Probabilistic Methods in Concurrency

Lecture 8

Encoding the π-calculus into the probabilistic asynchronous π-calculus

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Page of the course:
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Encoding π into π_{pa}

- **[[ ]] : $\pi \rightarrow \pi_{pa}$**

- **Fully distributed**

  [[P | Q]] = [[P]] | [[Q]]

- **Uniform**

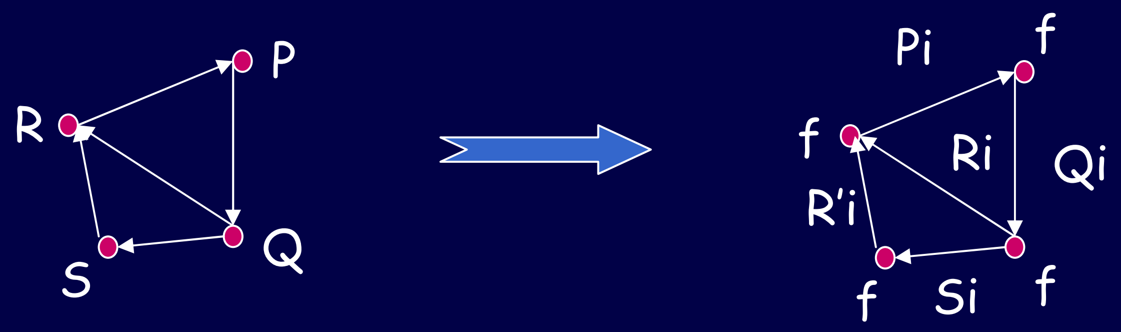
  [[P σ]] = [[P]] σ

- **Correct wrt a notion of probabilistic testing semantics**

  P must O iff [[P]] must [[O]] with prob 1

Encoding π into π_{pa}

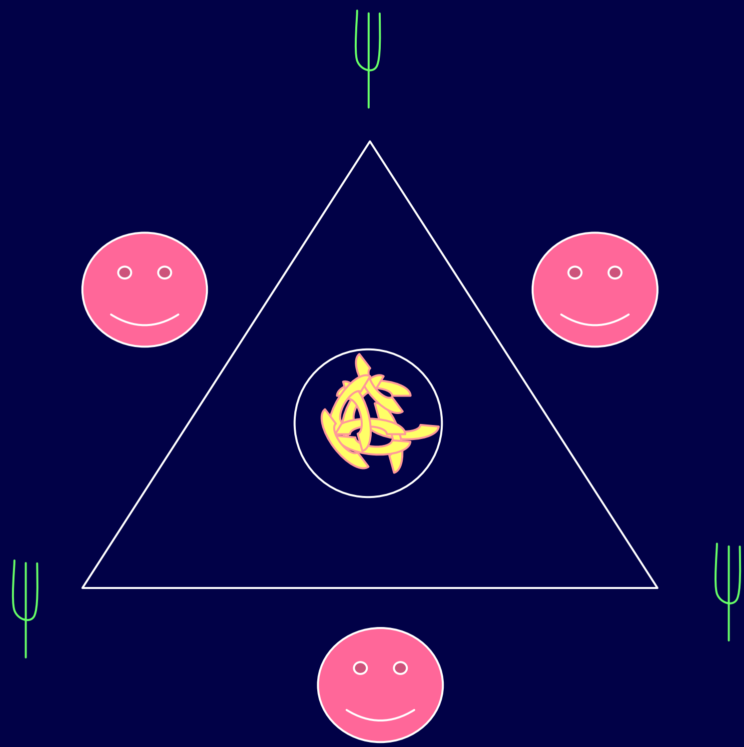
- Idea:
  - Every mixed choice is translated into a parallel comp. of processes corresponding to the branches, plus a lock f
  - The input processes compete for acquiring both its own lock and the lock of the partner
  - The input process which succeeds first, establishes the communication. The other alternatives are discarded



The problem is reduced to a generalized dining philosophers problem where each fork (lock) can be adjacent to more than two philosophers

Dining Philosophers: classic case

Each fork is shared by exactly two philosophers



The algorithm of Lehmann and Rabin

1. Think
2. choose first_fork in {left,right}   %commit
3. if taken(first_fork) then *goto* 3
4. take(first_fork)
5. if taken(first_fork) then {release(firstfork); *goto* 2}
6. take(second_fork)
7. eat
8. release(second_fork)
9. release(first_fork)
10. *goto* 1

Problems

- Wrt to our encoding goal, the algorithm of Lehmann and Rabin has two problems:

  1. It only works for certain kinds of graphs

  2. It works only for **fair** schedulers

- Problem 2 however can be solved by replacing the busy waiting in step 3 with suspension.

  [Duflot, Friburg, Picaronny 2002] – see also Herescu's PhD thesis

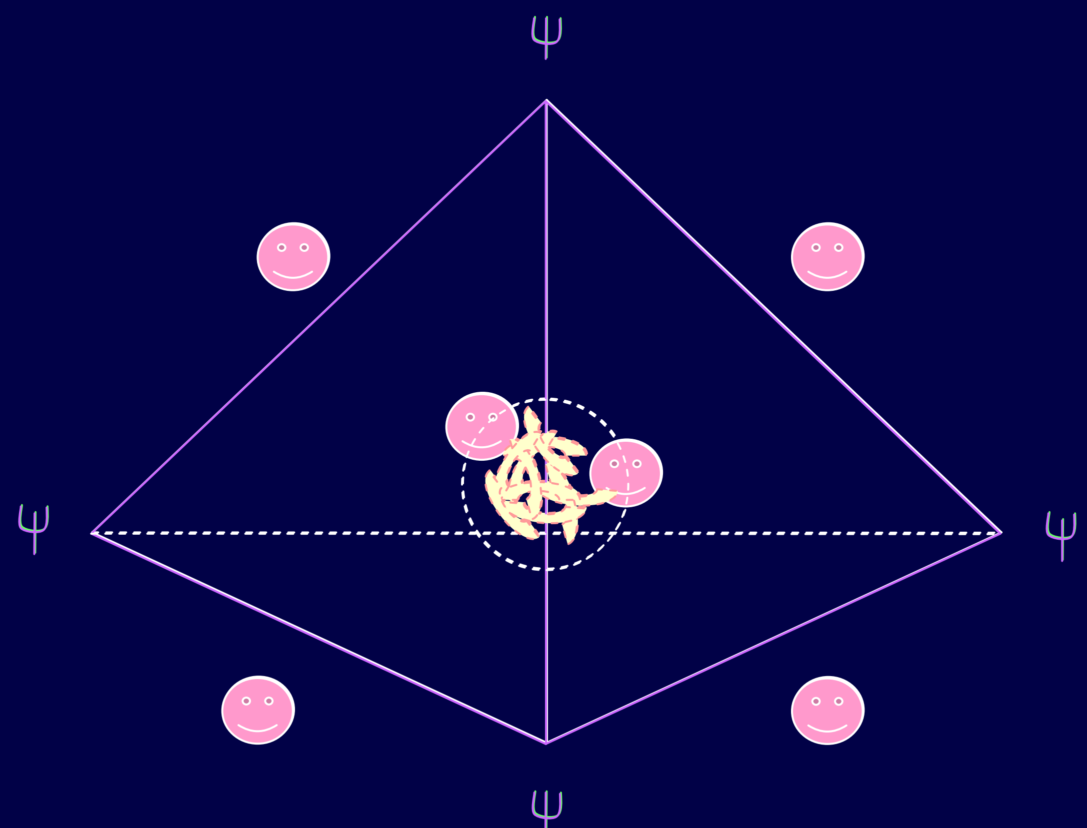
The algorithm of Lehmann and Rabin

Modified so to avoid the need for fairness

1. Think
2. choose first_fork in {left,right}   %commit
3. if taken(first_fork) then wait
   if taken(first_fork) then goto 3
4. take(first_fork)
5. if taken(first_fork) then goto 2
6. take(second_fork)
7. eat
8. release(second_fork)
9. release(first_fork)
10. goto 1

Dining Phils: generalized case

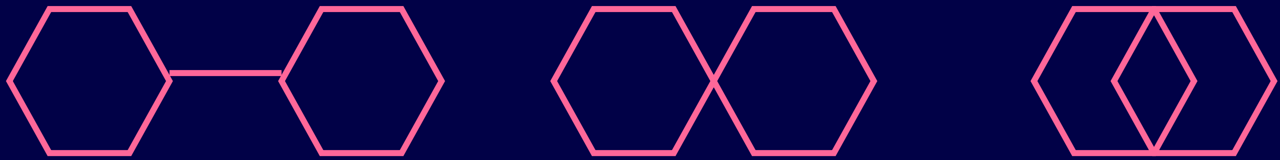
Each fork can be shared by more than two philosophers



Dining Phils: generalized case

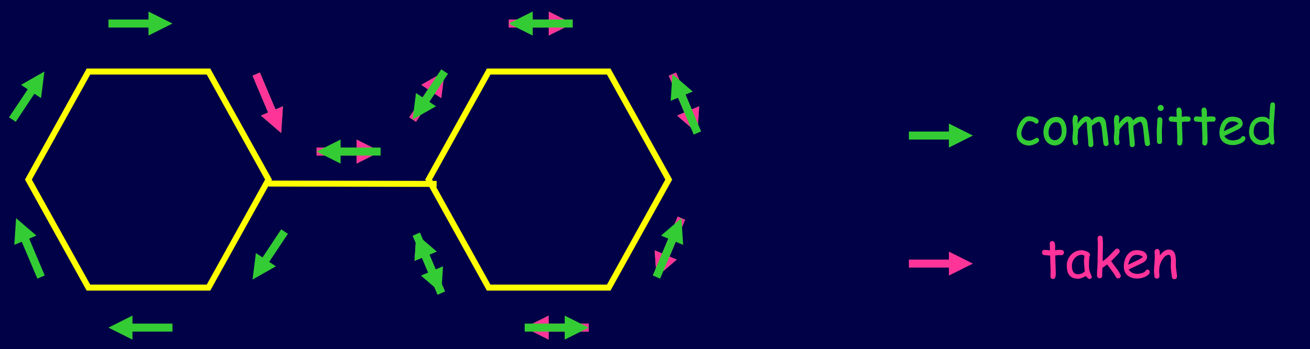
- **Theorem:** The algorithm of Lehmann and Rabin is deadlock-free if and only if all cycles are pairwise disconnected

- There are essentially three ways in which two cycles can be connected:



Proof of the theorem

- If part) Each cycle can be considered separately. On each of them the classic algorithm is deadlock-free. Some additional care must be taken for the arcs that are not part of the cycle.
- Only if part) By analysis of the three possible cases. Actually they are all similar. We illustrate the first case



Proof of the theorem

- The initial situation has probability p > 0
- The scheduler forces the processes to loop
- Hence the system has a deadlock (livelock) with probability p

- Note that this scheduler is **not fair**. However we can define even a fair scheduler which induces an infinite loop with probability > 0. The idea is to have a scheduler that "gives up" after n attempts when the process keep choosing the "wrong" fork, but that increases (by f) its "stubborness" at every round.

- With a suitable choice of n and f we have that the probability of a loop is p/4

Solution for the Generalized DP

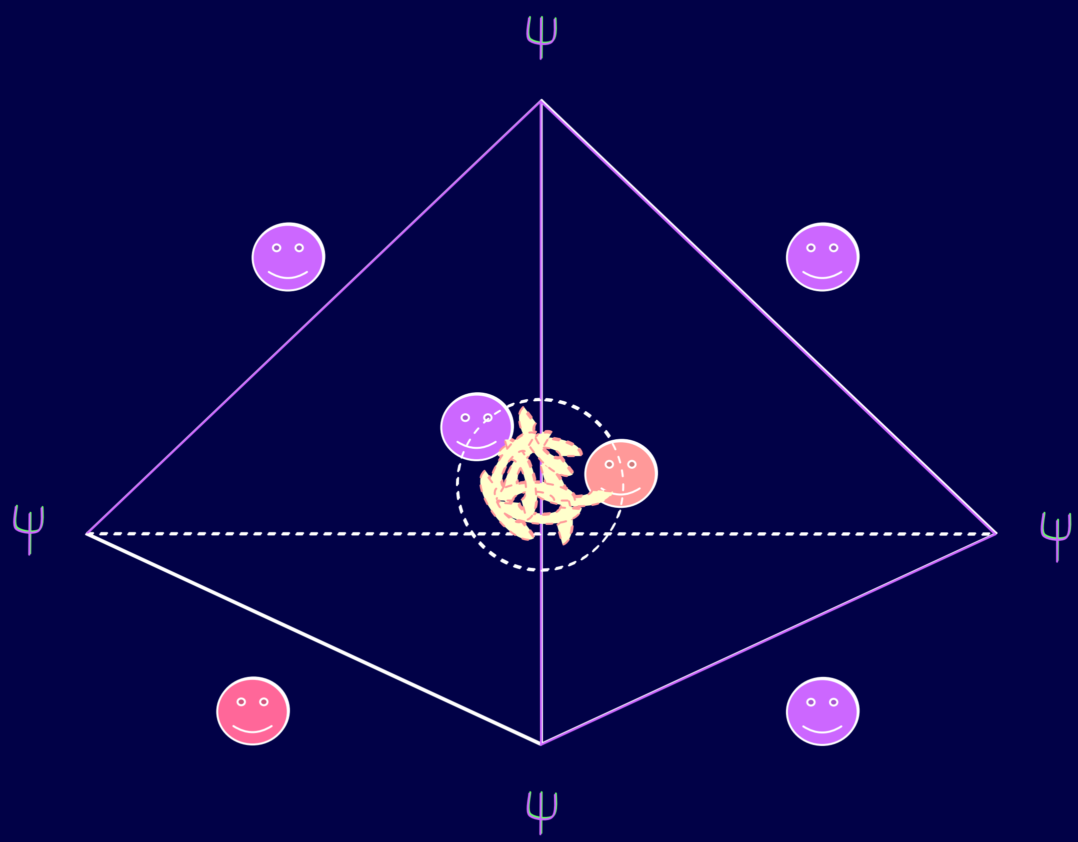
- As we have seen, the algorithm of Lehmann and Rabin does not work on general graphs
- However, it is easy to modify the algorithm so that it works in general
- The idea is to reduce the problem to the pairwise disconnected cycles case:

  Each fork is initially associated with one token. Each phil needs to acquire a token in order to participate to the competition. After this initial phase, the algorithm is the same as the Lehmann & Rabin's

  Theorem: The competing phils determine a graph in which all cycles are pairwise disconnected

  Proof: By case analysis. To have a situation with two connected cycles we would need a node with two tokens.

Dining Phils: generalized case



Reduction to the classic case: each fork is initially associated with a token. Each phil needs to acquire a token in order to participate to the competition. The competing phils determine a set of subgraphs in which each subgraph contains at most one cycle