CSE 520, Fall 2000

Midterm Exam

24 Oct 2000

- **Please write your name and SSN at the top of the sheet.**
- The exam consists of 5 exercises, totalling 100 points. Please write your answers directly on these pages.

Exercise 1 (10 pts)

Define a lambda term M such that, for every lambda term N and P,

$$NM(PN) = MNP$$

holds.

Exercise 2 (20 pts)

Define the function *factorial* in lambda calculus. Namely, define a lambda term [*factorial*] such that, for every natural number n:

$$[\mathit{factorial}][n] = [n!]$$

You can use the lambda encodings already seen in class, like the encoding of the multiplication.

Exercise 3 (28 pts)

Let Ω be the lambda term $(\lambda x.xx)(\lambda y.yy)$. For each of the statements below, say whether it is true or not. Justify your answer.

1. There exists a lambda term M such that $M\Omega$ is strongly normalizing.

2. There exists a lambda term M such that $M\Omega$ is weakly normalizing.

3. There exists a lambda term M such that ΩM is strongly normalizing.

4. There exists a lambda term M such that ΩM is weakly normalizing.

Exercise 4 (21 pts)

For each of the following terms, say whether it is typeable or not. If it is typeable, give the principal type.

- $\lambda x.x(\lambda y.x)$

- $\lambda xy.xyy$

- $\lambda xyz.xy(yz)$

Exercise 5 (21 pts)

For each of the following types, say whether it is inhabited or not. Justify your answer.

- $(\beta \to \alpha) \to \alpha$

- $\alpha \to \beta \to \beta \to \alpha$

- $(\alpha \to \beta) \to \beta \to \alpha$