Probabilistic Methods in Concurrency

Lecture 4

Problems in distributed systems for which only randomized solutions exist

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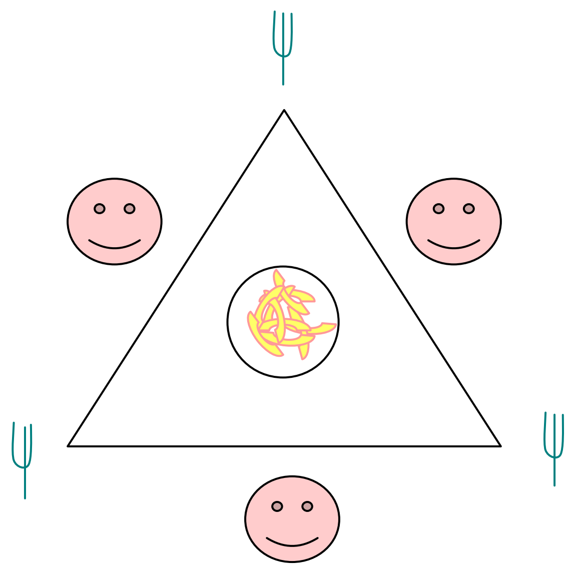
(1) The dining philosophers

- Each philosopher needs exactly two forks
- Each fork is shared by exactly two philosophers
- A philosopher can access only one fork at the time



Intended properties of solution

- **Deadlock freedom** (aka **progress**): if there is a hungry philosopher, a philosopher will eventually eat
- **Starvation freedom**: every hungry philosopher will eventually eat (but we won't consider this property here)
- **Robustness wrt a large class of adversaries**: Adversaries decide who does the next move (schedulers)
- **Fully distributed**: no centralized control or memory
- **Symmetric**:
  - All philosophers run the same code and are in the same initial state
  - The same holds for the forks

Non-existence of a “deterministic” solution

- Lehman and Rabin have shown that there does not exist a “deterministic” (i.e. non-probabilistic) solution to the dining philosophers, satisfying all properties listed in previous slide.

- The proof proceeds by proving that for every possible program we can define an adversary (scheduler) which preserves the initial symmetry

- **Note:** Francez and Rodeh did propose a “deterministic” solution using CSP. The solution to this apparent contradiction is that CSP cannot be implemented in a fully distributed way

The algorithm of Lehmann and Rabin

```
1.  Think
2.  randomly choose fork in {left,right}   %commit
3.  if taken(fork) then goto 3
4.                      else take(fork)
5.  if taken(other(fork)) then {release(fork); goto 2}
6.                             else take(other(fork))
7.  eat
8.  release(other(fork))
9.  release(fork)
10. goto 1
```

Correctness of the algorithm of Lehmann and Rabin

- **Theorem:** for every **fair** adversary, if a philosopher becomes hungry, then a philosopher (not necessarily the same) will eventually eat with probability 1.
- **Question:** why the fairness requirement? Can we write a variant of the algorithm which does not require fairness?

(2) The committee coordination problem

- **Description of the problem:** In a certain university, professors have organized themselves into committees. Each committee has a fixed membership roster of two or more professors. From time to time a professor may decide to attend a committee meeting. He then starts waiting and continues to wait until a meeting of a committee in which he is member is established.

- **Requirements**:
  - **Mutual exclusion:** No two committees meet simultaneously if they have a common member
  - **Weak Interaction Fairness (WIF) :** if all professors of a committee are waiting (i.e. the committee meeting is enabled) , then eventually some professor will attend a committee meeting (not necessarily the same).

    or
  - **Strong Interaction Fairness (SIF):** A committee meetig that is enabled infinitely often will be established infinitely often

The committee coordination problem

- **Question:** for which requirement among WIF and SIF do we have a correspondence with the synchronization mechanisms used in process calculi, like (the theory of) CSP and π?
  - General case equivalent to **multiway synchronization**, like the mechanism used in (the Theory of) CSP
  - Binary case equivalent to the **synchronization among two partners**, like in CCS and π

The algorithm of Joung and Smolka

```
while waiting do {
   randomly choose a committee M ;
   if TEST&OP(C_M,inc,inc) = n_M - 1
       then % a committee meeting is established
                attend the meeting M
       else { wait δ_M time ;
                if TEST&OP(C_M,no-op,dec) = 0
                    then % a committee meeting is established
                            attend the meeting M
                % else try another committee    }
```

Correctness of the algoritm

- Assumption:
  - $\delta_M > \max_{prof}\{time_{2-3}(M,prof)\}$
- **Theorem:** if a committee is enabled then a professor will eventually attend a meeting with probability 1 (WIF)
- **Theorem:** if a professor's transition from thinking to waiting does not depend on the random draws performed by other professors, then a committee meeting which is enabled infinitely often will eventually be established (SIF)

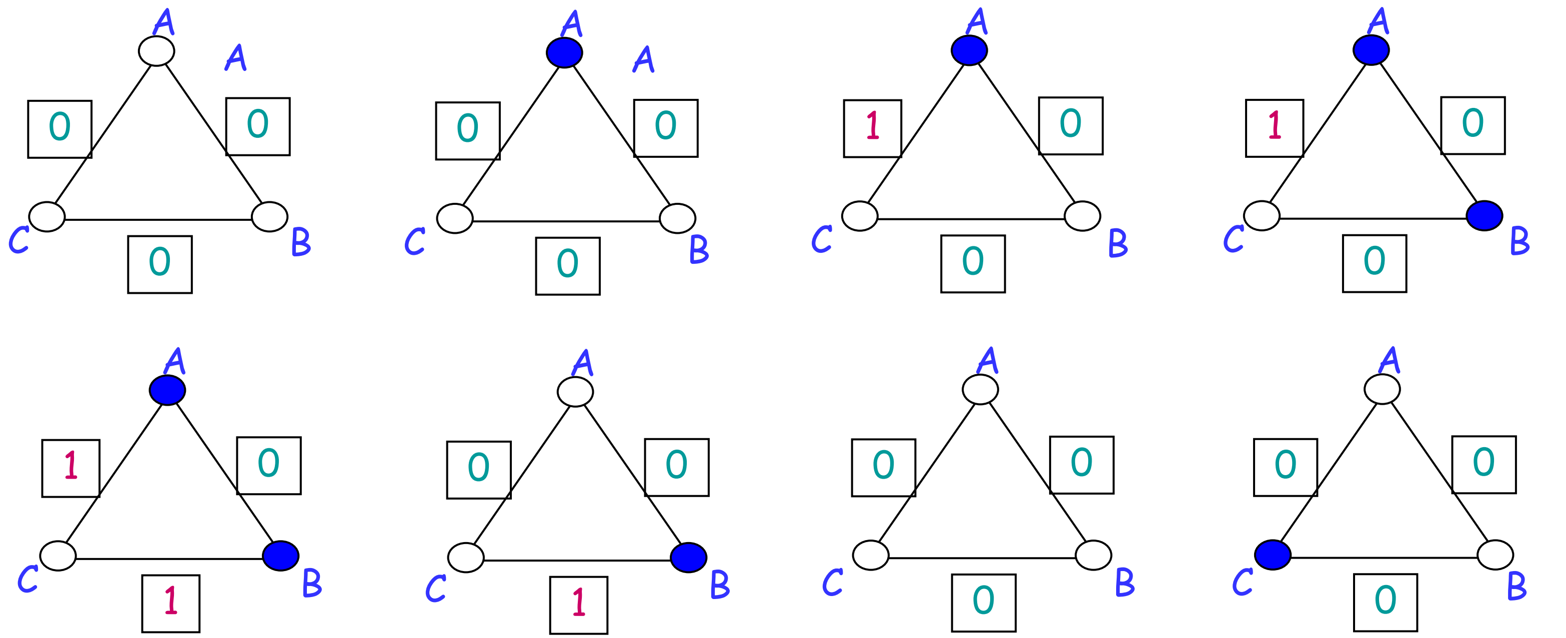
Importance of the assumption on δ

- The assumption on δ_M is an assumption about the degree of synchronism (in the sense of cooperation) of the system. In Distributed Algorithms there are three models of cooperation:
  1. Partially synchronous
  2. Asynchronous
  3. Synchronous (lockstep)

This assumption corresponds to (2)

- Hence this algorithm would not be suitable for implementing the synchronization mechanism of CSP or CCS in a fully distributed setting, since we need an asynchronous cooperation model.

Algorithm of Joung and Smolka:
Example of a livelock in absence of the assumption on δ



The states at the beginning of Lines 3, 5 and 6 are represented with a filled circle.
The states at the beginning of Line 1, 2 and 8 are represented with a white circle.
Lines 4 and 7 are never reached