CSE 468, Fall 1998

Final Exam

16 Dec 1998

- **Please write your name and SSN at the top of the sheet.**
- The exam consists of 4 exercises, totalling 100 points. Please write your answers directly on these pages.
- **Bonus for "No answer":** If you don't know the answer to a question, you may want to leave blank the corresponding space for the solution. This signals that you give no answer for that question. For each "no answer" you will receive a bonus of 10% of the maximum score for that question. Each question counts individually for the bonus, i.e. if an exercise is composed by more than one question, answering one question does not affect the possibility of receiving the bonus for another question in the same exercise. If you begin to write something and later wish you had left it blank, just draw a line over the text and write in large letters "BLANK".

Exercise 1 (30 pts)

For each of the following languages, say whether it is context-free or not. Justify your answer, namely: if the language is context-free, give a CFG which generates it or a PDA which recognizes it. If the language is not context-free, prove it by using the pumping lemma.

1. $L_1 = \{a^k b^m c^n | k, m, n \geq 0 \text{ and } (m = k \text{ or } m = n)\}$

2. $L_2 = \{a^k b^m c^n | k, m, n \geq 0 \text{ and } m = k + n\}$

3. $L_3 = \{a^k b^{2k} c^k | k \geq 0\}$

Exercise 2 (15 pts)

Give a deterministic PDA recognizing the language generated by the following grammar (whose starting symbol is S):

$$\begin{array}{lcl} S & \rightarrow & Sa \mid A \\ A & \rightarrow & bAc \mid bAd \mid \lambda \end{array}$$

Exercise 3 (15 pts)

Give a Turing machine M that accepts exactly the strings on $\{a, b\}$ with the same number of occurrences of a and b, i.e. $L(M) = \{x \in \{a, b\}^* | n_a(x) = n_b(x)\}$, where $n_a(x)$ = number of occurrences of a in x and $n_b(x)$ = number of occurrences of b in x.

Exercise 4 (40 pts)

Given a finite (and nonempty) alphabet Σ, let e be an encoding of Turing machines as strings on Σ, i.e. $e : TM \to \Sigma^*$. Define Σ^*_{even} to be the set of all strings in Σ of even length, i.e. $\Sigma^*_{even} = \{x \in \Sigma^* | \; |x| \text{ is even}\}$.

For each of the following sets say whether it is recursive, recursively enumerable, or not recursively enumerable. Justify your answer.

1. $L_1 = \{e(M) | M \text{ is a Turing machine and } \Sigma^*_{even} \subseteq L(M)\}$ (the set of TM's which accept all the strings of even length, an possibly more)

2. $L_2 = \{e(M)|M$ is a Turing machine and $L(M) \cap \Sigma^*_{even} \neq \emptyset\}$ (the set of TM's which accept at least one string of even length)

3. $L_3 = \{e(M)|M$ is a Turing machine and $L(M) \cap \Sigma^*_{even} = \emptyset\}$ (the set of TM's which accept no string of even length)

4. $L_4 = \{e(M)|M$ is a Turing machine and $e(M) \in \Sigma^*_{even}\}$ (the set of TM's whose code has even length)