

**Proposition** The intersection of an arbitrary collection of  $\sigma$ -fields on  $S$  is a  $\sigma$ -field on  $S$

**Proof:** Exercise

**Corollary** Given a set  $S$  and a set  $\mathcal{B}$  of subsets of  $S$ , there exists a least  $\sigma$ -field  $(S, \Sigma_{\mathcal{B}})$  containing  $\mathcal{B}$

We call  $\mathcal{B}$  *base* of  $(S, \Sigma_{\mathcal{B}})$  and we say that  $(S, \Sigma_{\mathcal{B}})$  *is generated by*  $\mathcal{B}$