MPRO — Programmation Mathématique Avancée — 2018/2019

Partie "Mixed Integer Nonlinear Programming": solution

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1 Problèmes de packing (3 points)

Variables

- $z_j \in \{0,1\}$ $\forall j = 1,\dots,c$ (1 if the object is inserted in the box, 0 otherwise)
- $x_j \geq 0$ $\forall j = 1,\dots,c$ (2-dimensional vector of coordinates of (the center of) object j)

$$\max_{x,z} \sum_{j=1}^{c} z_j$$
$$\|x_i - x_j\| \geq 2rz_iz_j \qquad \forall i,j = 1,\dots,c(i \neq j)$$
$$rz_j \leq x_j^1 \leq (L-r)z_j \qquad \forall j = 1,\dots,c$$
$$rz_j \leq x_j^2 \leq (W-r)z_j \qquad \forall j = 1,\dots,c$$

$\|\| = norme1$ pour le problème 1, $\|\| = norme2$ pour le problème 2, $\|\| = norme + \infty$ pour le problème 3

(b) non convex but the first and the last can be linearized. Spatial branch-and-bound.

2 Reformulations (4 points)

2.1 Reformulation exacte

$$\min \sum_{j \in J} c_j \sum_{k=0}^{u_j} k^2 y_{jk} \tag{1}$$
$$\sum_{j \in J} a_{ij} \sum_{k=0}^{u_j} k y_{jk} \leq b_i \qquad \forall i \in I \tag{2}$$
$$\sum_{k=0}^{u_j} y_{jk} = 1 \qquad \forall j \in J \tag{3}$$
$$y_{jk} \in \{0,1\} \qquad \forall j \in J, k = 0,\dots,u_j. \tag{4}$$

2.2 Relaxation

$0 \leq 4x_1 - 3x_3$ is equal to $x_3 \leq \frac{4}{3}x_1 = \frac{4}{3}$ as the upper bound on x_1 is 1.

To obtain a MILP:

- x_2x_3: apply Fortet (new variable name w)
- $x_1x_2x_3$: x_2x_3 is not w, thus it is sufficient to apply Mc Cormick to x_1w
- $x_2^2 = x_2$

2.3 Forme standard

$$\max\,(w_3 + w_5) \tag{5}$$
$$4x_1^2 \leq 2 \tag{6}$$
$$0 \leq 4x_1 - 3x_3 \leq 3 \tag{7}$$
$$0 \leq x_1 \leq 5 \tag{8}$$
$$-1 \leq x_2 \leq 1 \tag{9}$$
$$1 \leq x_3 \leq 3 \tag{10}$$
$$x_4 \in \{0, 1\}. \tag{11}$$
$$w_1 = x_1x_4 \tag{12}$$
$$w_2 = \frac{1}{x_3} \tag{13}$$
$$w_3 = w_1w_2 \tag{14}$$
$$w_4 = x_2x_3 \tag{15}$$
$$w_5 = w_1w_4 \tag{16}$$

MINLP convexe: Fortet pour (12), McCormick pour (14)-(16), and linear underestimator for $w_2 \leq \frac{1}{x_3}$