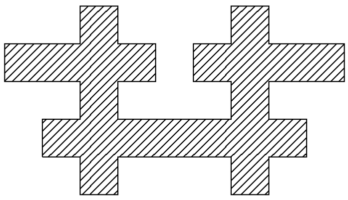
2.3.1: Concurrency

tuesday, the 2^{nd} of march 2021
duration: 3h

All the programs under consideration are supposed to be conservative.

Exercise 1: Denote the following (hatched) isothetic region by X:



1) Write a program whose forbidden region is X.

2) Draw the deadlock attractor of this program.

3) What are the maximal blocks of the forbidden region.

4) Give the prime factorization of the geometric model
(i.e. the complement of the hatched region).

5) Draw the category of components of the geometric model.

Exercise 2:

1) Give the prime decomposition of the following homogeneous languages (their underlying alphabet is $\{\mathtt{A}, \mathtt{B}, \mathtt{C}\}$).

		BABCA
CBAA	ABCABC	CBCAB
CAAB	AACCBB	ACBBA
ABCA	BBAACC	ACCBB
AACB	BAACCB	CBBAA
		BACCB

Let L be a homogeneous language of length $n \in \mathbb{N}$ (i.e. all the words of the language are of length n).

2a) Prove that if there exists $i \in \{1, \ldots, n\}$ such that all the words of L have the same i^{th} letter (i.e. for all words w and w' in L, $w_i = w'_i$), then L is not prime.

2b) Provide an example which proves that the converse of 2a) is false.

2c) Prove that if the cardinal of L (i.e. the number of words it contains) is a prime natural number, then the converse of 2a) is true.

Exercise 3: Assume that `a` is a synchronization barrier of arity 1 (it can stop a single process). Consider the 2 following programs (each being made of two processes):

```
x:=1;W(a) | W(a);x:=1          x:=1 | x:=1
```

1a) Give the forbidden region of each program (a picture suffice).

1b) Prove that in each program, the two processes are not model independent.

2a) For each program, give the list of the sequences of multi-instructions corresponding to the execution traces.

2b) For each program, are the two processes observationally independent ?

Consider the following program (the `print` instruction displays the corresponding character on the "terminal")

```
print '1'; Wa; Wa | Wa; print '2'; Wa | Wa; Wa; print '3'
```

3a) Prove that the character `'3'` cannot be displayed first.

3b) What are all the possible outputs on the screen (explain)?

3c) Assume that we replace the instructions `print '1'`, `print '2'`, `print '3'` by `x:=1`, `x:=2`, and `x:=3` respectively. What are the possible contents of the variable `x` at the end of the execution (explain)?

<u>Exercise 4</u>: Given a poset $(X, \sqsubseteq)$, one defines a category $\mathcal{C}$ as follows: the objects are the elements of X, the morphisms are the 2-tuples (a, b) such that $a \sqsubseteq b$.

1) Prove that $\mathcal{C}$ is loop-free.

2) Prove that any morphism of $\mathcal{C}$ is a *potential* weak isomorphisms (i.e. its preserves the future cones and the past cones).

Suppose that the poset $(X, \sqsubseteq)$ satisfies the following additional property: for all a, $b \in X$,

- $\{a, b\}$ has a lower bound, (i.e. there exists $x \in X$ such that $x \sqsubseteq a$ and $x \sqsubseteq b$),
- if $a \not\sqsubseteq b$ and $b \not\sqsubseteq a$, then $\{a, b\}$ has no upper bound (i.e. for all $x \in X$, $a \not\sqsubseteq x$ or $b \not\sqsubseteq x$)

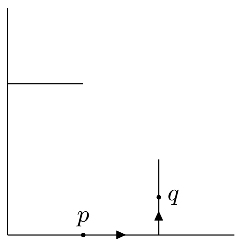
3a) Given a morphism (a, b) of $\mathcal{C}$, prove that if there exists $x \in X$ such that $b \not\sqsubseteq x$, $x \not\sqsubseteq b$, and $a \sqsubseteq x$, then (a, b) does not belong to any system of weak isomorphisms.

3b) Determine the greatest system of weak isomorphisms of $\mathcal{C}$.

4a) Describe the fundamental category $\overrightarrow{\pi_1} A$ of the subset

$$A \quad = \quad \{0\} \times [0, 3] \quad \cup \quad \{2\} \times [0, 1] \quad \cup \quad [0, 3] \times \{0\} \quad \cup \quad [0, 1] \times \{2\}$$

of $\mathbb{R}^2$ shown below (explain what are the directed paths on A, and provide the key argument allowing the computation of their dihomotopy classes).



4b) What is the greatest system of weak isomorphisms of $\overrightarrow{\pi_1} A$?