Drawing toroidal 3-connected graphs, via Schnyder woods

Luca Castelli Aleardi (Ecole Polytechnique, LIX)

Keywords: Computational geometry, graphs, combinatorics, algorithms.

Internship Advisor: Luca Castelli Aleardi (`amturing@lix.polytechnique.fr`)

Expected Knowledge of the Student: algorithms and data structures and discrete mathematics. Basic notions of algebraic topology are welcome.

Related material: for more details on research topics please visit the internship webpage

Thesis Description: Graphs embedded on surfaces are among the most important objects in several application domains (e.g. Computer Graphics), as they correspond to the combinatorics underlying 3D meshes. This internship focuses on the problem of efficiently processing and visualizing toroidal meshes: in particular we aim to study toroidal Schnyder woods (which are a special decomposition of the mesh edges into three vertex spanning trees) and to apply them in order to draw genus 1 graphs.

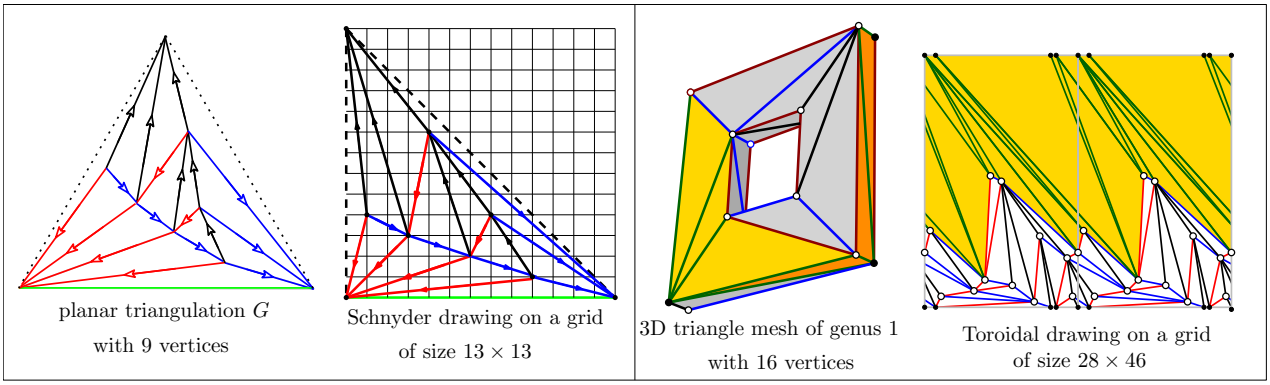
Drawing toroidal graphs (in the flat torus): related works. We consider the problem of devising a linear-time algorithm that computes a crossing-free layout of a graph with small integer coordinates (see pictures below): for the toroidal case a natural requirement is to ask the planar layout to be xy-periodic (in the flat torus). The existing solutions are either quite involved [3], and thus of mainly theoretical interest, or are restricted to a specific class of graphs (toroidal triangulations).

Goal of the internship: algorithms for drawing 3-connected toroidal graphs. In this internship we consider the more general case of toroidal 3-connected graphs (whose combinatorics correspond to general polygonal meshes). The main goal is to generalize the computation of crossing Schnyder woods [1] to the case of graphs which are not necessarily triangulated and to adapt the region-counting method devised for drawing toroidal triangulations [2]. We aim to draw general 3-connected toroidal graphs on a grid with small vertex coordinates: our hope is to improve the known bound on the grid size (currently $O(n^{\frac{3}{2}} \times n)$) and to achieve for the first time the optimal bound $O(n \times n)$ for a graph of size n.

References

[1] L. Castelli Aleardi, É. Fusy, J.-C. Ko, and R. Puscasu. Computation of toroidal schnyder woods made simple and fast: From theory to practice. In *41st International Symposium on Computational Geometry, SoCG 2025*, volume 332 of *LIPIcs*, pages 30:1–30:19, 2025.

[2] Luca Castelli Aleardi, Giselle Feng, and Éric Fusy. Schnyder drawing algorithm from cylindric to toroidal triangulations, with linear grid size. In *Symposium on Simplicity in Algorithms (SOSA 2026)*, 2026. to appear.



[3] D. Goncalves and B. Léveque. Toroidal maps : Schnyder woods, orthogonal surfaces and straight-line representations. *Discrete and Computational Geometry*, 51:67–131, 2014.