Mixed Integer Non Linear Optimization: Methods and Applications

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Introduction

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12/06/2023	16-17	Introduction Mathematical Programming, Linear Programming	(Simplified) Unit Commitment
	17-18	Introduction to AMPL	Activities Scheduling
13/06/2023	9-11	Linear Programming, Mixed Integer Linear Programming	Unit Commitment
	14-16	MILP and AMPL	Uncapacitated Facility Location, Knapsack Problem, Assignment Problem
14/06/2023	9-11	Convex Mixed Integer Non Linear Programming	Robust Portfolio Selection
	14-16	AMPL and MINLP 1/2	Robust Portfolio Selection, Nonlinear Knapsack Problem
15/06/2023	9-11	Non-convex Mixed Integer Non Linear Programming	Pooling Problem
	14-16	AMPL and MINLP 2/2	Support Vector Machine and MINLP, Algorithm Configuration
16/06/2023	9-11	Research Talks:	Perspective Formulation Comparison for Piecewise Convex Functions
			Urban Air Mobility

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`https://www.lix.polytechnique.fr/~dambrosio/teaching/Pisa/`

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- **Operations Research** : analytical methods to help better decisions

Euler, 1744

Nothing in the world takes place without optimization, and there is no doubt that all aspects of the world that have a rational basis can be explained by optimization methods.

Originally in Latin, briefly and very freely translated by Martin Grötschel in "Optimization Stories".

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- Algebraic Modeling Languages (AMPL) $\rightarrow$ exercise sessions

Mathematical Optimization (MO) or Mathematical Programming (MP)

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\min\limits_{x} & f(x) & & \\
& g_i(x) \leq & 0 & \forall i = 1, \dots, m \\
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- set $Z \subseteq \{1, 2, \ldots, n\}$ is the set of the indexes of the **integer variables**

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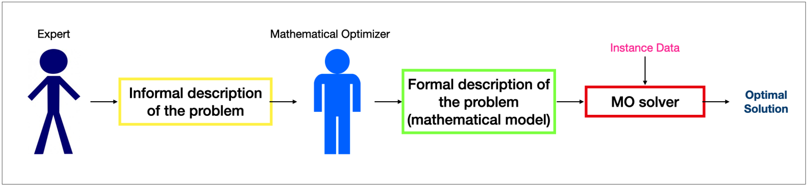
- $f(x)$ and $g(x)$ can be written in closed form
- $f(x)$ and $g_i(x)$ are given twice continuously differentiable functions of the variables $(\forall i = 1, \ldots, m)$

The Mathematical Optimizer's Job

Mathematical Optimisation is a knowledge-based approach



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$$\min_p \sum_{t \in T} \sum_{j \in J} c_j p_{jt} \tag{1}$$

$$\sum_{j \in J} p_{jt} = D_t \qquad \forall t \in T \tag{2}$$

$$0 \leq p_{jt} \leq \overline{p}_j \qquad \forall j \in J, t \in T \tag{3}$$

where (1) is the objective function minimizing the cost for producing electricity, (2) is the set of constraints satisfying the demand D_t at each time period $t \in T$, and (3) are the simple bounds on the electricity production by each unit j at each time period j.

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- **Instance** : when the expression of $f(x)$, $g(x)$ and the values of $\underline{x}$, $\overline{x}$, and Z are known. The set of instances of a MO problems is potentially infinite.

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NB: difference between optimization (decisions are made) and simulation (no decision made)

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Black Box Optimization: $f(x)$ or $g(x)$ $\rightarrow$ no closed form