# CSE 520, Fall 1999

Midterm Exam

19 Oct 1999

- **Please write your name and SSN at the top of the sheet.**
- The exam consists of 6 exercises, totalling 100 points. Please write your answers directly on these pages.

## Exercise 1 (10 pts)

Define a lambda term $M$ such that, for every lambda term $N$ and $P$,

$$MNP = PM(PNM)$$

holds.

## Exercise 2 (20 pts)

Define the function *is_even* in lambda calculus. Namely, define a lambda term $[is\_even]$ such that, for every natural number $n$:

$$[is\_even][n] = [true] \quad \text{if } n \text{ is even}$$

$$[is\_even][n] = [false] \quad \text{if } n \text{ is odd}$$

## Exercise 3 (20 pts)

For each of the following lambda terms $M$, say whether or not $M$ enjoys the properties (a) and (b) below

**(a)** For every $F$, $MF = F(MF)$ $\quad$ ($MF$ is lambda-convertible to $F(MF)$)

**(b)** For every $F$, $MF \twoheadrightarrow F(MF)$ $\quad$ ($MF$ is beta-reducible to $F(MF)$)

Justify your answer.

1. $M \stackrel{\text{def}}{=} \lambda f.(\lambda x.f(xx))(\lambda x.f(xx))$

   **(a)**

   **(b)**

2. $M \stackrel{\text{def}}{=} (\lambda f \lambda x.f(xxf))(\lambda f \lambda x.f(xxf))$

   **(a)**

   **(b)**

## Exercise 4 (15 pts)

For each of the following types, say whether it is inhabited or not. Justify your answer.

- $(\alpha \to \alpha) \to \alpha$

- $\alpha \to \beta \to \alpha * \beta$

- $\alpha \to (\alpha \to \beta) \to \beta$

## Exercise 5 (15 pts)

Given a natural number $n$, say what is the principal type of $[n]$ (the $n$-th Church numeral). Justify your answer.

## Exercise 6 (20 pts)

1. Define in ML a function $iterate : (\alpha \to \alpha) \to int \to \alpha \to \alpha$ such that, for every $f : \alpha \to \alpha$, $n : int$ with $n \geq 0$, and $x : \alpha$ we have

   $$iterate\ f\ n\ x = f^n x$$

   We are not interested in the resut of *iterate* on negative values of $n$.
   Hint: use recursion on $n$.

2. Use the function *iterate* to define a function $Church\_num$ which, given a natural number $n$, returns the $n$-th Church numeral $[n]$.