CSE 468, Fall 1998

Midterm Exam

27 Oct 1998

- **Please write your name and SSN at the top of the sheet.**
- The exam consists of 5 exercises, totalling 100 points. Please write your answers directly on these pages.
- **Bonus for "No answer":** If you don't know the answer to a question, you may want to leave blank the corresponding space for the solution. This signals that you give no answer for that question. For each "no answer" you will receive a bonus of 10% of the maximum score for that question. Each question counts individually for the bonus, i.e. if an exercise is composed by more than one question, answering one question does not affect the possibility of receiving the bonus for another question in the same exercise. If you begin to write something and later wish you had left it blank, just draw a line over the text and write in large letters "BLANK".

Exercise 1 (30 pts)

Consider the regular expression $E = ((a + ab)b)^*$ on the alphabet $\Sigma = \{a, b\}$.

1. (15 pts) Give the minimal (deterministic) Finite Automaton recognizing this language.

2. (15 pts) Prove that the above FA is indeed minimal.

Exercise 2 (20 pts)

For each of the following two languages, say whether it is regular or not. Justify your answer.

1. (10 pts) $L_1 = \{x \in \{a,b\}^* | \ |x| = 2k \text{ for some } k \geq 0\}$

2. (10 pts) $L_2 = \{x \in \{a,b\}^* | \ |x| = 2^k \text{ for some } k \geq 0\}$

Exercise 3 (10 pts)

Given an alphabet Σ, is it decidable whether a given FA M accepts all strings in Σ^* (i.e. whether $L(M) = \Sigma^*$)? Justify your answer.

Exercise 4 (40 pts)

Consider the following grammar G generating the strings of balanced parentheses

$$S \to \lambda \mid SS \mid (S)$$

Given $x \in \{(,)\}^*$, let $n_(x), n_)(x)$ denote the number of symbols "(" and ")" occurring in x, respectively

1. (15 pts) Prove that $\forall x \in L(G), n_(x) = n_)(x)$ holds.

2. (5 pts) Let $L = \{x \in \{(,)\}^* |\ n_(x) = n_)(x)\}$. Say whether $L = L(G)$ or not. Justify your answer.

3. (20 pst) Prove whether G is ambiguous or not.