MPRI 2004/05 – Cours 2-3 (Concurrency)

Exam

17 February 2005

Part I - CCS

Question 1 For each of the following pairs of processes, say which is the strongest relation which relates them, among $\sim$, $\approx$ and $\cong$. Justify your answer, namely prove that the relation holds and that no stronger relation holds. Note: In some case, it may be that none of the relations holds

1. $\tau.P$ and $P + \tau.P$
2. P and $P \mid P$
3. A and B, where A and B are defined recursively by $A = a.A$ and $B = a.B \mid a.B$.
4. $(\nu b)(a.b.c.0 \mid d.\overline{b}.e.0)$ and $a.c.0 \mid d.e.0$
5. $P + Q$ and $\tau.P + \tau.Q$
6. $(\nu a)(\overline{a} \mid (a.P + a.Q))$ and $\tau.P + \tau.Q$

Question 2 We say that P *simulates* Q, notation $Q \preceq P$, if there exists a relation $\mathcal{R}$ such that $(Q, P) \in \mathcal{R}$ and for every $(T, U) \in \mathcal{R}$ we have:

$$T \xrightarrow{\alpha} T' \textit{ implies } \exists U'.\, U \xrightarrow{\alpha} U' \textit{ and } (T', U') \in \mathcal{R}$$

Say whether it is the case that $P \sim Q$ if and only if $P \preceq Q$ and $Q \preceq P$. Justify your answer, namely prove the above equivalence or find a counterexample

Part II - The Pi-calculus

1. The monadic synchronous π-calculus has the following grammar:

$$P \quad ::= \quad \overline{x}y.P \quad | \quad x(y).P \quad | \quad \boldsymbol{\nu}x.P \quad | \quad \mathbf{0} \quad | \quad P \mid P \quad | \quad !P$$

2. The following labelled transition rules give the behavior of replication:

$$\frac{P \xrightarrow{\alpha} P'}{!P \xrightarrow{\alpha} P' \mid !P} \text{if } \mathsf{bn}(\alpha) \cap \mathsf{fn}(P) = \varnothing \qquad \textsf{(bang-spawn)}$$

$$\frac{P \xrightarrow{\overline{x}y} P' \qquad P \xrightarrow{xy} P''}{!P \xrightarrow{\tau} (P' \mid P'') \mid !P} \qquad \textsf{(bang-comm)}$$

$$\frac{P \xrightarrow{\overline{x}(y)} P' \qquad P \xrightarrow{xy} P''}{!P \xrightarrow{\tau} \boldsymbol{\nu}y.(P' \mid P'') \mid !P} \text{if } y \notin \mathsf{fn}(P) \qquad \textsf{(bang-close)}$$

Question 3 Define what it means for a relation to be a strong labelled bisimulation.

Question 4 In the monadic π-calculus, prove that strong labelled bisimilarity is preserved by replication, i.e. $P \sim_\ell Q$ implies $!P \sim_\ell !Q$. To do this, construct an explicit relation $\mathcal{R}$ that contains the latter pairs (amongst others) and prove that it is a strong labelled bisimulation.

Note: You may use the fact that $\sim_\ell$ is preserved by parallel composition and by new binding, but if you do, cite the these results clearly.

You may use "up to" techniques; if so explain clearly how and where you use them.

Part III - The Asynchronous Pi-calculus

1. The monadic *asynchronous* π-calculus has the following grammar:

$$P \quad ::= \quad \overline{x}y \;\mid\; x(y).P \;\mid\; \boldsymbol{\nu}x.P \;\mid\; \mathbf{0} \;\mid\; P \mid P \;\mid\; !P$$

2. The set of *evaluation contexts* $\mathcal{E}$ is defined as

$$\mathcal{E} = \{\boldsymbol{\nu}\vec{z}.(- \mid T) \quad / \quad T \text{ a process}\}$$

   where the *binding names* of $D = \boldsymbol{\nu}\vec{z}.(- \mid T) \in \mathcal{E}$ are $\mathsf{bind}(D) = \{\vec{z}\}$.

3. *Reduction* and *strong barbs* are both characterisable in terms of evaluation contexts and structural congruence:

   - $P \longrightarrow P'$ iff there exists $D \in \mathcal{E}$, names x, v, and u, and a process P_0 such that $P \equiv D[\overline{x}v \mid x(u).P_0]$ and $P' \equiv D[\{v/u\}P_0]$.
   - $P{\downarrow}x$ iff there exists $D \in \mathcal{E}$ and names x and v such that $P \equiv D[\overline{x}v]$ and $x \notin \mathsf{bind}(D)$.

4. *Weak barbed bisimulation* $\dot{\approx}$ is the largest relation $\mathcal{R}$ that is symmetric and for which for all $(P, Q) \in \mathcal{R}$

   - if $P \longrightarrow P'$, there exists Q' such that $Q \longrightarrow^* Q'$ and $(P', Q') \in \mathcal{R}$;
   - if $P{\downarrow}x$ then $Q{\Downarrow}x$.

Question 5

1. Define the weak barb relation $Q{\Downarrow}x$ used above in the definition of $\dot{\approx}$.

2. Show why $\dot{\approx}$ is not preserved by parallel composition, i.e. find P, Q, R such that $P \dot{\approx} Q$ but not $P \mid R \dot{\approx} Q \mid R$.

3. Prove that $P \longrightarrow P'$ implies $\{x/y\}P \longrightarrow \{x/y\}P'$.

4. Consider an *equator* process

$$E_{x,y} = !x(u).\overline{y}u \mid !y(u).\overline{x}u$$

5. Prove that $E_{x,y} \mid D[\overline{x}v] \longrightarrow E_{x,y} \mid D[\overline{y}v]$ for $x, y \notin \mathsf{bind}(D)$.

6. Show that equators can act like substitutions, i.e. $\boldsymbol{\nu}y.(E_{x,y} \mid P) \dot{\approx} \{x/y\}P$. To do this, you need to demonstrate that there exists a weak barbed bisimulation containing the pair of processes in question.

7. Show that in the monadic *synchronous* π-calculus, i.e. the calculus where output prefixing $\overline{w}z.Q$ is allowed, that this is false. To do this exhibit a concrete P for which the barbed behaviour of $\boldsymbol{\nu}y.(E_{x,y} \mid P)$ differs from that of $\{x/y\}P$.