1993 Paper 8 Question 12

Concurrency

State the expansion law for observation congruence between CCS agents.

[4 marks]

A counter which can assume any of the natural numbers as its state is specified as a CCS agent by:

$$\begin{aligned} \mathit{Count}_0 &\stackrel{\text{def}}{=} \mathit{inc}.\mathit{Count}_1 + \mathit{zero}.\mathit{Count}_0 \\ \mathit{Count}_n &\stackrel{\text{def}}{=} \mathit{inc}.\mathit{Count}_{n+1} + \mathit{dec}.\mathit{Count}_{n-1} \qquad \text{for } n > 0 \end{aligned}$$

It is required to implement the counter by linking together several copies of an agent C and one agent B, where

$$\begin{aligned} C &\stackrel{\text{def}}{=} \mathit{inc}.(C \frown C) + \mathit{dec}.D \\ D &\stackrel{\text{def}}{=} \overline{d}.C + \overline{z}.B \\ B &\stackrel{\text{def}}{=} \mathit{inc}.(C \frown B) + \mathit{zero}.B \end{aligned}$$

Here the *linking*, $P \frown Q$, of two agents P and Q is an abbreviation for

$$(P[f] \mid Q[g]) \setminus L$$

where

$$L = \{i', z', d'\} \quad \text{and} \quad f(\ell) = \begin{cases} i' & \text{if } \ell = i \\ z' & \text{if } \ell = z \\ d' & \text{if } \ell = d \\ \ell & \text{otherwise} \end{cases} \quad \text{and} \quad g(\ell) = \begin{cases} i' & \text{if } \ell = \mathit{inc} \\ z' & \text{if } \ell = \mathit{zero} \\ d' & \text{if } \ell = \mathit{dec} \\ \ell & \text{otherwise} \end{cases}$$

(*a*) Use the expansion law to prove that $D \frown C \approx C \frown D$ and $D \frown B \approx B \frown B$. (You may assume without proof that $\tau.P \approx P$, for any P.) [5 marks]

(*b*) Letting $C^{(0)}$ stand for B and $C^{(n)}$ (when $n > 0$) stand for

$$\underbrace{(C \frown \cdots (C \frown (C}_{n \text{ copies of } C} \frown B)) \cdots)$$

show that $D \frown C^{(n)} \approx C^{(n)}$. Deduce that $C^{(n)}$ satisfies the defining equations for Count_n up to observation congruence. (You may assume without proof that $B \frown B \approx B$, that observation equivalence is a congruence relation for the operation of linking and that linking is associative up to observation equivalence. State carefully any other general principles you use.) [11 marks]