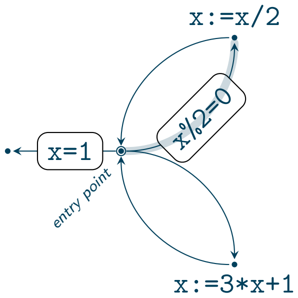
of the Hasse-Syracuse algorithm



the current value of x is 16

An execution trace on a control flow graph

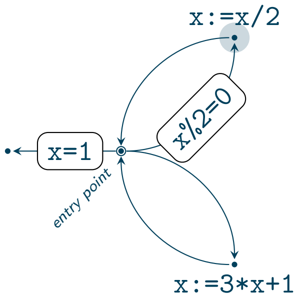
of the Hasse-Syracuse algorithm



the current value of `x` is 16

An execution trace on a control flow graph

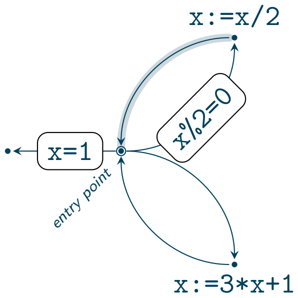
of the Hasse-Syracuse algorithm



the current value of `x` is 8

An execution trace on a control flow graph

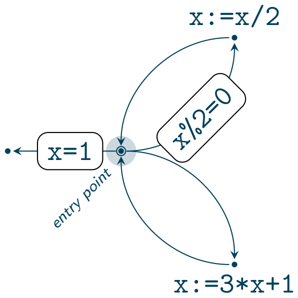
of the Hasse-Syracuse algorithm



the current value of `x` is 8

An execution trace on a control flow graph

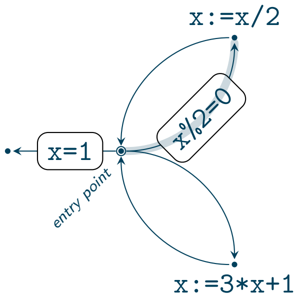
of the Hasse-Syracuse algorithm



the current value of x is 8

An execution trace on a control flow graph

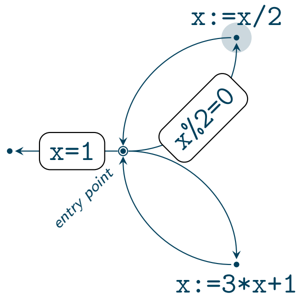
of the Hasse-Syracuse algorithm



the current value of `x` is 8

An execution trace on a control flow graph

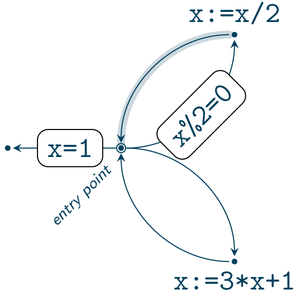
of the Hasse-Syracuse algorithm



the current value of x is 4

An execution trace on a control flow graph

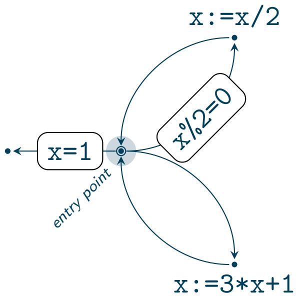
of the Hasse-Syracuse algorithm



the current value of `x` is 4

An execution trace on a control flow graph

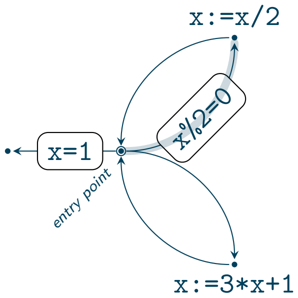
of the Hasse-Syracuse algorithm



the current value of x is 4

An execution trace on a control flow graph

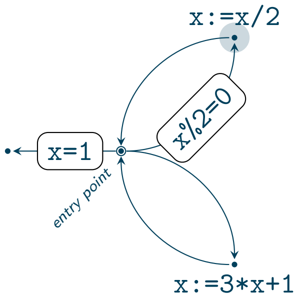
of the Hasse-Syracuse algorithm



the current value of `x` is 4

An execution trace on a control flow graph

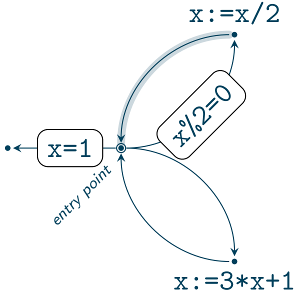
of the Hasse-Syracuse algorithm



the current value of `x` is 2

An execution trace on a control flow graph

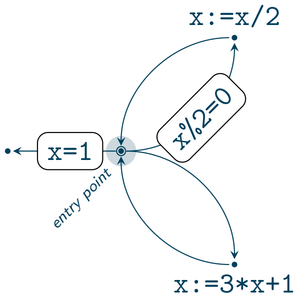
of the Hasse-Syracuse algorithm



the current value of `x` is 2

An execution trace on a control flow graph

of the Hasse-Syracuse algorithm



the current value of x is 2

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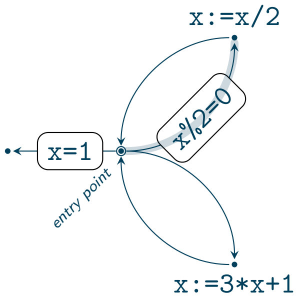

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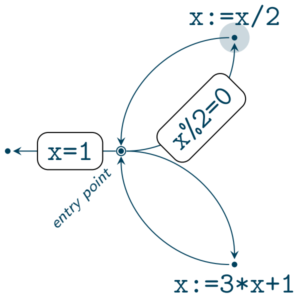

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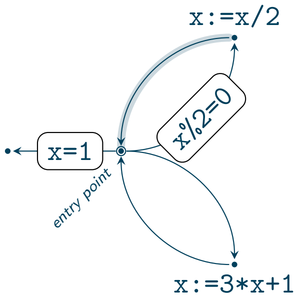

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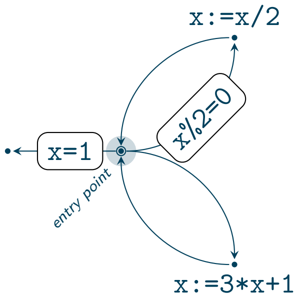

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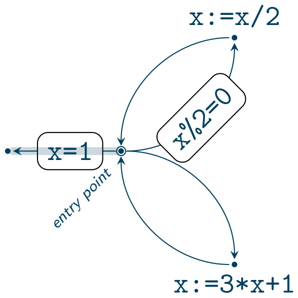

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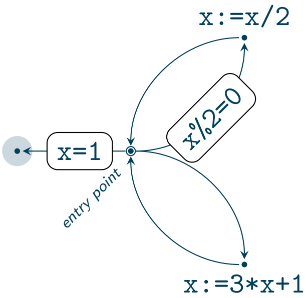

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Execution traces as paths over a control flow graph

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- The (infinite) collection of paths is entirely determined by the (finite) control flow graph

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One of the goal of the course it to provide such a structure for a large class of PAML programs.

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- The arity of resources cannot by changed at runtime
- There is no pointer arithmetics

Abstract Machine

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a conditional branching at vertex $v \in V$ is a mapping

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- The synchronisation primitives $P(s)$, $V(s)$, and $W(b)$ for $s \in \mathcal{S}$ and $b \in \mathcal{B}$

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The arrows are interpreted as intermediate positions of the instruction pointer so a point on a control flow graph is either a vertex or an arrow.

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- The tuple $(G_1, \ldots, G_n)$ of processes which are launched simultaneously at the beginning of each execution of the program.

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- A point of $(G_1, \ldots, G_n)$ is an n-tuple p whose i^{th} component, namely p_i, is a point of G_i.
- A multi-instruction is a partial map $\mu : \{1, \ldots, n\} \to \{\text{instructions}\}$.

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- for all $b \in \mathcal{B}$, $\mathrm{card}\{\mathrm{i} \in \mathrm{dom}(\mu) \mid \mu(\mathrm{i}) = \mathrm{W(b)}\} \notin \{1, \ldots, \alpha(\mathrm{b})\}$

Action of a multi-instruction on a state

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- For all $s \in \mathcal{S}$ the multiset $(\sigma \cdot \mu)(s)$, seen as a mapping from $\{1, \ldots, n\}$ to $\mathbb{N}$, is given by

$$i \quad \mapsto \quad \begin{cases} \sigma(s)(i) + 1 & \text{if } i \in \mathrm{dom}(\mu) \text{ and } \mu(i) = P(s) \\ \sigma(s)(i) - 1 & \text{if } i \in \mathrm{dom}(\mu) \text{ and } \mu(i) = V(s) \\ \sigma(s)(i) & \text{in all other cases} \end{cases}$$

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A sequence $\mu_0, \ldots, \mu_{q-1}$ of multi-intructions is said to be admissible at state σ when for all $k \in \{0, \ldots, q-1\}$ the multi-instruction μ_k is admissible at state $\sigma \cdot \mu_0 \cdots \mu_{k-1}$.

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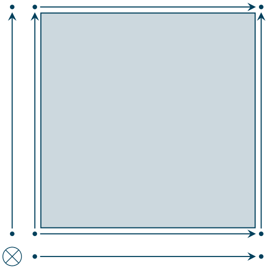
- $\mu_k(i) = \lambda_i(\gamma_i(k+1))$ for all $k \in \{0, \ldots, q-1\}$ and all $i \in \mathrm{dom}(\mu_k)$

Discrete paths are “continuous”



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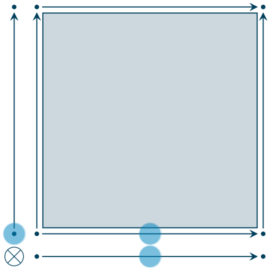

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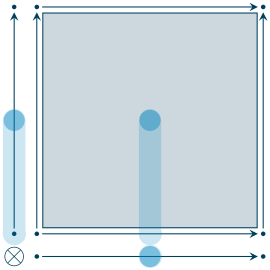

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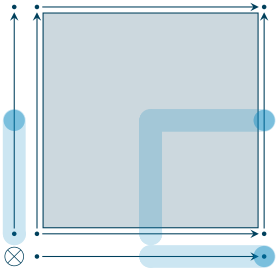
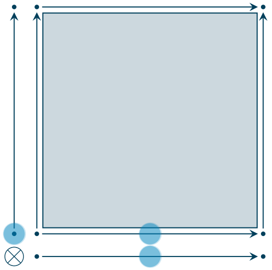


Discrete paths are "continuous"



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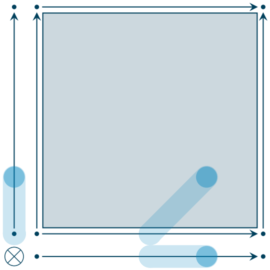

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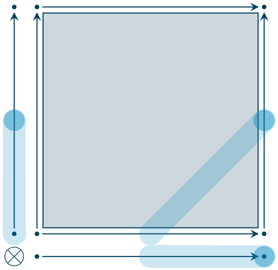
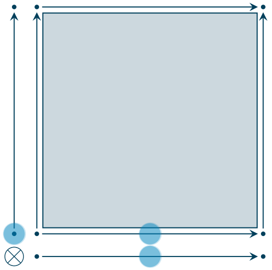


Discrete paths are “continuous”



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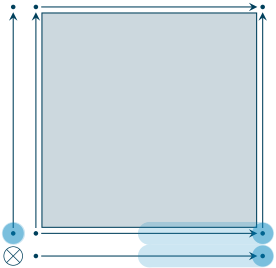
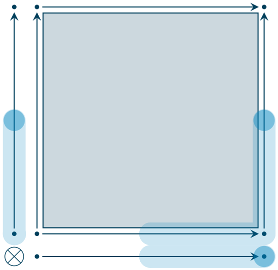
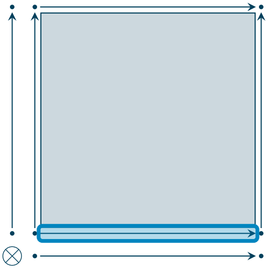


Discrete paths are "continuous"



Discrete paths are “continuous”



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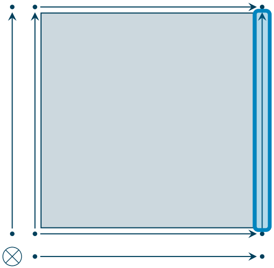

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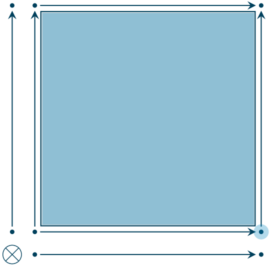

Admissible paths and execution traces

Admissible paths and execution traces

Given σ a state of the program, a directed path is said to be admissible at σ when so is its associated sequence of multi-instructions at state σ. In this case we define the action of γ on the right of σ as follows.

$$\sigma \cdot \gamma \quad = \quad \sigma \cdot \mu_0 \cdots \mu_{q-1}$$

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An admissible path is an execution trace when all the conditional branchings met along the way are respected: for all $k \in \{0, \ldots, q-2\}$ and all $i \in \{1, \ldots, n\}$ such that $\mu_k(i)$, which is by definition $\lambda_i(\gamma_i(k+1))$, is a branching, we have

$$\big(\mu_k(i)\big)(\sigma \cdot \mu_0 \cdots \mu_{k-1}) \quad = \quad \gamma_i(k+2)$$

Concurrent access

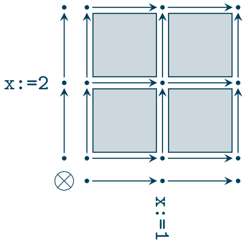
```
var x = 0
```

```
proc p = x:=1

proc q = x:=2
```

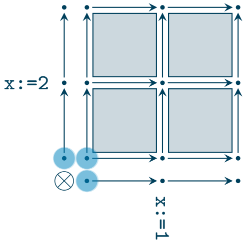
```
init p q
```

Admissible execution trace



the value of x is 0

Admissible execution trace



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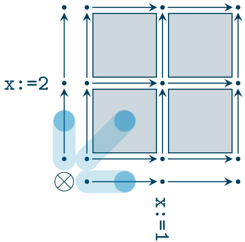

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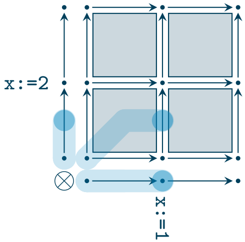
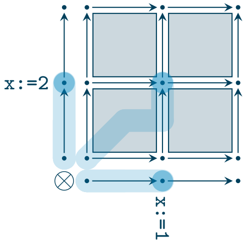


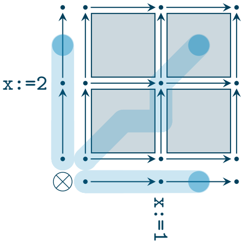
the value of x is 1

Admissible execution trace



the value of x is 2

Admissible execution trace



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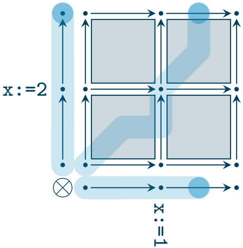
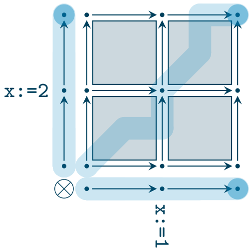


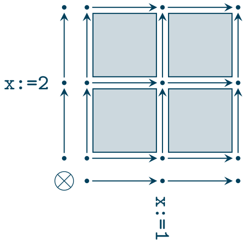
the value of x is 2

Admissible execution trace



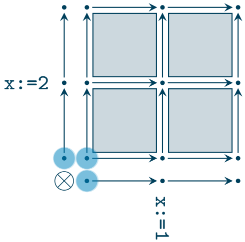
the value of x is 2

Not admissible execution trace



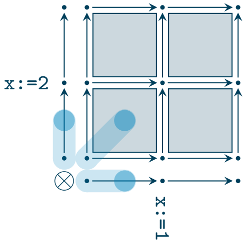
the value of x is 0

Not admissible execution trace



the value of `x` is 0

Not admissible execution trace



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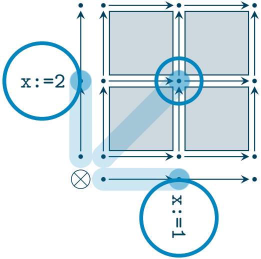
Not admissible execution trace



the value of x is ?

Lack of resources

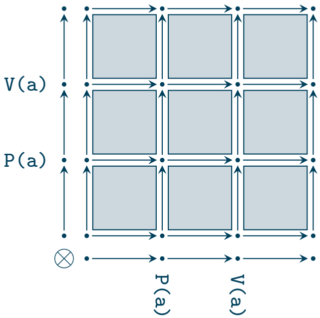
```
sem 1 a
```

```
proc p = P(a);V(a)
```

```
init 2p
```

Admissible concurrent execution trace

`sem 1 a`



Admissible concurrent execution trace

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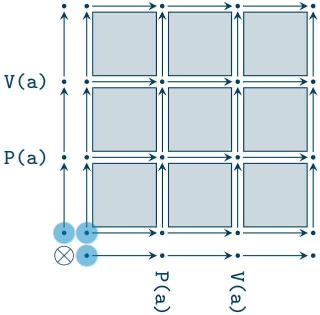

Admissible concurrent execution trace

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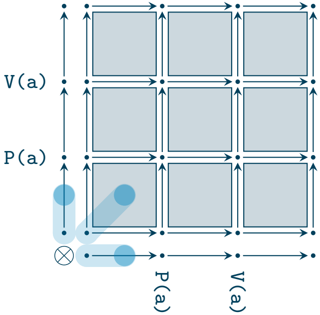

Admissible concurrent execution trace

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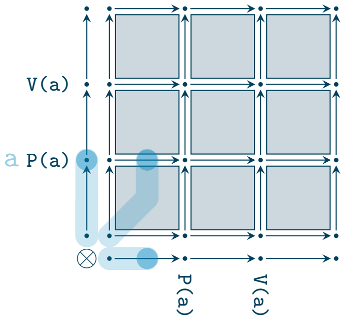
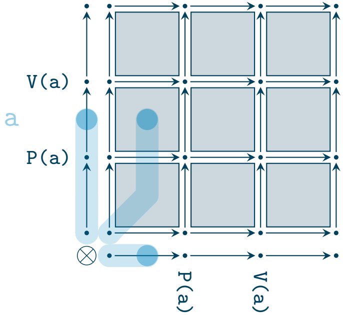


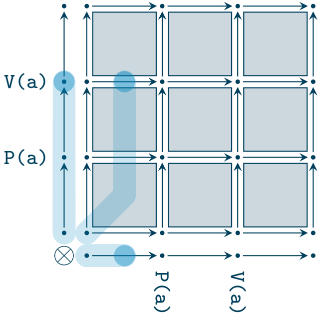
Admissible concurrent execution trace

`sem 1 a`



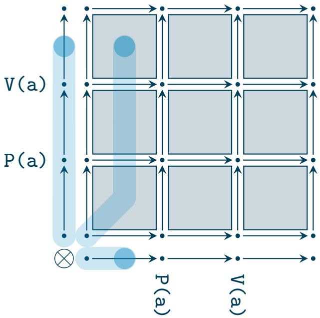
Admissible concurrent execution trace

`sem 1 a`



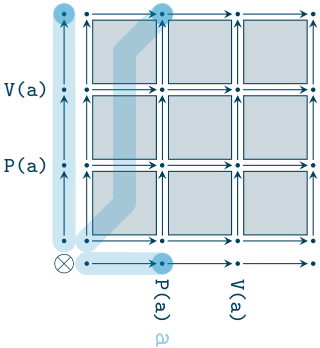
Admissible concurrent execution trace

`sem 1 a`



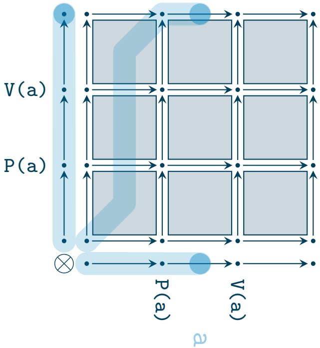
Admissible concurrent execution trace

`sem 1 a`



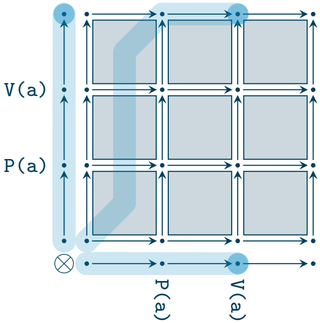
Admissible concurrent execution trace

`sem 1 a`



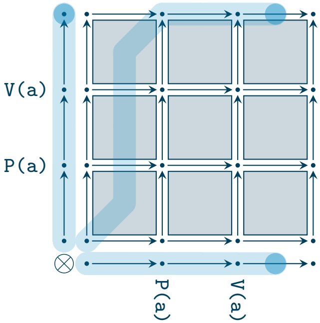
Admissible concurrent execution trace

`sem 1 a`



Admissible concurrent execution trace

`sem 1 a`



Admissible concurrent execution trace

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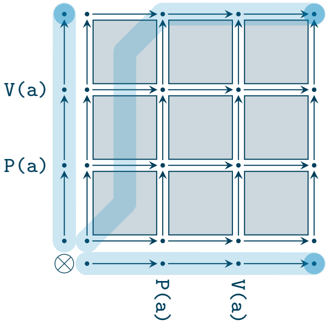
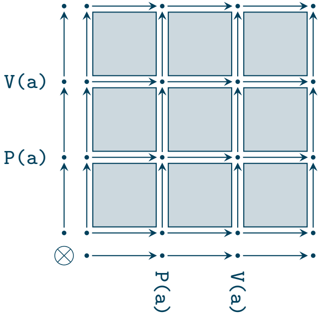


Not admissible concurrent execution trace

`sem 1 a`



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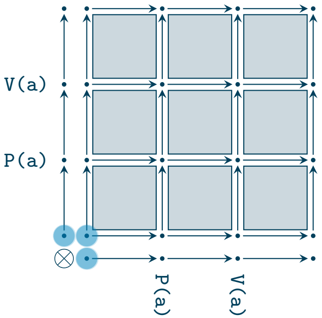

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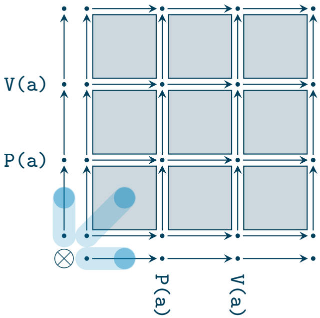

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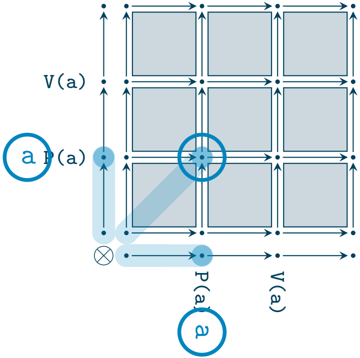


Synchronisation

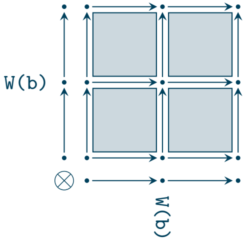
```
sync 1 b
```

```
proc p = W(b)
```

```
init 2p
```

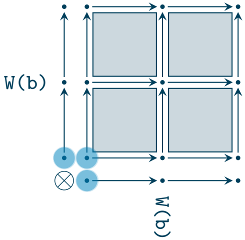
Concurrent execution trace

`sync 1 b`



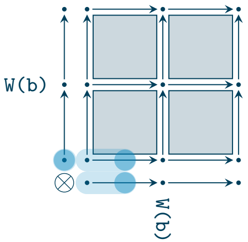
Concurrent execution trace

`sync 1 b`



Concurrent execution trace

`sync 1 b`



Concurrent execution trace

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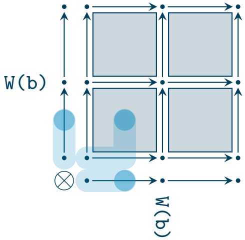

Concurrent execution trace

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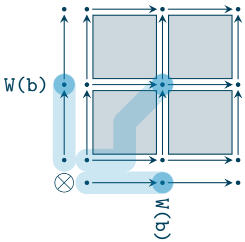

Concurrent execution trace

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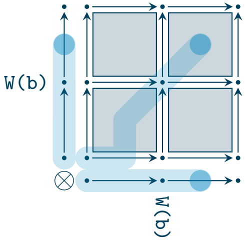
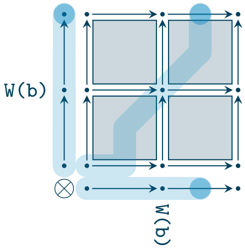


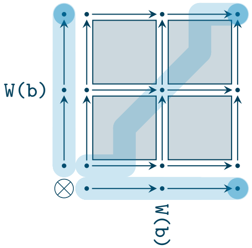
Concurrent execution trace

`sync 1 b`



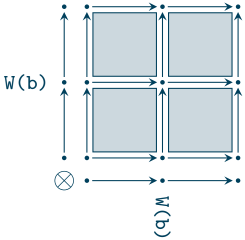
Concurrent execution trace

`sync 1 b`



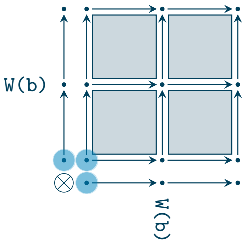
Not admissible concurrent execution trace

`sync 1 b`



Not admissible concurrent execution trace

`sync 1 b`



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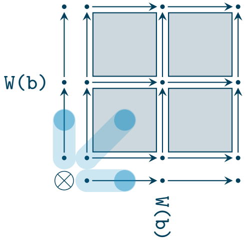

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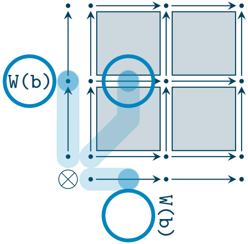

Next goal

Encode admissibility into a model.

CONSERVATIVE PROGRAMS

Potential Functions

The potential functions of processes and programs

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A program $\Pi = (G_1, \ldots, G_n)$ is conservative when for all directed paths starting at the origin, the amount of semaphores held by the program at the end of the path only depends on its arrival point.

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$$\partial^{+}\gamma = \partial^{+}\gamma' \quad \Rightarrow \quad \sigma \cdot \gamma|_{\mathcal{S}} = \sigma \cdot \gamma'|_{\mathcal{S}}$$

In particular, the program Π comes with a potential function

$$F_{\Pi} \ : \ \{\text{semaphores}\} \times \{\text{points}\} \to \mathbb{N} \quad \cong \quad \{\text{points}\} \to \{\text{multisets over } \mathcal{S}\}$$

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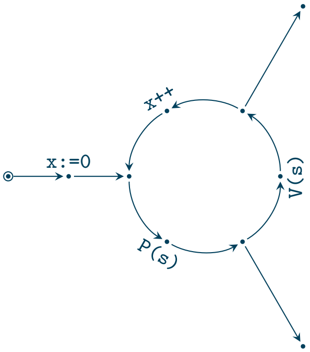
Proposition: The program Π is conservative if and only if so are its processes $G_1, \ldots, G_n$ and its potential function is given by

$$F_\Pi(p_1, \ldots, p_n) \quad = \quad \sum_{k=1}^{n} F_{G_k}(p_k)$$

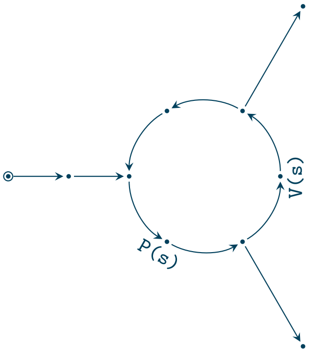
Conservative process

example



Conservative process

example



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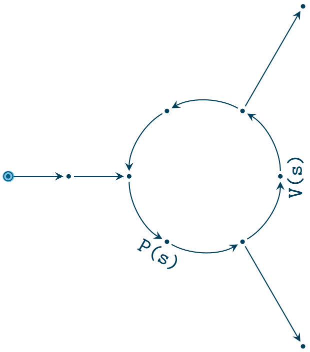

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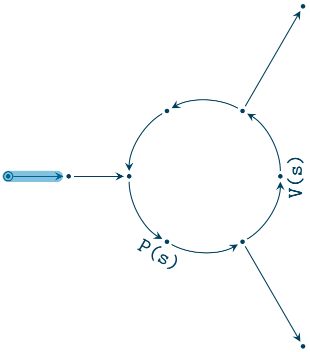
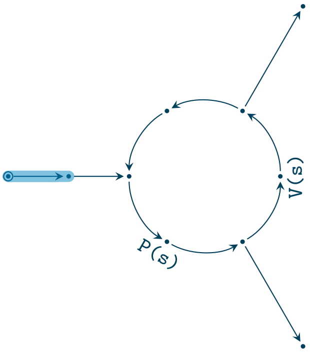


Conservative process

example



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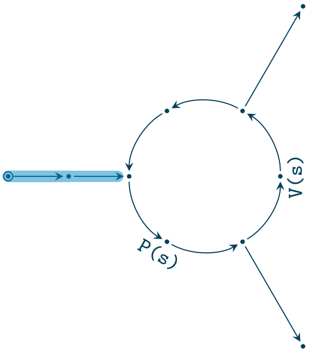

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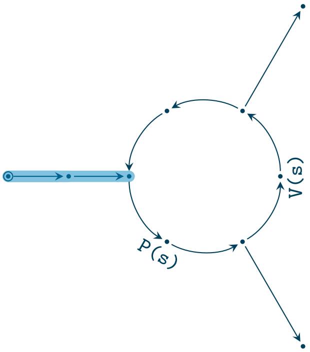

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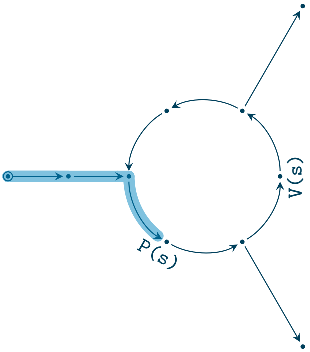

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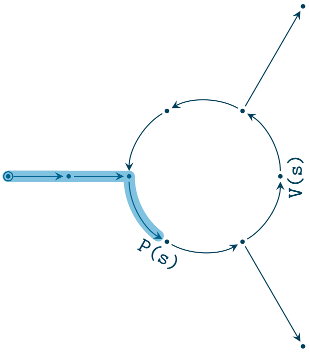

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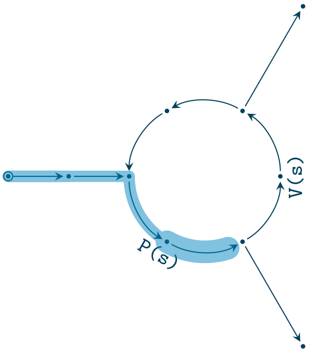

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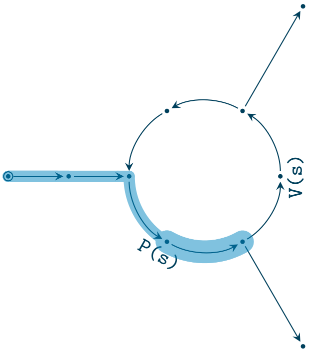

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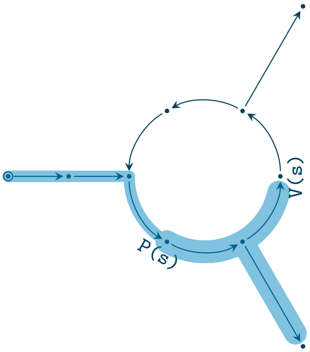

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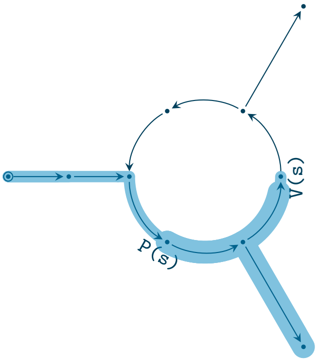

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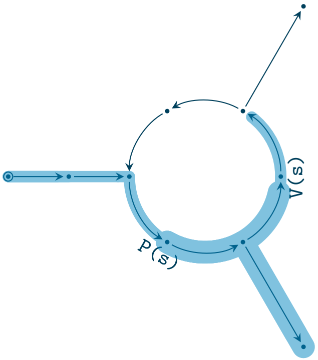

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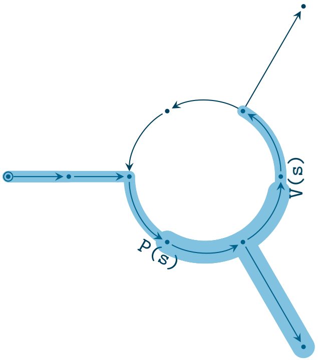

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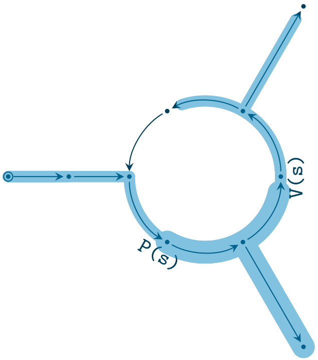

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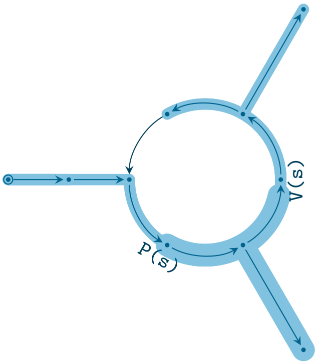

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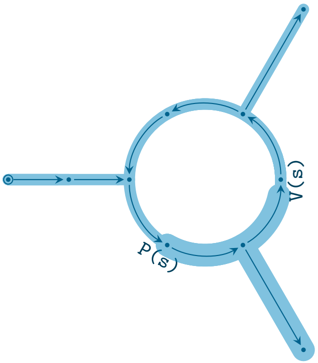

Not conservative process

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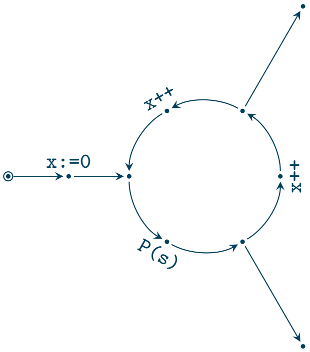

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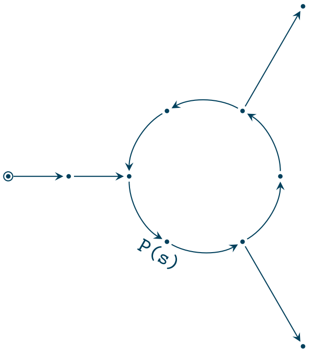

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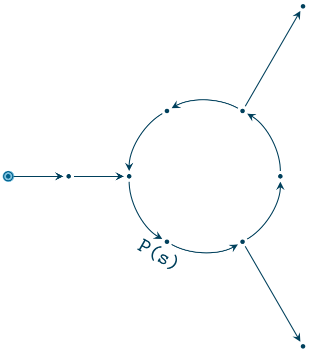

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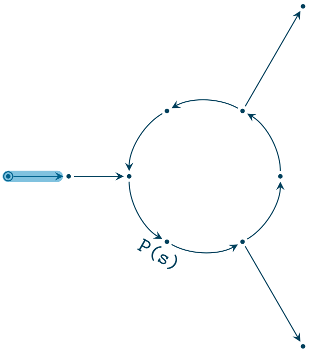

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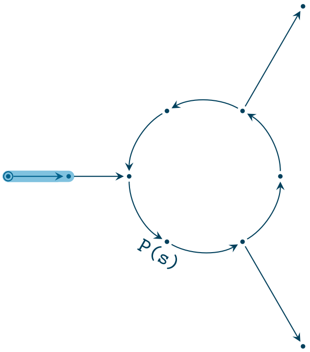

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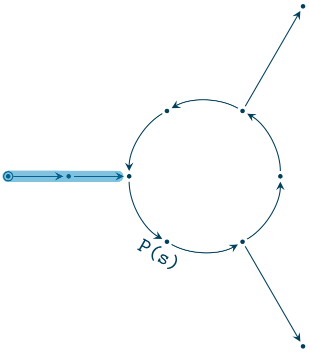

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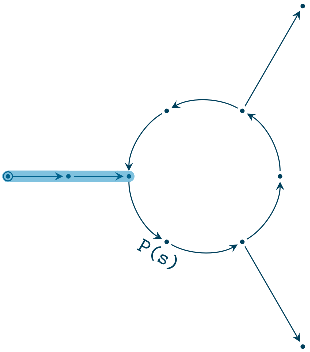

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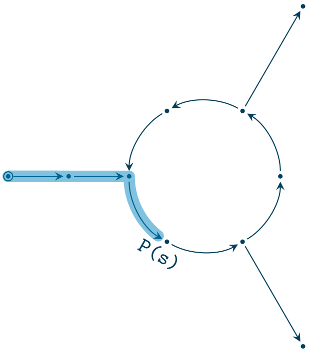

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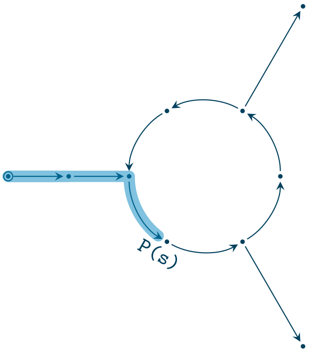

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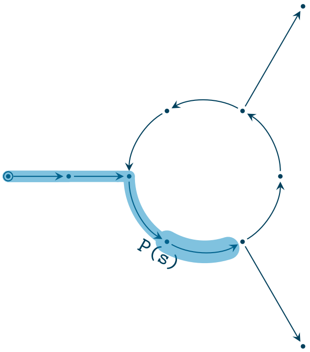

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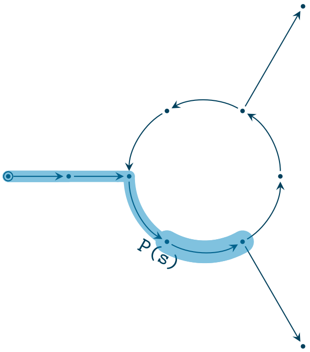

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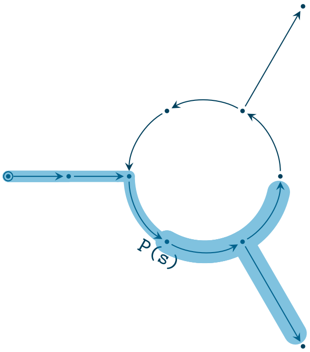

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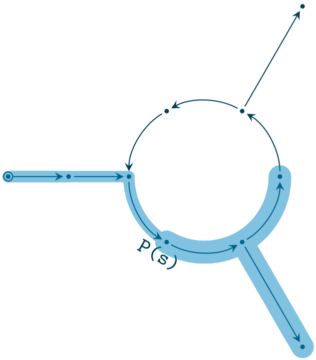

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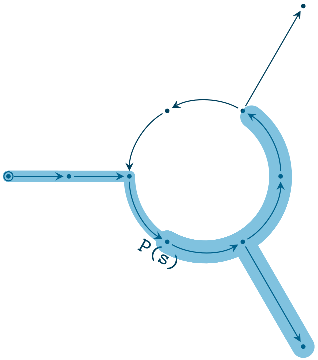

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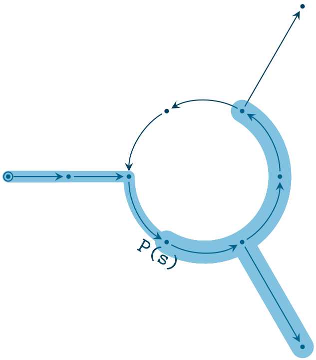

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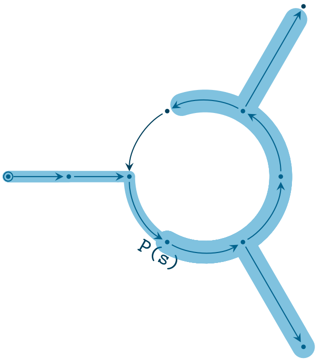

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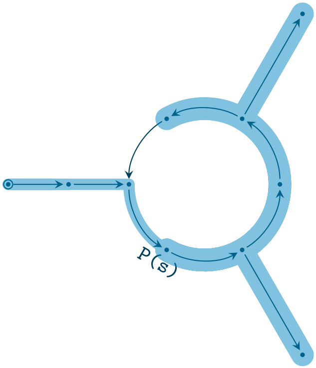

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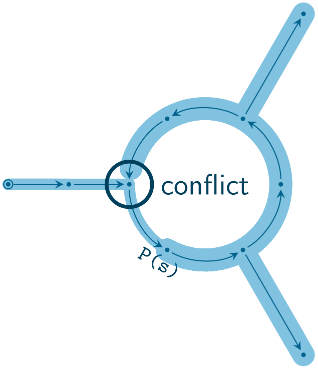

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Discrete Models

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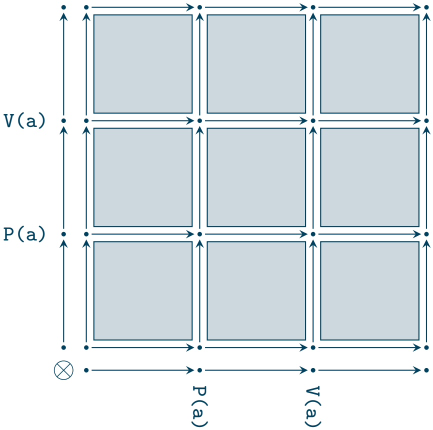
The discrete model is the complement of its forbidden set.

$$\{\text{points of the program}\} \setminus \{\text{forbidden points}\}$$

Discrete model

`sem 1 a`



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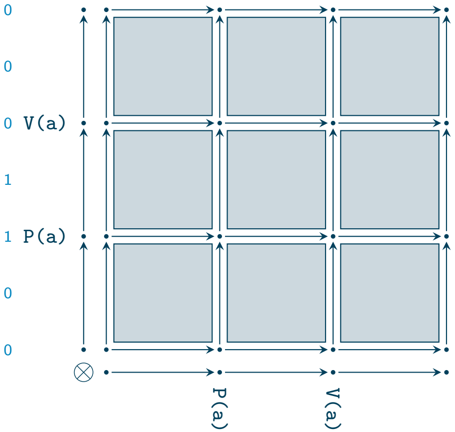

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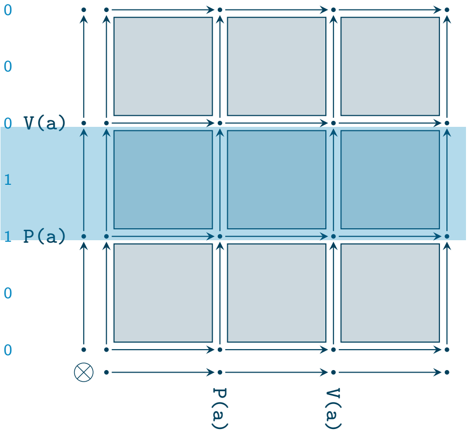

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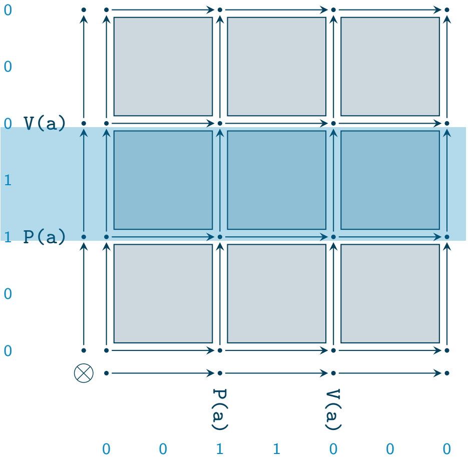

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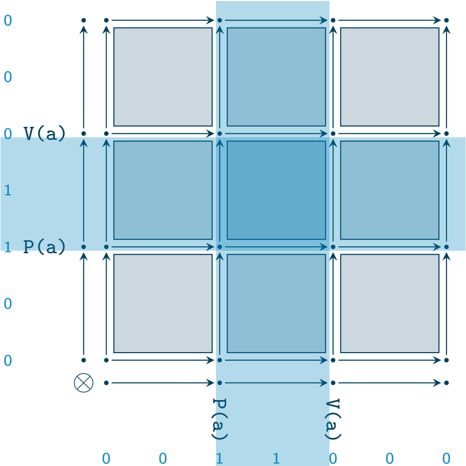

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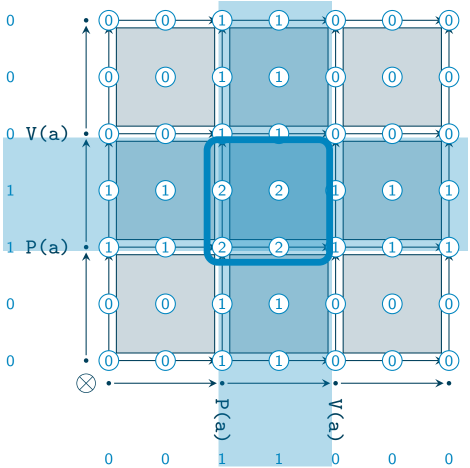

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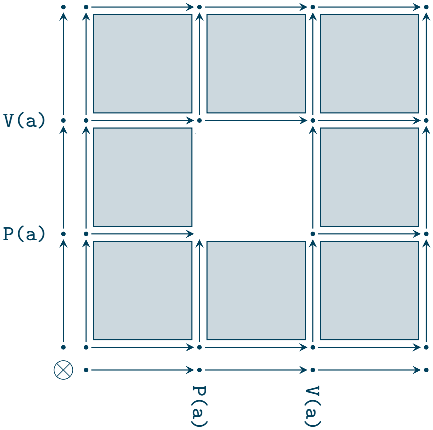

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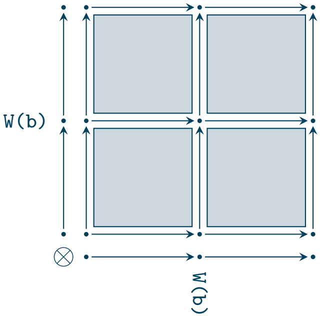

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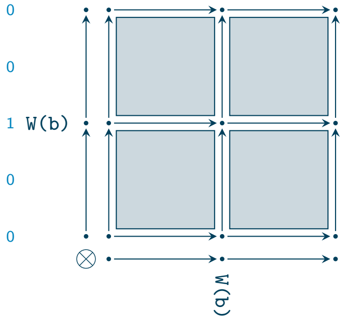

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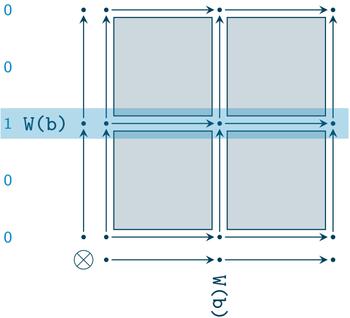

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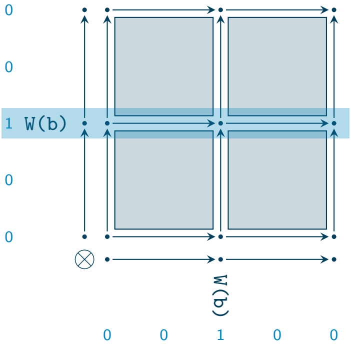

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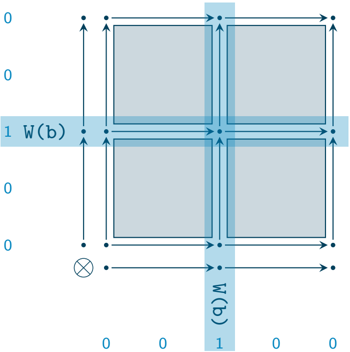

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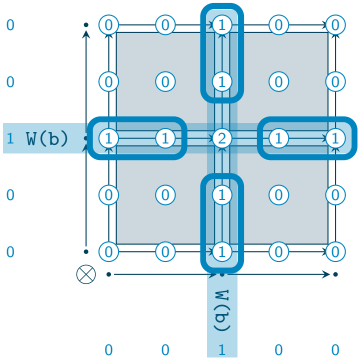

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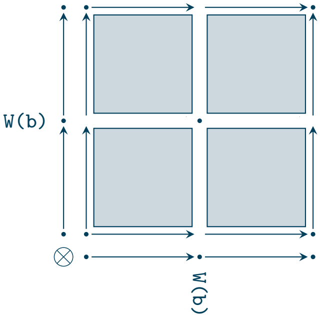

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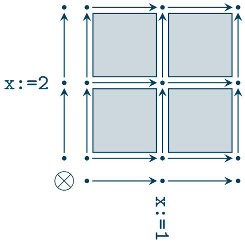
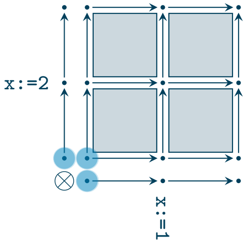
- Completeness: for each admissible path which meets a forbidden point there exists a directed path which avoids them and such that both directed paths induce the same sequence of multi-instructions.

Admissible execution trace



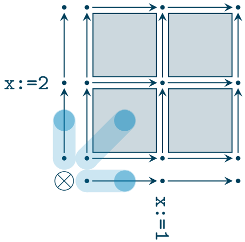
the value of x is 0

Admissible execution trace



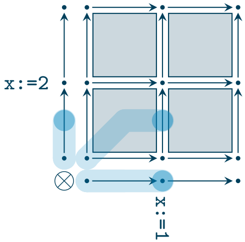
the value of x is 0

Admissible execution trace



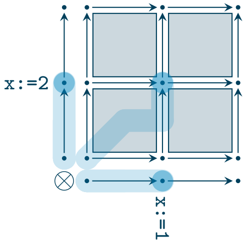
the value of $\mathtt{x}$ is 0

Admissible execution trace



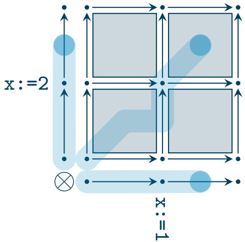
the value of x is 1

Admissible execution trace



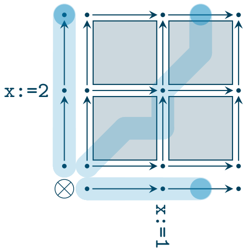
the value of x is 2

Admissible execution trace



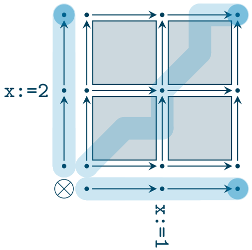
the value of x is 2

Admissible execution trace



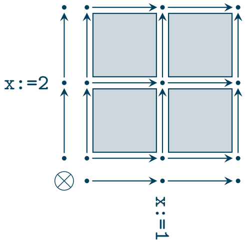
the value of x is 2

Admissible execution trace



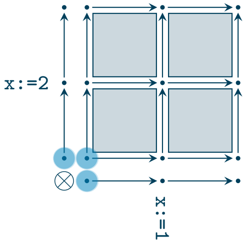
the value of $\mathtt{x}$ is 2

Admissible execution trace avoiding forbidden points



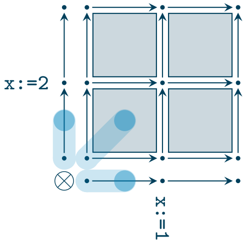
the value of x is 0

Admissible execution trace avoiding forbidden points



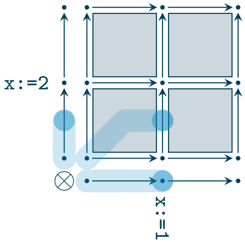
the value of x is 0

Admissible execution trace avoiding forbidden points



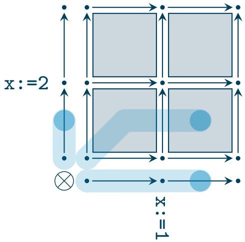
the value of x is 0

Admissible execution trace avoiding forbidden points



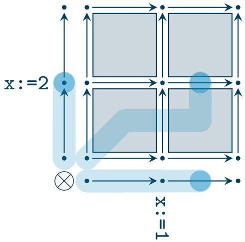
the value of x is 1

Admissible execution trace avoiding forbidden points



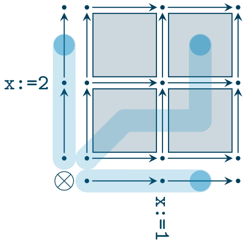
the value of x is 1

Admissible execution trace avoiding forbidden points



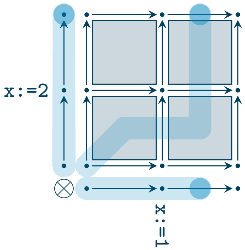
the value of x is 2

Admissible execution trace avoiding forbidden points



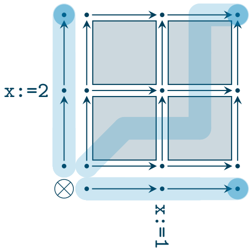
the value of x is 2

Admissible execution trace avoiding forbidden points



the value of x is 2

Admissible execution trace avoiding forbidden points



the value of $\mathtt{x}$ is 2

Replacement