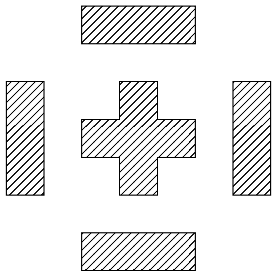
2.3.1: Concurrency

Friday, the 8^{th} of march 2024
duration: 3h

Exercise 1: Denote the following (hashed) isothetic region by X:



1) Write a program whose forbidden region is X.

2) Draw the deadlock attractor of the complement of X.

3a) What are the maximal blocks of the forbidden region?

3b) How many maximal blocks are there in the *complement* of forbidden region?

4) Give the prime factorization of the geometric model
(i.e. the complement of the hashed region).

5) Draw the category of components of the geometric model.

Exercise 2: Consider the following programs:

```
sync 1 b              sync 2 b              sync 1 b
proc x = W(b)         proc x = W(b)         sem 1 a
init 2x               init 2x               proc y = P(a);W(b);V(a)
                                            init 2p
```

1a) Draw the geometric models of these programs.

1b) What if we change `sync 1 b` into `sync 2 b` in the third program?

2) Which of these models are prime (explain).

3) Draw the categories of components of these models.

4) For each program P above:

4a) Give the list of all the sequences of multi-instructions corresponding to execution traces of P.

4b) Are the two processes of P observationally independent? (explain)

5) Guess a property Π about conservative programs such that for any $P_1|\cdots|P_n$ satisfying Π, and any partition $\{i_1,\ldots,i_k\}\sqcup\{j_1,\ldots,j_{n-k}\}$ of $\{1,\ldots,n\}$, the subprograms $P_{i_1}|\cdots|P_{i_k}$ and $P_{j_1}|\cdots|P_{j_{n-k}}$ are observationally independent.

Turn the page →

Exercise 3: Posets can be defined as a loop-free categories in which there is at most one arrow from an object to another. A *lattice* is a poset in which any two elements has a least upper bound and a greatest lower bound.

1) Prove that every morphism of a poset preserves its future and its past cones.

2a) Prove that in a poset, the pushout of $z \to x$ and $z \to y$ exists if, and only if, the least upper bound of x and y exists in the poset.

2b) Give the corresponding statement for pullbacks.

3) Prove that the category of components of a loop-free category has a single object if, and only if, it is a lattice.

Exercise 4: The dimension of a homogeneous language is the common length of its words. For every n-dimensional homogeneous language H on Σ, we denote by $[H]$ the equivalence class of H under the action of the group of permutations of $\{1, \ldots, n\}$. The free commutative monoid of (equivalence classes of) homogeneous languages over Σ is denoted by $\mathcal{H}(\Sigma)$. We denote by Σ^* the set of all words on Σ.

1a) Give the prime decomposition of every nonunit $X \in \mathcal{H}(\Sigma)$ whose cardinal is 1.

1b) Let X be a *non-prime* element of $\mathcal{H}(\Sigma)$ whose cardinal is *prime* in $(\mathbb{N} \setminus \{0\}, \times, 1)$. Prove that every matrix representing X has a constant column (i.e. all its entries are equal).

Let $f : \Sigma' \to \Sigma^*$ be a map such that the language $\{f(x) \mid x \in \Sigma'\}$ is homogeneous; we define the dimension of f as the dimension of this language.

2a) Prove that there is a 'canonical' morphism of monoids $f^* : \mathcal{H}(\Sigma') \to \mathcal{H}(\Sigma)$ such that for all $x \in \Sigma'$, $f^*([\{x\}]) = [\{f(x)\}]$.

2b) What is the dimension of $f^*(X')$ for a given $X' \in \mathcal{H}(\Sigma')$?

3) Decompose the following elements of $\mathcal{H}(\{\texttt{0,1,...,5}\})$ into prime factors.

143	040314	143
042	240334	052
052	024314	043
043	224334	142
142	224231	152
152	040211	053
053	024211	153
153	240231	