CSE 428 **Semantics for an Imperative Language** **Fall 1998**

Notation:

ρ**:** Environments (mapping identifiers to locations)

σ**:** Store (mapping locations to values)

I,O**:** Input and Output streams

$\mathcal{S}$**:** State consisting of a store, an input stream, and an output stream, written $\langle\sigma, I, O\rangle$

Operations:

Adding/Changing a binding: $\rho[\mathtt{x} \mapsto \ell]$, $\sigma[\ell \mapsto v]$

Streams: $a{::}I$ denotes an input stream whose next element (head) is a and whose remaining elements (tail) is I. So reading from the input stream $a{::}I$ yields element a and the new input stream is just I. Similarly, if O is the current output stream (denoting everything written so far), then writing v yields the new output stream $v{::}O$.

Judgment: Our semantics uses the judgments:

$\rho, \mathcal{S} \vdash \mathtt{c} \hookrightarrow_c \mathcal{S}'$**:** Evaluation of command `c` with respect to environment ρ and state $\mathcal{S}$ yields new state $\mathcal{S}'$;

$\rho, \mathcal{S} \vdash \mathtt{Seqc} \hookrightarrow_{seqc} \mathcal{S}'$**:** Evaluation of a sequence of commands `Seqc` with respect to environment ρ and state $\mathcal{S}$ yields new state $\mathcal{S}'$;

$\rho, \mathcal{S} \vdash \mathtt{e} \hookrightarrow_e v$**:** Evaluation of expression `e` with respect to environment ρ and state $\mathcal{S}$ yields value v.

$\rho, \sigma \vdash \mathtt{d} \hookrightarrow_d \rho', \sigma'$**:** Elaboration of single declaration `d` with respect to environment ρ and store σ yields new environment ρ' and new store σ'.

$\rho, \sigma \vdash \mathtt{Seqd} \hookrightarrow_{seqd} \rho', \sigma'$**:** Elaboration of a sequence of declarations `Seqd` with respect to environment ρ and store σ yields new environment ρ' and new store σ'.

Inference System:

$$\frac{}{\rho, \mathcal{S} \vdash \texttt{n} \hookrightarrow_e n} \qquad \frac{}{\rho, \langle \sigma, I, O \rangle \vdash \texttt{x} \hookrightarrow_e \sigma(\rho(\texttt{x}))}$$

$$\frac{}{\rho, \mathcal{S} \vdash \texttt{true} \hookrightarrow_e true} \qquad \frac{}{\rho, \mathcal{S} \vdash \texttt{false} \hookrightarrow_e false}$$

$$\frac{\rho, \mathcal{S} \vdash \texttt{e1} \hookrightarrow_e n_1 \quad \rho, \mathcal{S} \vdash \texttt{e2} \hookrightarrow_e n_2}{\rho, \mathcal{S} \vdash \texttt{e1 op e2} \hookrightarrow_e n_1 op n_2}$$

$$\frac{\rho, \langle \sigma, I, O \rangle \vdash \texttt{e} \hookrightarrow_c v \quad \rho(x) = \ell}{\rho, \langle \sigma, I, O \rangle \vdash \texttt{x := e} \hookrightarrow_c \langle \sigma[\ell \mapsto v], I, O \rangle}$$

$$\frac{\rho(x) = \ell}{\rho, \langle \sigma, a :: I, O \rangle \vdash \texttt{read(x)} \hookrightarrow_c \langle \sigma[\ell \mapsto a], I, O \rangle} \qquad \frac{\rho, \langle \sigma, I, O \rangle \vdash \texttt{e} \hookrightarrow_c v}{\rho, \langle \sigma, I, O \rangle \vdash \texttt{print(e)} \hookrightarrow_c \langle \sigma, I, v :: O \rangle}$$

$$\frac{\rho, \mathcal{S} \vdash \texttt{e} \hookrightarrow_c true \quad \rho, \mathcal{S} \vdash \texttt{c1} \hookrightarrow_c \mathcal{S}'}{\rho, \mathcal{S} \vdash \texttt{if e then c1 else c2} \hookrightarrow_c \mathcal{S}'} \qquad \frac{\rho, \mathcal{S} \vdash \texttt{e} \hookrightarrow_c false \quad \rho, \mathcal{S} \vdash \texttt{c2} \hookrightarrow_c \mathcal{S}'}{\rho, \mathcal{S} \vdash \texttt{if e then c1 else c2} \hookrightarrow_c \mathcal{S}'}$$

$$\frac{\rho, \mathcal{S} \vdash \texttt{Seqc} \hookrightarrow_{seqc} \mathcal{S}'}{\rho, \mathcal{S} \vdash \texttt{begin Seqc end} \hookrightarrow_c \mathcal{S}'}$$

$$\frac{\rho, \sigma \vdash \texttt{Seqd} \hookrightarrow_{seqd} \rho', \sigma' \quad \rho', \langle \sigma', I, O \rangle \vdash \texttt{Seqc} \hookrightarrow_{seqc} \mathcal{S}'}{\rho, \langle \sigma, I, O \rangle \vdash \texttt{begin Seqd in Seqc end} \hookrightarrow_c \mathcal{S}'}$$

$$\frac{\rho, \mathcal{S} \vdash \texttt{e} \hookrightarrow_c false}{\rho, \mathcal{S} \vdash \texttt{while e do c} \hookrightarrow_c \mathcal{S}}$$

$$\frac{\rho, \mathcal{S} \vdash \texttt{e} \hookrightarrow_c true \quad \rho, \mathcal{S} \vdash \texttt{c} \hookrightarrow_c \mathcal{S}' \quad \rho, \mathcal{S}' \vdash \texttt{while e do c} \hookrightarrow_c \mathcal{S}''}{\rho, \mathcal{S} \vdash \texttt{while e do c} \hookrightarrow_c \mathcal{S}''}$$

$$\frac{\rho, \mathcal{S} \vdash \texttt{c} \hookrightarrow_c \mathcal{S}'}{\rho, \mathcal{S} \vdash \texttt{c} \hookrightarrow_{seqc} \mathcal{S}'} \qquad \frac{\rho, \mathcal{S} \vdash \texttt{Seqc} \hookrightarrow_{seqc} \mathcal{S}' \quad \rho, \mathcal{S}' \vdash \texttt{c} \hookrightarrow_c \mathcal{S}''}{\rho, \mathcal{S} \vdash \texttt{Seqc ; c} \hookrightarrow_{seqc} \mathcal{S}''}$$

$$\frac{}{\rho, \sigma \vdash \texttt{x : t} \hookrightarrow_d \rho[\texttt{x} \mapsto \ell], \sigma[\ell \mapsto ?]} \ \ell \notin \mathsf{dom}(\sigma)$$

$$\frac{\rho, \sigma \vdash \texttt{Seqd} \hookrightarrow_{seqd} \rho', \sigma' \quad \rho', \sigma' \vdash \texttt{d} \hookrightarrow_d \rho'', \sigma''}{\rho, \sigma \vdash \texttt{Seqd ; d} \hookrightarrow_{seqd} \rho'', \sigma''}$$