2003 Paper 9 Question 15

Topics in Concurrency

This question assumes familiarity with the higher-order process language HOPLA, which has prefix types, function types, sum types and recursive types. Subject to suitable typings, HOPLA has transitions $t \xrightarrow{p} t'$ between closed terms t, t' and action p given by the following rules:

$$\frac{t[rec\,x\,t/x] \xrightarrow{p} t'}{rec\,x\,t \xrightarrow{p} t'} \qquad \frac{t_j \xrightarrow{p} t'}{\Sigma_{i\in I}\, t_i \xrightarrow{p} t'}\ j \in I$$

$$\frac{}{.t \xrightarrow{\bullet} t} \qquad \frac{u \xrightarrow{\bullet} u' \quad t[u'/x] \xrightarrow{p} t'}{[u > .x \Rightarrow t] \xrightarrow{p} t'}$$

$$\frac{t[u/x] \xrightarrow{p} t'}{\lambda x\, t \xrightarrow{u \mapsto p} t'} \qquad \frac{t \xrightarrow{u \mapsto p} t'}{t\ u \xrightarrow{p} t'} \qquad \frac{t \xrightarrow{p} t'}{a\ t \xrightarrow{a\,p} t'} \qquad \frac{t \xrightarrow{a\,p} t'}{\pi_a(t) \xrightarrow{p} t'}$$

(a) Write down the typing rule for $[u > .x \Rightarrow t]$. [2 marks]

(b) Let t be a term of type $\mathbb{P}$ with one free variable x of type $\mathbb{Q}$. Say t is *linear in* x iff for any sum of closed terms $\Sigma_{i\in I} u_i$ of type $\mathbb{Q}$

$$t[\Sigma_{i\in I} u_i/x] \ \sim_{\mathbb{P}}\ \Sigma_{i\in I} t[u_i/x].$$

(The relation $\sim_{\mathbb{P}}$ is that of bisimilarity between terms of type $\mathbb{P}$.)

Show from the transition semantics that the following terms, assumed well-typed and to have only x as free variable, are all linear in x:

$$\pi_a(x) \qquad x\ u \qquad [x > .y \Rightarrow u] \qquad (x \text{ is not free in } u).$$

Show that the prefix term $.x$ is not linear in x. (Here you may assume that x has any type that is convenient.) [7 marks]

(c) For u of sum type, let $[u > a.x \Rightarrow t]$ abbreviate $[\pi_a(u) > .x \Rightarrow t]$. Derive a rule for the transitions of $[u > a.x \Rightarrow t]$. [2 marks]

(d) Describe the type you would use to interpret CCS in HOPLA. Write down a HOPLA term that realises the parallel composition of CCS. What is its type? [5 marks]

(e) Write down the HOPLA types you would use to interpret variants of CCS (i) with value passing, and (ii) with process passing. [4 marks]