Comments on “Equivalence Constraint Satisfaction Problems”

Manuel Bodirsky

Institut für Algebra
TU Dresden, Germarny
Manuel.Bodirsky@tu-dresden.de

1 Counterexample

The conjecture about countably infinite ω-categorical structures Γ is not true in the form stated in the abstract of the paper “Equivalence Constraint Satisfaction Problems” by Bodirsky and Wrona. A counterexample can already be found among the reducts of the structure $(\mathbb{Q};<)$. Consider the structure $\Gamma = (\mathbb{Q}; I, <)$ where I is the four-ary relation $\{(x,y,u,v) \mid x = y \Rightarrow u = v\}$. This structure clearly is a model-complete core. Every function that depends on all of its arguments and preserves I must be injective. Let $d_0 := f(1,0,\ldots,0), d_1 := f(0,1,0,\ldots,0), \ldots, d_{n-1} := f(0,\ldots,0,1)$. We claim that for $n \geq 2$ there is no endomorphism a of Γ such that $d_i = a(d_{i+1})$ for all indices $i \in \mathbb{Z}/n\mathbb{Z}$. If $d_0 < d_1$, we would obtain that $d_0 < d_1 < d_2 < \cdots < d_{n-1}$, a contradiction. Similarly, we obtain a contradiction if $d_0 > d_1$.

On the other hand, no such counterexamples are known for the variant of the conjecture where we are looking for a polymorphism f and endomorphisms $a_1, \ldots, a_n$ satisfying the equations

$$a_1(f(y,x,\ldots,x)) = a_2(f(x,y,x,\ldots,x)) = \cdots = a_n(f(x,\ldots,x,y)),$$

that is, for *weak near unanimity polymorphisms modulo endomorphisms*.

2 Proof of Corrected Form of Conjecture

At the BIRS-Workshop Homogeneous Structures, Banff, Canada, November 2015, Michael Pinsker announced that in joint work with Libor Barto they proved the following variant of the conjecture (in adapted terminology) for ω-categorical structures Γ: either the model-complete core Δ of Γ has an expansion by finitely many constants such that the pseudovariety generated by its polymorphism algebra contains a two-element algebra all of whose operations are projections, or there is a 6-ary *Siggers polymorphism modulo endomorphisms*, that is, a 6-ary polymorphism f and endomorphisms a_1, a_2 satisfying $a_1(f(x,x,y,y,z,z)) = a_2(f(y,z,x,z,x,y))$ for all elements x, y, z of Δ.