AN "UNSIGNED" STOCHASTIC LOT-SIZING PROBLEM

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PROBLEM FORMULATION

We consider the following version of the dynamic lot-sizing problem:

$$
\begin{array}{lll}
\textbf{minimize} & \mathbb{E}\left[\sum_{t=1}^{T} P_t x_t + \psi(s_T)\right] & \\
\text{s.t.} & s_t = s_{t-1} + x_t - D_t & \forall t \\
 & s_t \overset{\text{a.s}}{\in} [0, S] & \forall t \\
 & |x_t - D_t| \overset{\text{a.s}}{\in} [-C^{\max}, C^{\max}] & \forall t \\
\textbf{non-anticipativity} & \left\{ \begin{array}{l} s_t \text{ a function of } (P_{[t-1]}, D_{[t]}) \\ x_t \text{ a function of } (P_{[t-1]}, D_{[t-1]}) \end{array} \right. & \forall t
\end{array}
$$

(Note that x_t is decided before D_t is revealed.)

Parameters:

- T: horizon
- S: maximal inventory
- $C^{\max}$: maximal inventory variation
- D_t: random demand
- P_t: random price
- ψ: final cost

Decision variables:

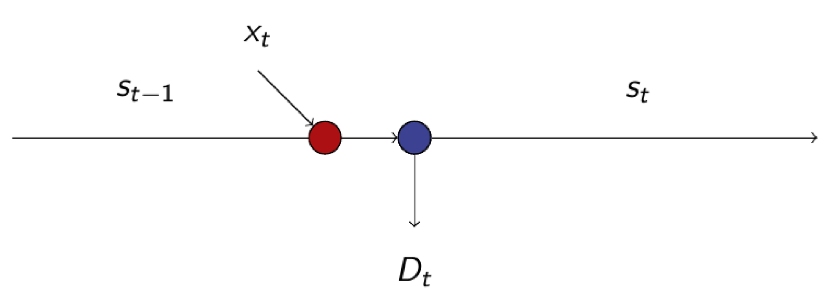
- s_t: inventory
- x_t: order size

MOTIVATION AND DYNAMICS

The motivation comes from a collaboration with the French electricity provider EDF eDF : **how to manage optimally a battery on the electricity "intraday" market?**

Interpretation in the electricity context:

- Horizon $\iff$ T time steps of one hour
- Inventory $\iff$ energy level
- Order size $\iff$ electricity charge or discharge
- Demand $\iff$ reserve activation: the battery can be activated by the transmission system operator (TSO) to stabilize the network.
- ψ $\iff$ to anticipate the next day



PRELIMINARY REMARKS

- The optimal value is convex with respect to the initial inventory (easy to prove).
- Feasible initial inventories form an interval (easy to prove as well).

NUMERICAL RESULTS

Realistic instances: P_t auto-regressive, D_t uniform

Lower bound given by Theorem 2 (better than removing non-anticipativity)

s_0	$C^{\max}$	**Anticipate**	**Thm 2**
30	20	-12521	-2501
30	80	-20055	-2501
50	20	-13939	-2696
50	80	-22526	-2696
70	20	-15270	-2892
70	80	-22274	-2892

Table 1: Experimental results for S

CONTRIBUTIONS

For the following results, we assume that the essential supremum and infimum of D_t are known. The *policy* $\pi_t(s_{t-1}, P_{[t-1]})$ decides x_t.

Proposition 1 (Meunier and S. E. 2024). *Deciding whether s_t is a feasible inventory at time t can be done in linear time.*

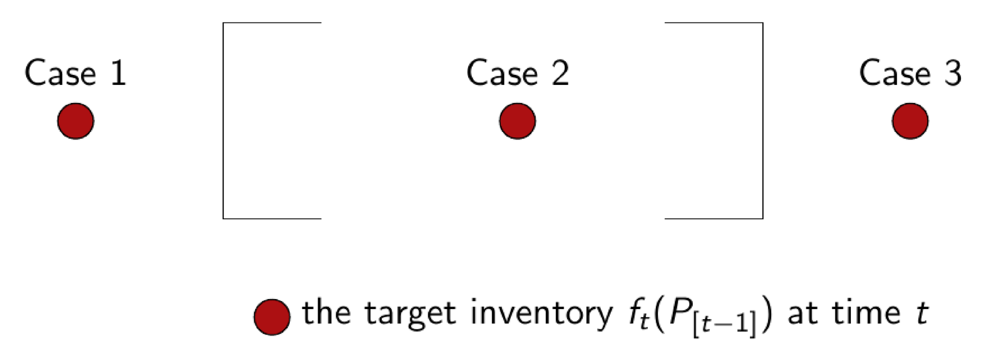
Theorem 1 (Meunier and S. E. 2024). *There exist measurable functions $\ell_t, u_t \colon [0, S] \to \mathbb{R}$ and $f_t \colon \mathbb{R}^{t-1} \to \overline{\mathbb{R}}$ such that an optimal policy is given when s_{t-1} is feasible by*

$$
\pi_t^\star(s_{t-1}, P_{[t-1]}) = \begin{cases} \ell_t(s_{t-1}) - s_{t-1} & \text{if } \ell_t(s_{t-1}) > f_t(P_{[t-1]}), \\ u_t(s_{t-1}) - s_{t-1} & \text{if } u_t(s_{t-1}) < f_t(P_{[t-1]}), \\ f_t(P_{[t-1]}) - s_{t-1} & \text{otherwise.} \end{cases}
$$

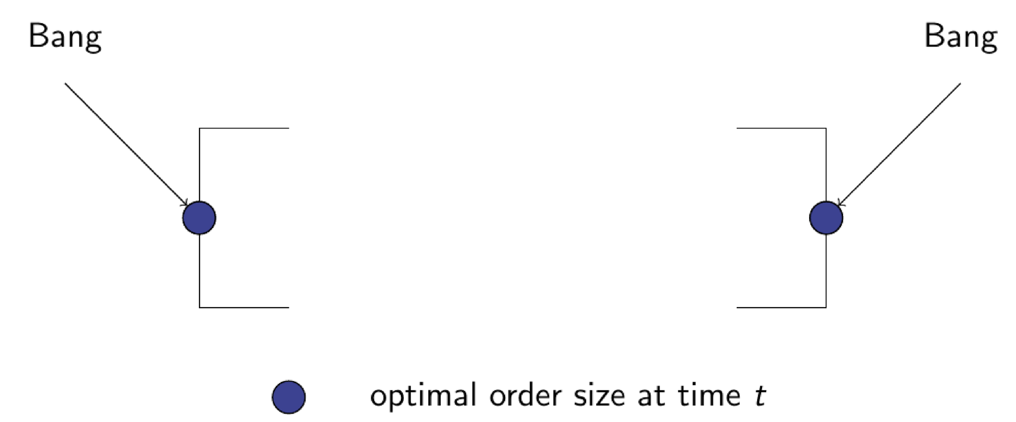
Up to an extra assumption, the previous theorem can be made more explicit.

Theorem 2 (Meunier and S. E. 2024). *Suppose $C^{\max} = +\infty$. Then the value function depends affinely on the current inventory and an optimal policy when $t < T$ is given by*

$$
\pi_t^\star(s_{t-1}, P_{[t-1]}) = \begin{cases} \operatorname{ess\,sup}(D_t) - s_{t-1} & \text{if } \mathbb{E}[P_{t+1} - P_t | P_{[t-1]}] \geqslant 0, \\ S + \operatorname{ess\,inf}(D_t) - s_{t-1} & \text{otherwise.} \end{cases}
$$



Theorem 1



Theorem 2

Proposition 2 (Meunier and S. E. 2024). *Suppose $\psi = 0$. Assume furthermore that the prices form a martingale and all have the same sign with probability one. Up to a linear-time precomputation, an optimal policy at time t can be computed in constant time.*

OPEN QUESTION

The functions ℓ_t and u_t of Theorem 1 actually admit closed-form expressions. This is not the case for the functions f_t.

Open question. *Under which conditions the quantity $f_t(P_{[t-1]})$ in Theorem 1 can be efficiently computed?*

Theorem 2 and Proposition 2 provide examples of such conditions.