

$$\begin{aligned}
I_\infty(S; O) &= H_\infty(S) - H_\infty(S|O) \\
&= -\log \max_s p(s) - (-\log(\sum_o \max_s (p(o|s) p(s)))) \\
&= \log \frac{\sum_o \max_s (p(o|s) p(s))}{\max_s p(s)} \\
&\leq \log \frac{\sum_o (\max_s p(o|s)) (\max_s p(s))}{\max_s p(s)} \\
&= \log \sum_o \max_s p(o|s)
\end{aligned}$$