Mixed Integer Non Linear Optimization: Methods and Applications Introduction to AMPL

Day 4

Exercises session

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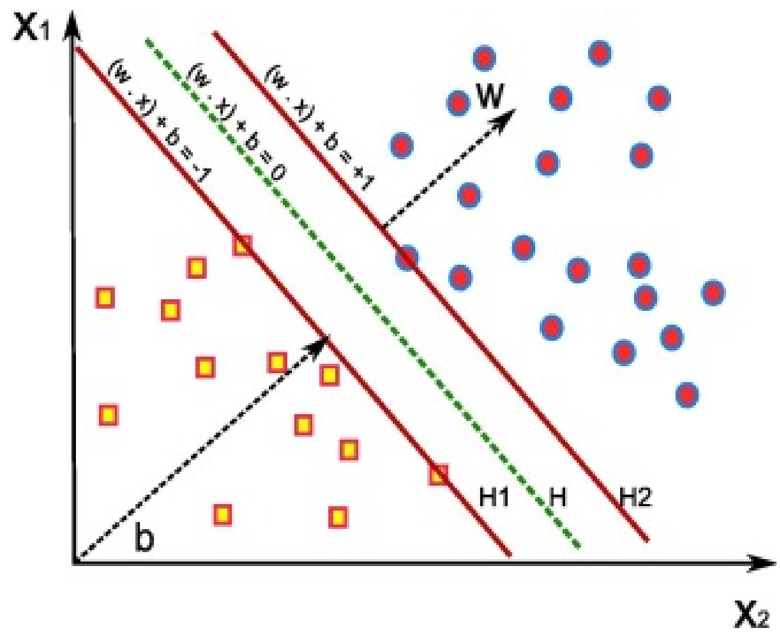
Exercise 1: Support Vector Machine

The Support Vector Machine (SVM) methods aim at separating a set of points in two classes by means of a hyperplane, $\omega^\top x + b = 0$.



Let n be the size of the sample used to build the classifier. Each of the sample points are represented by (x_i, y_i) where $x_i \in \mathbb{R}^d$ and $y_i \in \{-1, +1\}$ is the label for point x_i (thus, it is a supervised learning). We wish to find $\omega \in \mathbb{R}^d$ and $b \in \mathbb{R}$ such that we minimize the following function:

$$\frac{1}{2}\sum_{j=1}^{d}\omega_j^2 + \frac{C}{n}\sum_{i=1}^{n} p_i(\omega, b, x_i)$$

where C is a tradeoff parameter for tuning (input data) and $p_i(\omega, b, x_i)$ is the penalty function for misclassification of the i-th element described below:

When $y_i = +1$

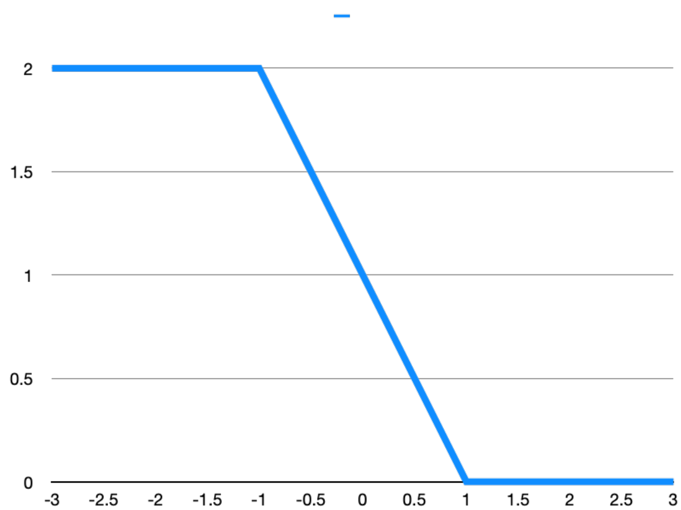


Figure 1: x-axis: $\omega^\top x_i + b$; y-axis: $p_i(\omega, b, x_i)$

When $y_i = -1$

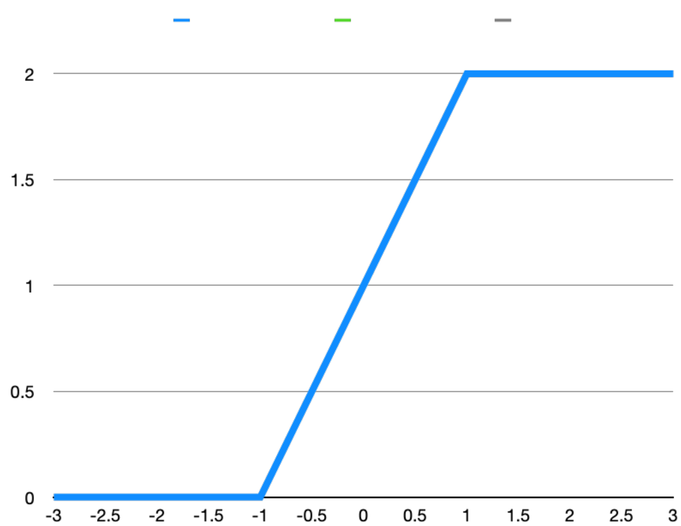


Figure 2: x-axis: $\omega^\top x_i + b$; y-axis: $p_i(\omega, b, x_i)$

To do

- Write on paper a MINLP that models the problem described above.
- Write the same model in AMPL.
- Solve the provided instance, see `https://www.lix.polytechnique.fr/~dambrosio/teaching/Pisa/exercises/Day4/` with a MINLP solver (e.g., baron)