Firas Ben Jedidia

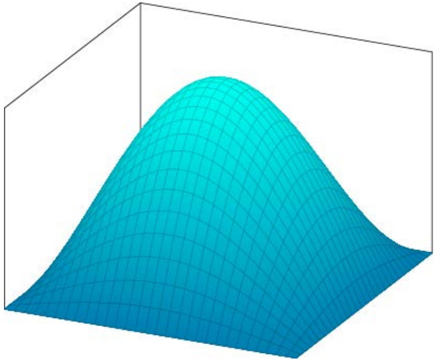
Benjamin Doerr

Martin S. Krejca

Estimation-of-Distribution Algorithms for Multi-Valued Decision Variables

Background

Estimation-of-Distribution Algorithms



probabilistic model

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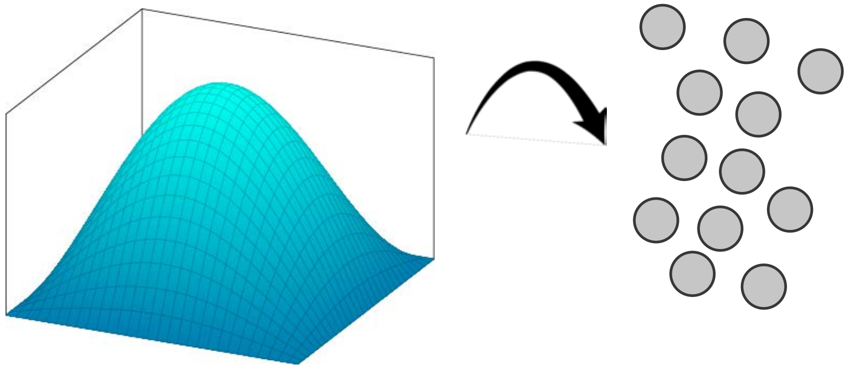
Estimation-of-Distribution Algorithms



probabilistic model

samples

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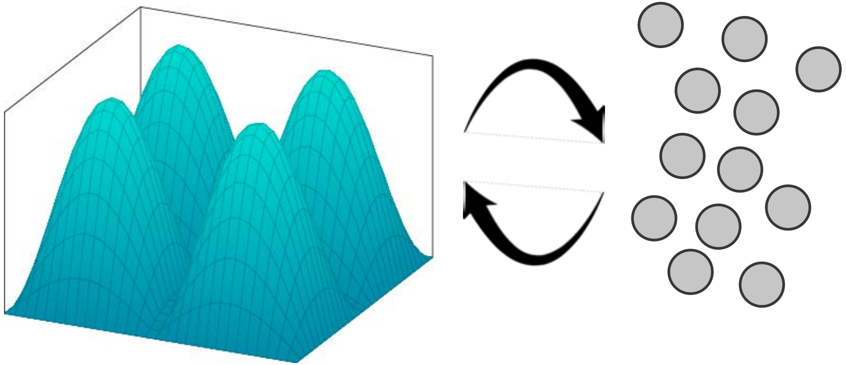
Estimation-of-Distribution Algorithms



probabilistic model   samples

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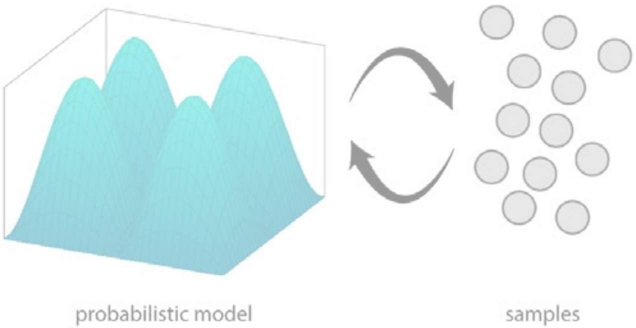

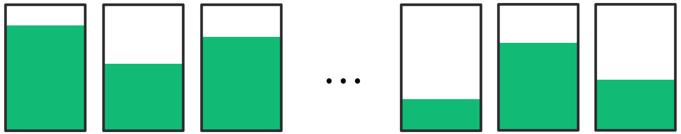

univariate model

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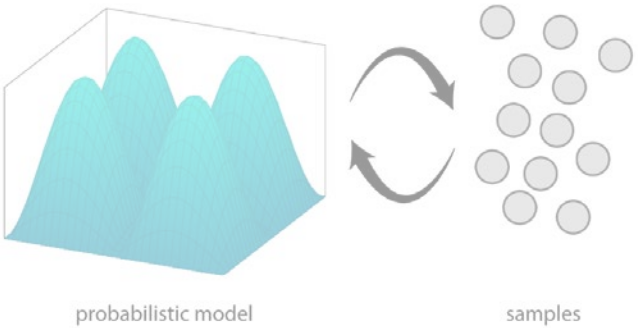
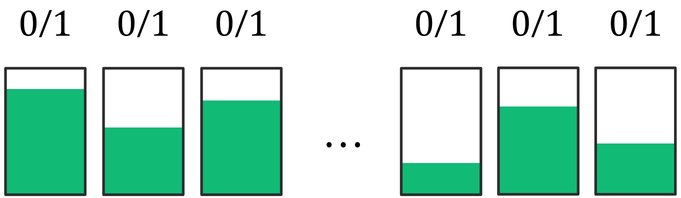
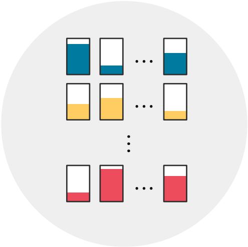




univariate model

Our Contribution



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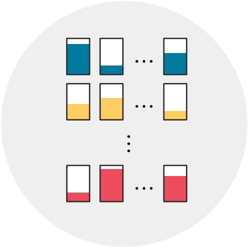
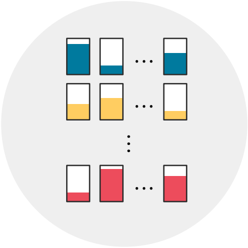


1. Our Model

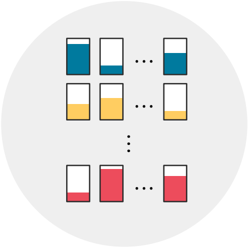
Our Contribution



1. Our Model

2. Properties

Our Contribution



1. Our Model

2. Properties

3. Run Time Analysis

Our Model

(Univariate) Multi-Valued EDAs

$$[0..r-1]^n$$

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$\boldsymbol{p} \in [0,1]^{n \times r}$ *(frequency matrix)*

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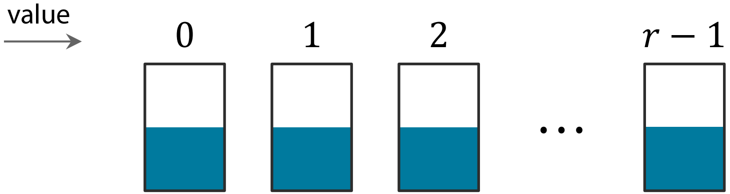

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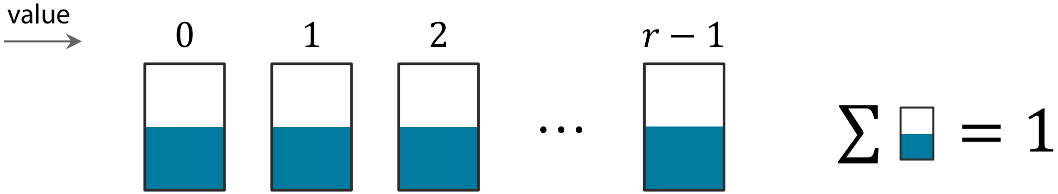

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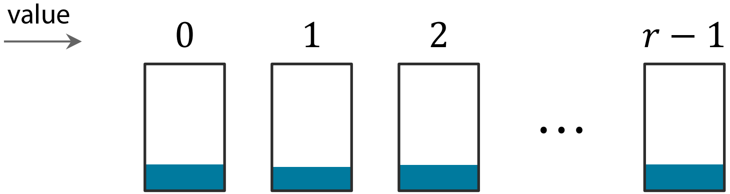
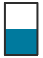


$\sum$  $= 1$ $\quad \boldsymbol{p}_{i,j}^{(0)} = \frac{1}{r}$

Our Model

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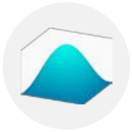

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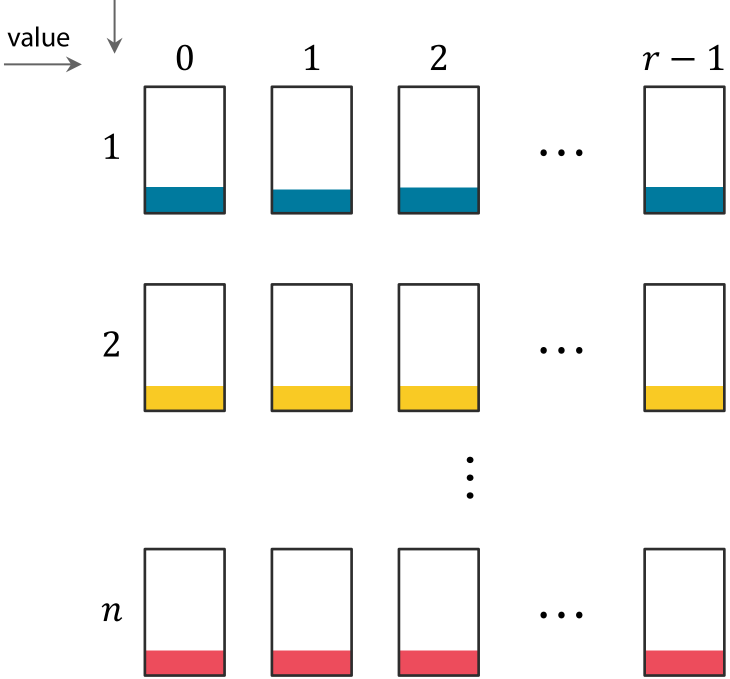
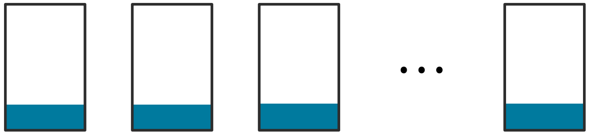


$\sum \square = 1 \qquad \boldsymbol{p}_{i,j}^{(0)} = \frac{1}{r}$

Our Model

Rounding Frequencies



Our Model

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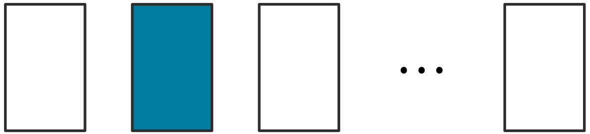

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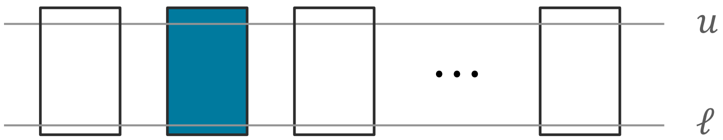

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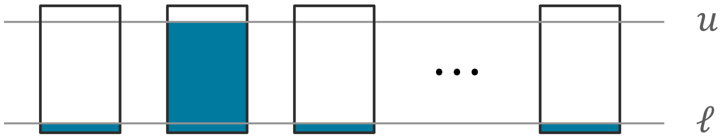

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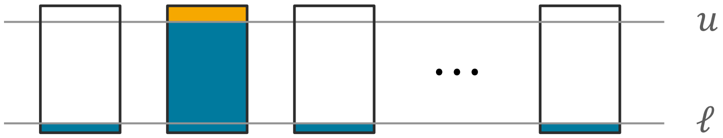

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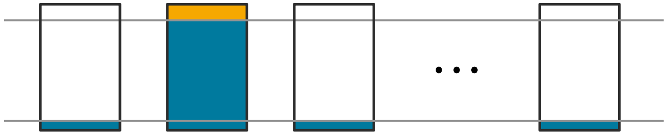

$$u = 1 - \frac{1}{n}$$

$$\ell = \frac{1}{r-1}\frac{1}{n}$$

Our Model

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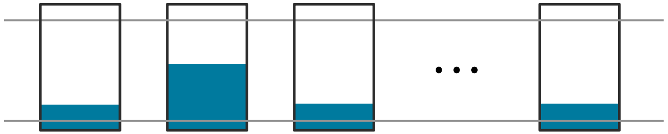

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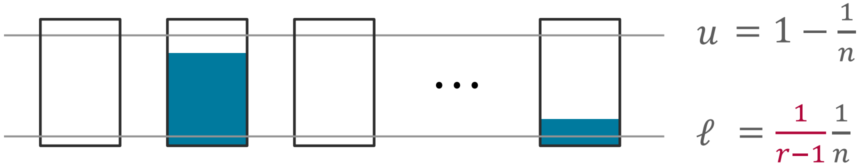

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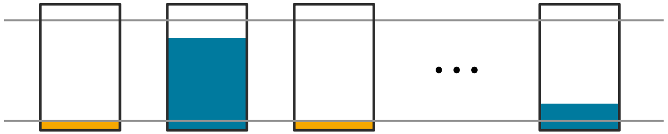

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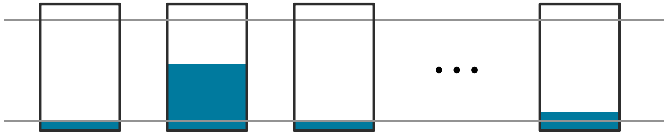


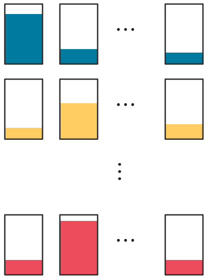
$$u = 1 - \frac{1}{n}$$

$$\ell = \frac{1}{r-1}\frac{1}{n}$$

$$\boldsymbol{p}_{i,j}^{\text{new}} = \left(\boldsymbol{p}_{i,j}^{\text{after rounding}} - \ell\right)\frac{1 - r\ell}{1 - r\ell + \square} + \ell$$

Our Model

r-UMDA



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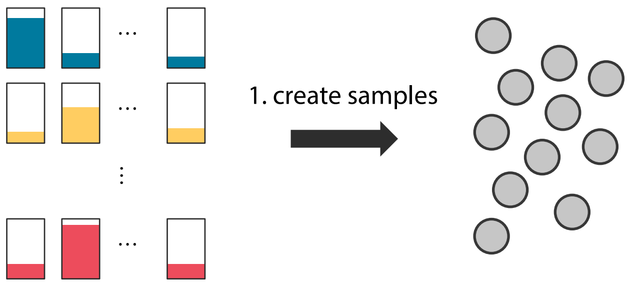

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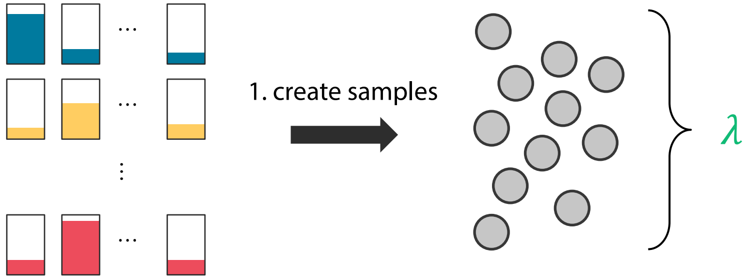

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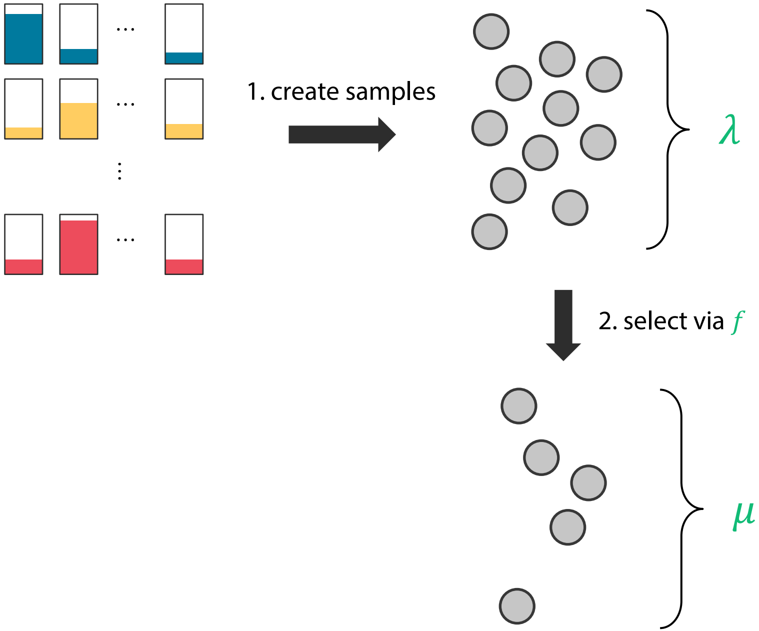

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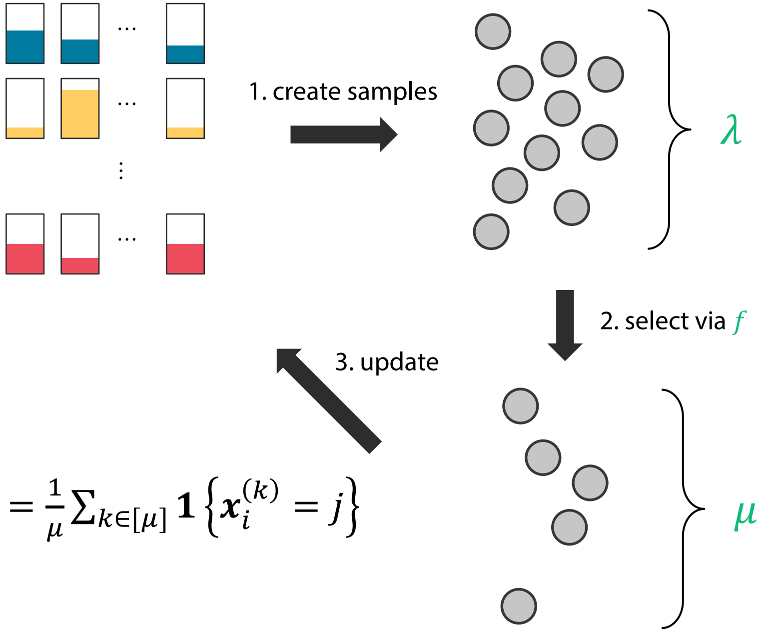

$$\boldsymbol{p}_{i,j}^{\text{b.round.}} = \frac{1}{\mu}\sum_{k\in[\mu]} \mathbf{1}\left\{\boldsymbol{x}_i^{(k)} = j\right\}$$

Balanced Frequencies

$$\mathrm{E}\left[\boldsymbol{p}_{i,j}^{(t+1)} \,|\boldsymbol{p}_{i,j}^{(t)}\right] = \boldsymbol{p}_{i,j}^{(t)}$$ *(balanced frequency)*

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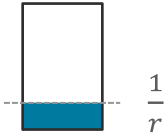

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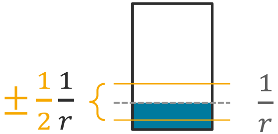

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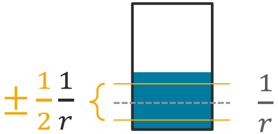

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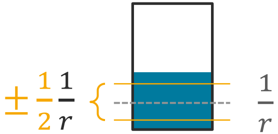

$$\Pr\left[\max_{s\in[0..T]}\left|\boldsymbol{p}_{i,j}^{(s)} - \frac{1}{r}\right| \geq \frac{1}{2}\frac{1}{r}\right] \lesssim \exp\left(-\frac{\mu}{Tr}\right)$$ *(r-UMDA)*

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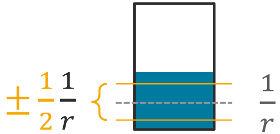

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Theoretician's Trick

(# iter.)

$T\ldots$ intended run time

$\mu = Tr\log(n)$

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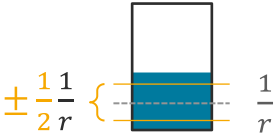

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(r-UMDA)

Theoretician's Trick

$T\ldots$ intended run time *(# iter.)*

$\mu = Tr\log(n)$

$$\Rightarrow \Pr[\ldots] \lesssim \frac{1}{n^{O(1)}}$$

Run Time Analysis

r-LeadingOnes

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$$(0,15,0,0,0,r-1) \mapsto 1$$

$$\left(0,0,0,1,3,\frac{r}{2}\right) \mapsto 3$$

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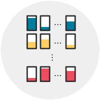

r-UMDA

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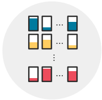

r-UMDA

Our Result

Run Time Analysis

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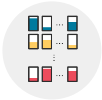

r-UMDA

Our Result

$T \lesssim n \log(r)$ w.h.p. **if:** $\mu \gtrsim n \log(r)\, r \log(n)$

$\lambda \gtrsim \mu$

Run Time Analysis

Proof Sketch

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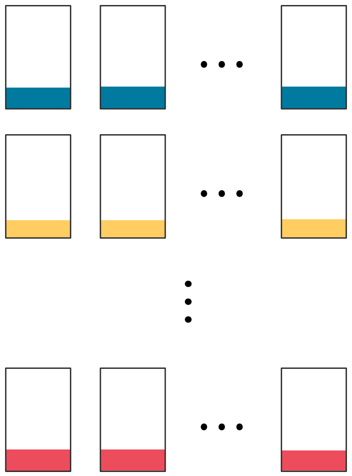

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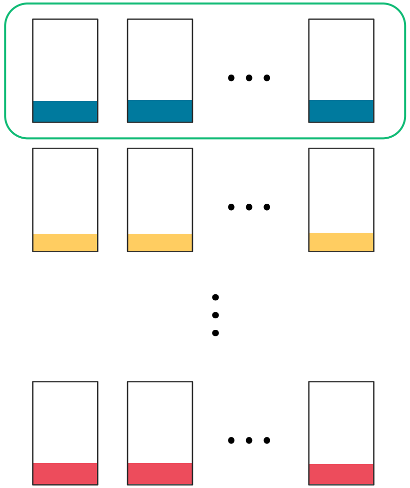

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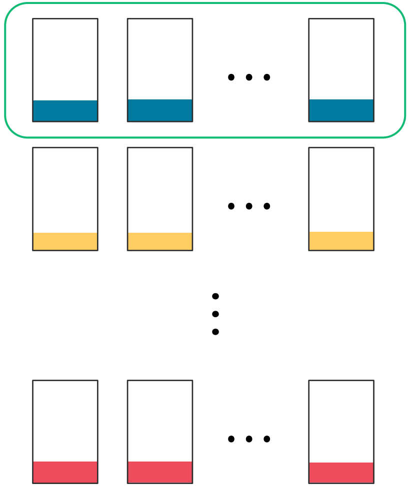

$(0,15,0,0,0,r-1)$ $(2,1,12,3,r-5,0)$

$(0,0,7,r-2,4,2)$ $(3,0,0,7,18,23)$

Run Time Analysis

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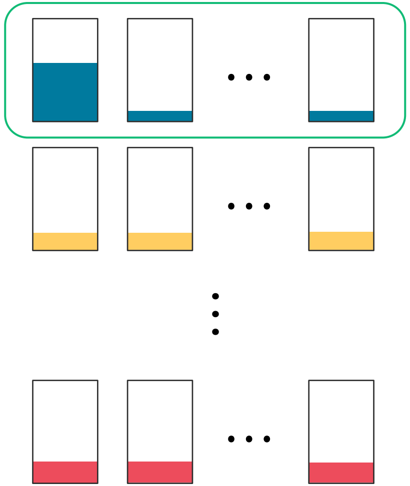

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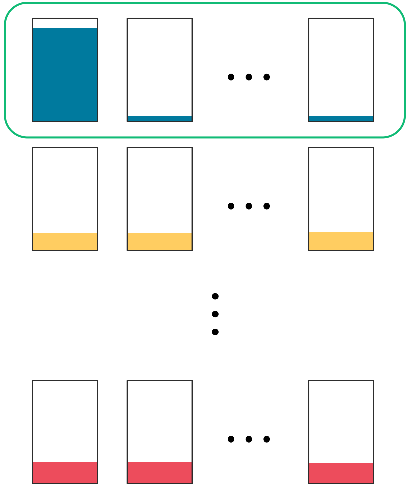

$\approx \log(r)$

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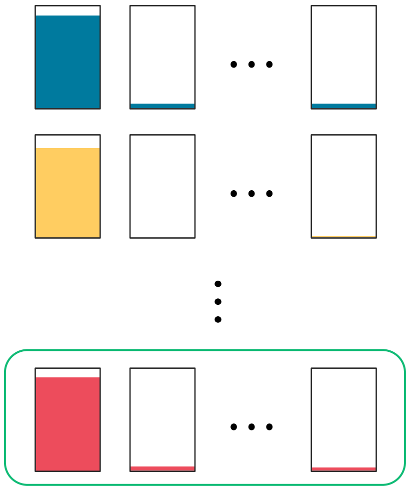

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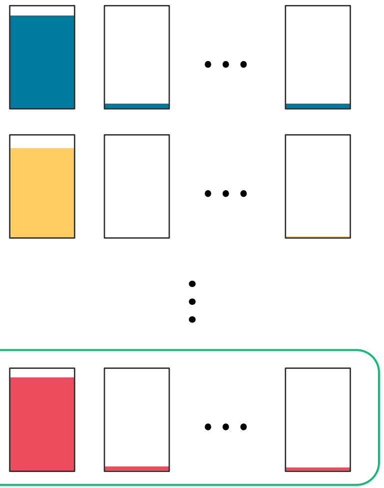

$\approx \log(r)$

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fin

» *Runtime Analysis of a Multi-Valued Compact Genetic Algorithm on Generalized OneMax* [Adak, Witt'24]