CSE 468, Fall 2000

Final Exam, 15 Dec 2000

- **Please write your name and SSN at the top of the sheet.**
- The exam consists of 5 exercises, totalling 100 points. Please write your answers directly on these pages.

Exercise 1 (30 pts)

Consider the following grammar G:

$$S \to \lambda \mid aSbSa \mid aba$$

Given $x \in \{a, b\}^*$, let $n_a(x), n_b(x)$ denote the number of symbols "a" and "b" occurring in x, respectively

1. (15 pts) Prove that $\forall x \in L(G),\ n_a(x) = 2n_b(x)$ holds.

2. (5 pts) Let $L = \{x \in \{a, b\}^* \mid n_a(x) = 2n_b(x)\}$. Say whether $L = L(G)$ or not. Justify your answer.

3. (10 pst) Say whether G is ambiguous or not, and prove your answer. If it is ambiguous, please define an equivalent unambiguous grammar.

Exercise 2 (30 pts)

For each of the following languages, say whether it is context-free or not. Justify your answer, namely: if the language is context-free, then give a CFG which generates it or a PDA which recognizes it. If the language is not context-free, then prove it by using the pumping lemma.

1. (10 pst) $L_1 = \{a^k b^m c^n | k, m, n \geq 0 \text{ and } (k = n \text{ or } m = k + n)\}$

2. (10 pst) $L_2 = \{a^k b^n c^k d^n | k, n \geq 0\}$

3. (10 pst) $L_3 = \{a^k b^k c^n d^n | k, n \geq 0\}$

Exercise 3 (15 pts)

Give a deterministic PDA recognizing the language $L = \{xc \mid x \in \{a, b\}^* \text{ and } n_a(x) = n_b(x)\}$, where $n_a(x), n_b(x)$ are defined as in Exercise 1. Note that L is the set of strings which have the same number of a's and b's, and terminate with a c. Examples: c, abc, $aabbc$, $babac$, $babbaac$, etc.

Exercise 4 (15 pts)

Give a Turing machine that recognizes the language $L = \{a^k b^k c^k | k \geq 0\}$. Please write some comments to explain how your Turing machine works.

Exercise 5 (10 pts)

Let e be an encoding of Turing machines as strings on $\{0,1\}$, i.e. $e : TM \to \{0,1\}^*$. Consider the language

$$L = \{e(M) | M \text{ is a Turing machine and } 00 \in L(M)\}$$

Say which of the following alternatives is correct, and justify your answer

1. L is recursive,
2. L is recursively enumerable, but not recursive
3. L is not recursively enumerable.