2.3.1: Concurrency

wednesday the 1st of march 2017
duration: 2h

Exercise 1:

```
sem 1 a
proc p = P(a);V(a)
init:  3p
```

1) Draw the geometric model X of the program above.

2) Let p and q be the lower and upper corners of the geometric model X. How many elements the set $\overrightarrow{\pi_1}X(p,q)$ (the set of directed homotopy classes of directed paths from p to q) contains.

3) Prove that two directed paths on that model are weakly dihomotopic iff their underlying paths are homotopic in the usual "undirected" sense.

4) Make a comment about the following program.

```
sem 1 a
sem 2 b
proc p = P(b);P(a);V(a);V(b)
init:  3p
```

Exercise 2:

Given $n \geqslant 2$, the n-philosophers diner is the following program.

```
sem 1 a1 ... an
proc p1 = P(a1);P(a2);V(a1);V(a2)
        ⋮
proc pk = P(ak);P(ak+1);V(ak);V(ak+1)
        ⋮
proc pn = P(an);P(a1);V(an);V(a1)
init:  p1 ... pn
```

1) Draw the geometric model for $n = 2$ and its category of components.

An execution trace is said to be maximal when it cannot be extended to a longer execution trace. To every execution trace one associates an admissible sequence of multi-instructions.

2) For $n = 2$, give all the sequences of multi-instructions coming from maximal execution traces of the program.

3) For all n, give all the sequences of multi-instructions (coming from an execution trace) that end up in a deadlock.

4a) For all n, give the maximal block covering of the forbidden region.

4b) Compute the prime decomposition of the geometric model of the program.

Let $\mu_0, \dots, \mu_q$ be the sequence of multi-instructions corresponding to some partial execution trace γ of the n-philosophers diner. Considering all the execution traces extending γ, we obtain the finite set S of all the multi-instructions μ_{q+1} that could be executed after the sequence $\mu_0, \dots, \mu_q$.

Suppose that, at each step of the execution, μ_{q+1} is chosen at random in S (all the element of S occurring with the same probability).

5) Compute the limit of the probability that a deadlock occurs when the number of philosophers (i.e. n) goes to infinity.

Exercise 3:
1) Prove there exists infinitely many precubical sets whose realizations in $\mathcal{Top}$ are homeomorphic to the compact unit segment $[0,1]$ though their realizations in $d\mathcal{Top}$ are pairwise non isomorphic.

2) Given a finite precubical set K, prove there exists infinitely many precubical sets whose realizations in $\mathcal{Top}$ are homeomorphic to that of K though their realizations in $d\mathcal{Top}$ are pairwise non isomorphic.

Exercise 4:
1) Find a program whose geometric model (as defined in the course) is not connected.

2) Comment the claim: "we can suppose that the geometric model of a program is connected".

Exercise 5:
We denote by the running processes of a program by $G_1, \ldots, G_n$.

1) Explain why the set of execution traces of a program is open in the space of directed paths on its geometric model.

2) Suppose the set of execution traces is arc-connected, what can be said about the program?

3) Find a program such that the set of admissible paths on $\rceil G_1 \lfloor \times \cdots \times \rceil G_n \lfloor$ is neither open nor closed in the space of directed paths on $\rceil G_1 \lfloor \times \cdots \times \rceil G_n \lfloor$.

Exercise 6:
The *initial* object of a category $\mathcal{C}$ is (when it exists) an object I such that $\mathcal{C}(I, X)$ is a singleton for all objects X of $\mathcal{C}$.

1) Prove that if the *cubical* set K is not the initial object of $c\mathcal{Set}$, then $K_n \neq \emptyset$ for all $n \in \mathbb{N}$. Prove that it does not hold for *precubical* sets.

The terminal object of a category $\mathcal{C}$ is (when it exists) an object T such that $\mathcal{C}(X, T)$ is a singleton for all objects X of $\mathcal{C}$.

2) Compute the realization in $\mathcal{Top}$ of the terminal *cubical* set.

3) Give a simple description of the underlying set of the realization in $\mathcal{Top}$ of the terminal *precubical* set.

Exercise 7:
1) Find a prime isothetic region whose category of component is not prime.

2) Find infinitely many non-isomorphic prime isothetic regions whose categories of components are not prime.