CSE 520, Fall 2001

Midterm Exam

18 Oct 2001

- **Please write your name and SSN at the top of the sheet.**
- The exam consists of 6 exercises, totalling 100 points. Please write your answers directly on these pages.

Exercise 1 (10 pts)

Define a lambda term M such that, for every lambda terms N and P,

$$M([true]NP) = NMN$$

holds.

Exercise 2 (20 pts)

Define the function *Fibonacci* in lambda calculus. Namely, define a lambda term $[\mathit{fibo}]$ such that, for every natural number n:

- $[\mathit{fibo}][0] = [1]$
- $[\mathit{fibo}][1] = [1]$
- $[\mathit{fibo}][n+2] = [\mathit{plus}]([\mathit{fibo}][n])([\mathit{fibo}][n+1])$

You can use the lambda encodings already seen in class, like the encoding of the predecessor.

Exercise 3 (12 pts)

Assume that a term M is beta-reducible to itself, namely

$$M \to M$$

For each of the following sentences, say whether it is true or not. Justify your answer.

1. M cannot be strongly normalizing.

2. M cannot be weakly normalizing.

3. M cannot be typeable.

Exercise 4 (16 pts)

Let $M_1 : T_1$, $M_2 : T_2$ in the system of Curry and assume that M_1 and M_2 are lambda-convertible. What can we say about the types T_1 and T_2:

1. They must be identical.
2. They must be instaces of a common type, i.e. there must exist a type T, and substitutions θ_1 and θ_2 such that $T_1 = T\theta_1$ and $T_2 = T\theta_2$.
3. They must be unifiable, i.e. there must exist a type T, and substitutions θ_1 and θ_2 such that $T = T_1\theta_1$ and $T = T_2\theta_2$.
4. They may be completely unrelated.

Please justify your answer.

Exercise 5 (21 pts)

For each of the following terms, say whether it is typeable or not. If it is typeable, give the principal type.

- $\lambda x.x[true]$

- $\lambda xy.xxy$

- $\lambda xyz.yzx$

Exercise 6 (21 pts)

For each of the following types, say whether it is inhabited or not. Justify your answer.

- $\alpha \to (\alpha \to \alpha) \to \alpha$

- $(\alpha \to \beta \to \alpha) \to \beta$

- $(\alpha \to \beta \to \gamma) \to \beta \to \alpha \to \gamma$