Mixed Integer Non Linear Optimization: Methods and Applications

LP real-world application

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### Reminder...



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#### Mathematical Optimisation is a knowledge-based approach



Source https://ec.europa.eu/research/infocentre/article\_en.cfm?artid=52665

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#### Informal description of the problem

#### A company produces solar panels.



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#### Informal description of the problem

A company produces solar panels. The production process of each of them is composed of different steps and some of these steps have to be processed after other steps. Each step has a weight and a duration. Supposing that the first step starts at 0, determine the start times of each step so as to minimize their weighted average.

### Formal description of the problem

Sets

► *S*: set of steps to perform to produce a solar panel

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#### **Decision variables**

determine the start times of each step

• 
$$t_i$$
 (for  $i \in S$ ): starting time of step  $i$ 

### Objective function

minimize their weighted average



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### **Objective function**

minimize their weighted average

$$\min_t \sum_{i \in S} w_i t_i$$

Constraints

Supposing that the first step starts at 0

$$t_1 = 0$$

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Variables bounds

 $t_i \geq 0 \quad \forall i \in S$ 

Formal description (or Mathematical Model/Formulation) of the problem

The final Linear Programming model

$$\min_{t} \sum_{i \in S} w_{i}t_{i}$$

$$t_{1} = 0$$

$$t_{i} \geq t_{p} + d_{p} \qquad \forall i \in S, p \in P_{i}$$

$$t_{i} \geq 0 \qquad \forall i \in S$$

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**Instance(s)** of the problem: valid input values of a problem.

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$$S = \{1, 2, 3, 4\}$$

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What is the optimal solution?

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We can solve this instance of the problem graphically (3 variables  $(t_2, t_3, t_4)$ , as  $t_1 = 0$ ).

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What if |S| > 4?

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What if |S| > 4?  $\rightarrow$  algebraic modeling languages + MO solver

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