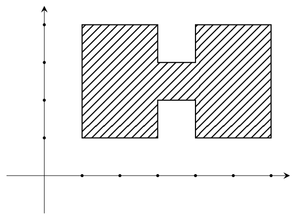
2.3.1: Concurrency

Friday, the 7^{th} of march 2025
duration: 3h

Exercise 1: Denote the following (hashed) isothetic region by X:



1) Write a program whose forbidden region is X.

2) Draw the deadlock attractor of the complement of X.

3a) What are the maximal blocks of the forbidden region?

3b) How many maximal blocks are there in the *complement* of forbidden region?

4) Give the prime factorization of the geometric model
(i.e. the complement of the hashed region).

5) Draw the category of components of the geometric model.

Exercise 2:

1) Let P be a program made of two processes `x` and `y`. Assuming that `x` and `y` are observationally independent and that the sequence of multi-instruction

x:	···	I_1	–	···
y:	···	–	I_2	···

is an execution trace of P, give another execution trace of P.

Consider the following programs:

```
sem  1 a
sync 1 b
proc x = W(b);P(a);V(a)
proc y = P(a);V(a);W(b)
init x y
```

```
sem  1 a
sync 1 b
proc x = P(a);W(b);V(a)
proc y = P(a);V(a);W(b)
init x y
```

2) Draw the geometric models of these programs.

3) Prove that in both programs, the processes `x` and `y` are not model independent.

For each of the two programs above:

4a) What are the execution traces?

4b) Are the processes `x` and `y` observationally independent? (explain)

Turn the page →

Exercise 3: We denote by $\#S$ the cardinal of a set S. For every nonempty subset H of Σ^n with $n \in \mathbb{N}$ (i.e. H homogeneous language of dimension n on Σ), we put $\dim(H) = n$, $\bar{H} = \Sigma^n \backslash H$, and we denote by $[H]$ the equivalence class of H under the action of $\mathfrak{S}_n$ the permutation group of $\{1, \ldots, n\}$. For $w \in \Sigma^n$ and $i \in \{1, \ldots, n\}$ we denote by w_i the i^{th} letter of w. The free commutative monoid of (equivalence classes of) homogeneous languages over Σ is denoted by $\mathcal{H}(\Sigma)$.

Let $X \in \mathcal{H}(\Sigma)$ and let H, H' be homogeneous languages of dimension n on Σ.

1) Prove that if $\#X = \#\Sigma^{\dim(X)}$ then X is not prime.

2a) Suppose that $\Sigma = \{0, 1\}$ and for every $i \in \{1, \ldots, n\}$ we have

$$1 \leqslant \sum_{w \in H} w_i \leqslant \#H - 1$$

Prove that if $\#H$ is prime in $(\mathbb{N}, \times, 1)$ then $[H]$ is prime in $\mathcal{H}(\{0, 1\})$.

2b) Decompose the following elements of $\mathcal{H}(\{\mathtt{0},\mathtt{1}\})$ into prime factors (explain):

010101	0111000010101100100110010010001010101001001001
100101	1101101110100011101010011110100110110101010011
011101	1001100100001010001001010010010100011111101101
010010	0001111010110101110001101010101010010001111110
011010	1110000111100011100001100111001111011110011011
101101	1001011110001001111010110001101100101110101010
100010	1110011001111001010111101000011101011101000111
101010	

3) Find homogeneous languages A, A', B, and B' such that $[A] = [A']$, $[B] = [B']$, but $[A \cup B] \neq [A' \cup B']$. Same question with $[A \cap B] \neq [A' \cap B']$.

4a) Given $\sigma \in \mathfrak{S}_n$, prove that $\sigma\bar{H} = \overline{\sigma H}$.

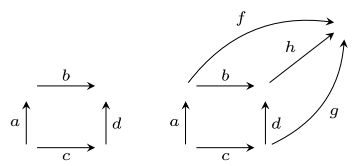
4b) Prove that if H and H' are equivalent, then so are $\bar{H}$ and $\overline{H'}$.

Following 4b) if $X = [H]$ then we define $\bar{X}$ as $[\bar{H}]$.

4c) Compute $\bar{X}$ for $X = [\{\mathtt{01}\}]$ in $\mathcal{H}(\{0, 1\})$ and in $\mathcal{H}(\{0, 1, 2\})$.

4d) Suppose that $\#\Sigma \geqslant 2$. Prove that for every $X \in \mathcal{H}(\Sigma)$, if $\#X = 1$ then $\bar{X}$ is prime in $\mathcal{H}(\Sigma)$.

Exercise 4: Let X be the pospace $[-1, 1]^3 \setminus]-1, 1[^3$ (i.e. the boundary of a cube) with the standard product order. We denote the fundamental category of X by $\vec{\pi}_1(X)$. A square in $\vec{\pi}_1(X)$ is a diagram of morphisms of $\vec{\pi}_1(X)$ as follows (left):



The square is said to be *commutative* when $b \circ a = d \circ c$. An *equalizer* of the square is a morphism e (starting at the upper corner of the square) such that $e \circ b \circ a = e \circ d \circ c$. The square is a *pushout* when it is commutative and for every f, g satisfying $f \circ a = g \circ c$ there exists a unique h such that $f = h \circ b$ and $g = h \circ d$.

1) Give a square in $\vec{\pi}_1(X)$ that is not commutative but which has an equalizer. (make a drawing and explain)

2) Give a commutative square in $\vec{\pi}_1(X)$ that is not a pushout. (make a drawing and explain)