How to deal with nondifferentiability of Hazen-Williams equation

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The notation follows
http://www.minlp.org/library/problem/index.php?i=134&lib=MINLP.

The Hazen-Williams equation can be written as follows:

$$H(i)-H(j) = sgn(Q(e))|Q(e)|^{1.852} \cdot 10.7 \cdot len(e) \cdot k(e)^{-1.852} \left(\frac{\pi}{4}\right)^{2.435} / A(e)^{2.435}$$

.
An alternative way to deal with the nondifferentiability of sgn(Q(e)) wrt the one presented in the url mentioned above is the following.
Let us introduce two additional sets of positive variables $Q^+(e)$ and $Q^-(e)$ $foralle \in E$, the additional set of binary variables $Y(e)$ $\forall e \in E$ and let us define the following sets of constraints:

$$\begin{aligned} Q(e) = Q^+(e) - Q^-(e) & \qquad \forall e \in E \\ 0 \le Q^+(e) \le Y(e)(v_{max}(e)\frac{\pi}{4}d^2_{max}(e)) & \qquad \forall e \in E \\ 0 \le Q^-(e) \le (1-Y(e))(v_{max}(e)\frac{\pi}{4}d^2_{max}(e)) & \qquad \forall e \in E \\ Y(e) \in \{0,1\} & \qquad \forall e \in E \end{aligned}$$

The nondifferentiable part can now be written as:

$$sgn(Q(e))|Q(e)|^{1.852} = (Q^+(e))^{1.852} - (Q^-(e))^{1.852}.$$

For details, see P. Belotti, C. Kirches, S. Leyffer, J. Linderoth, J. Luedtke, A. Mahajan. Mixed-integer nonlinear optimization, Acta Numerica: 22, pages: 1–131, 2013.