Mixed Integer Non Linear Optimization: Methods and Applications Introduction to AMPL

Day 3

Exercises session

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Exercises 1

Let us consider the robust portfolio selection formulation we have seen this morning. A small instance is provided here: `https://www.lix.polytechnique.fr/~dambrosio/teaching/Pisa/exercises/Day3/`, see file portfolio_selection.dat.

- Model in AMPL the formulation we have seen this morning
- Solve the small instance with different solvers, like knitro, baron, gurobi
- Now add the constraints that guarantee that each x_i is in $\{0\} \cup [0.1, 1]$, i.e., it is not allowed to take less than 0.1, besides the value 0. For modeling this condition, you can check the first slides of yesterday.
- Solve the small instance with different solvers, like knitro, baron, gurobi

Exercises 2

Implement in AMPL example where outer approximation takes an exponential number of iterations we have seen this morning. Formally,

$$\begin{array}{rcl} & \min 0 & \\ \displaystyle\sum_{i=1}^{n}\left(x_i - \frac{1}{2}\right)^2 & \leq & \dfrac{n-1}{4} \\ x & \in & \{0,1\}^n \end{array}$$

Solve the problem for increasing values of n with the solver Knitro, with the option mip_method=2. When does it become intractable?
Now try the following reformulation:

$$\begin{array}{rcll} \min 0 & & & \\ \displaystyle\sum_{i=1}^{n} y_i & \leq & \dfrac{n-1}{4} & \\ y_i & \geq & \left(x_i - \dfrac{1}{2}\right)^2 & \forall i = 1,\ldots,n \\ x & \in & \{0,1\}^n & \end{array}$$

Do you see the same performance? Why?

Exercises 3

The continuous, non linear knapsack problem (NLKP) is defined as follows: we are given N items, each of U_j maximum availability (for $j = 1, \ldots, N$). The knapsack capacity is represented by C. We have to decide how much to take of each of the items, so that the knapsack capacity is respected (the unit weight of each of the items is 1) and maximize the total profit. The profit function for each item is described below.

1. Implement the knapsack problem model by considering the profit of each item j equal to $a_j + b_j x_j + c_j x_j^2 + d_j x_j^3$ and solve the instance nlkp.dat (you can download it from the course web page) with KNITRO and with BARON.

2. Are the optimal solutions found by the two solvers equivalent?

3. Considering that $a, b, c > 0$ and $d < 0$, is the problem convex? Why?

4. Implement the knapsack problem model by considering the profit of each item j equal to $d_j + c_j x_j + b_j x_j^2 + a_j x_j^3$ and solve the instance nlkp.dat with KNITRO and with BARON. Answer to questions 2. and 3.

5. Now use $c[j]/(1 + b[j] * exp(-a[j] * (x[j] + d[j])))$ as profit function for item j and solve the instance nlkp.dat with KNITRO and with BARON. Answer to questions 2. and 3.