Another Look at Shape-Wilf-Equivalence and its Consequences

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(Joint with Sergi Elizalde & Dan Saracino)

Dartmouth College

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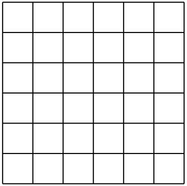

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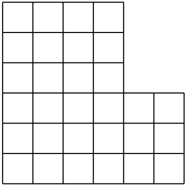

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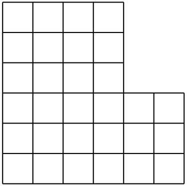
$F =$ 

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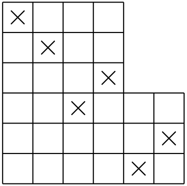

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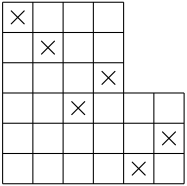
$F =$ 

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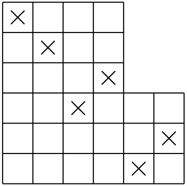
$F =$ 

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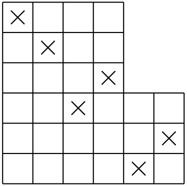
$F =$ 

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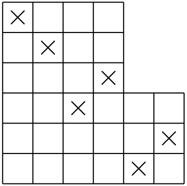
$F =$ 

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- $\mathcal{R}_F$ = set of all f.r.p.'s on F
- $\mathcal{R}_n = \bigcup_{F \in \mathcal{F}_n} \mathcal{R}_F$ - Analogous to S_n.

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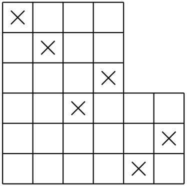

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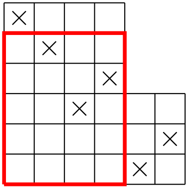

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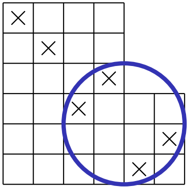

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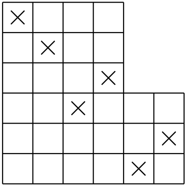

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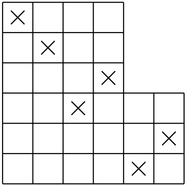

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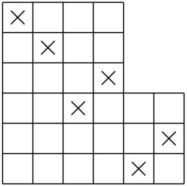

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Theorem (Backlin-West-Xin '01)
If $\sigma \sim \tau$ and ρ is any other permutation then

$$\sigma \oplus \rho \sim \tau \oplus \rho.$$

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There are 3 (shape-Wilf) equivalence classes:

$$231 \sim 312 \qquad < \qquad 123 \sim 321 \sim 213 \qquad < \qquad 132$$

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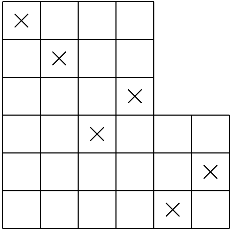

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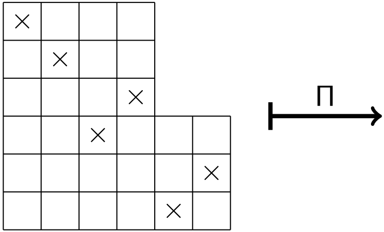

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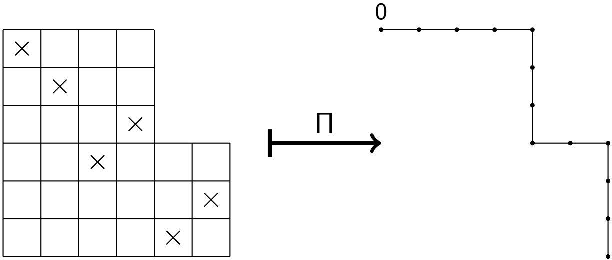

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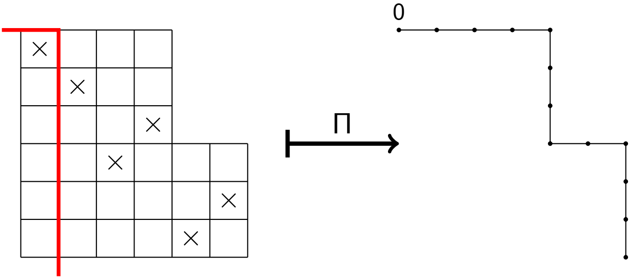

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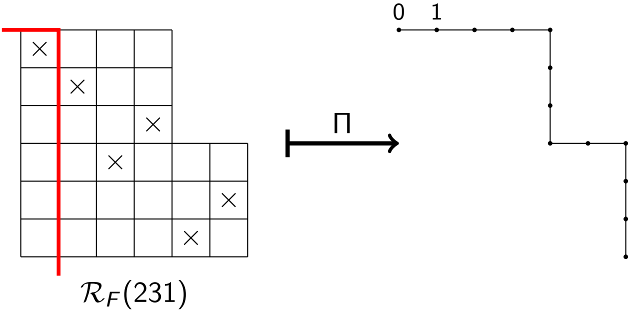

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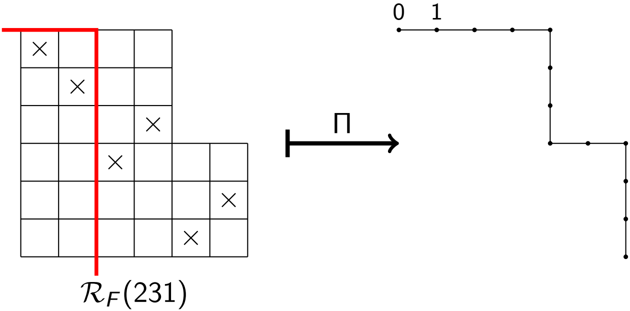

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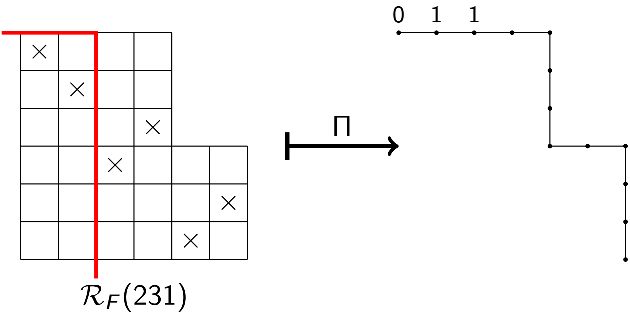

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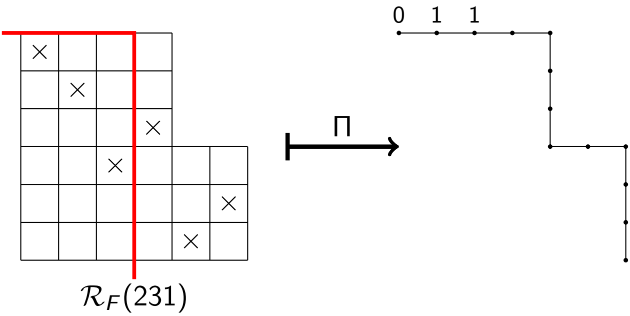

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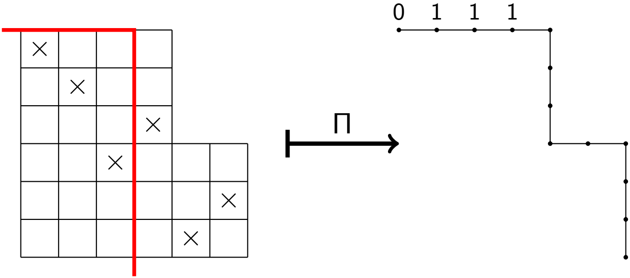

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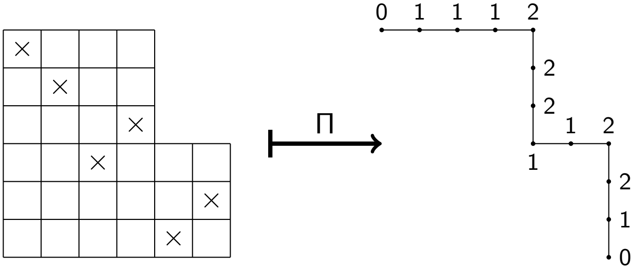

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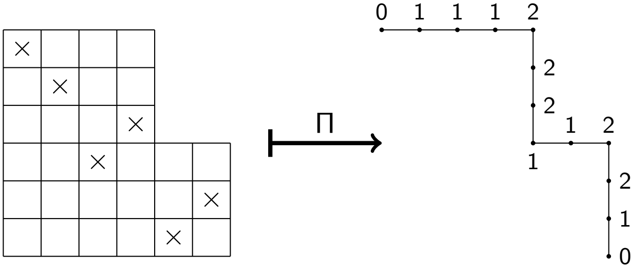

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Defining Properties of $\mathcal{L}_F(231)$

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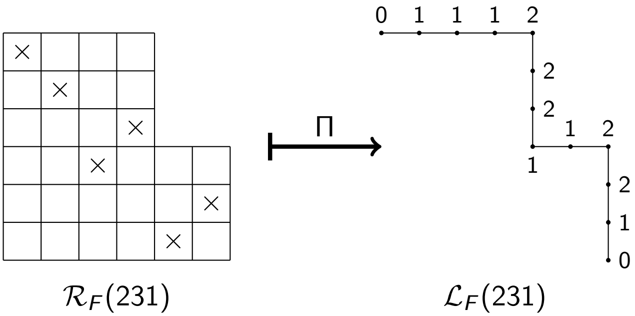

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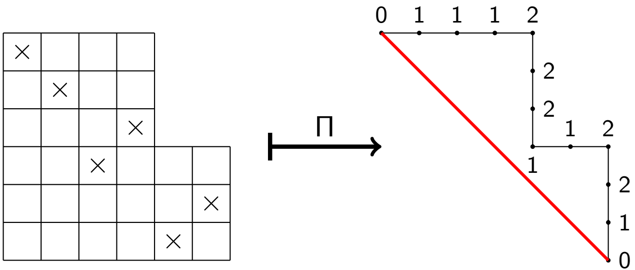

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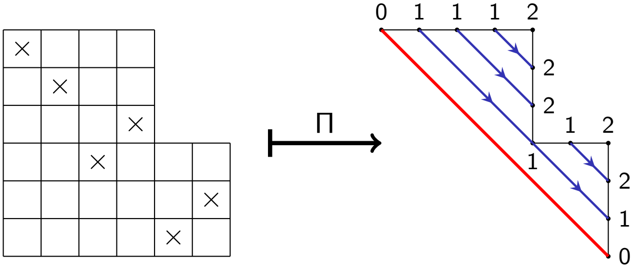

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- Diagonal Property:
  - Upper $\leq$ Lower

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Theorem (Bloom-Saracino '11)

The mapping

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where $\mathcal{L}_F(312) =$ *the set of labelings with the* **reverse** *diagonal property:*

$$\textit{Upper} \geq \textit{Lower}$$

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Corollary (Bloom-Saracino '11)

There exists an explicit (and simple) bijection between $\mathcal{R}_F(231)$ *and* $\mathcal{R}_F(312)$*, i.e.,* $231 \sim 312$*.*

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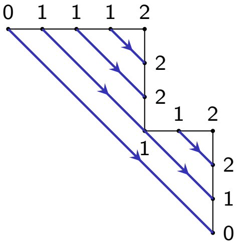

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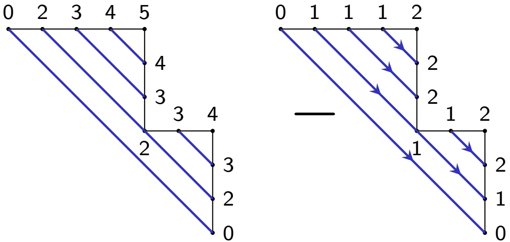

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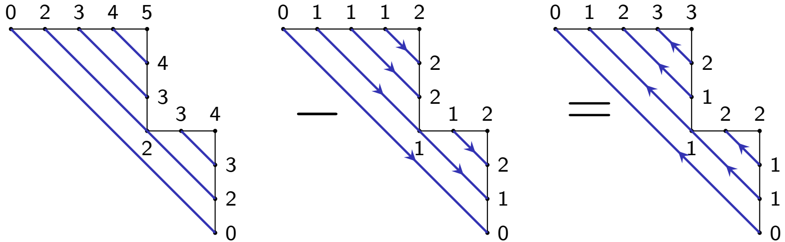

$\mathcal{L}_F(231)$ $\qquad$ $\mathcal{L}_F(312)$

Enumerative Results

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Theorem (Bloom-Elizalde '13)

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  - In fact the set of labelings with the monotone and diagonal properties are in bijection with rooted planar maps!

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321 (3-noncrossing)

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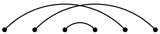

213

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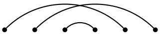

213

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Counting 2314–Avoiding Permutations

Counting 2314−Avoiding Permutations

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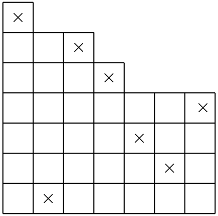

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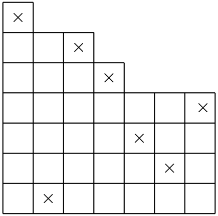

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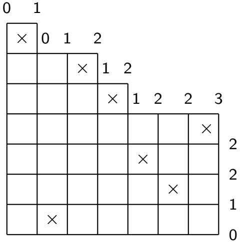

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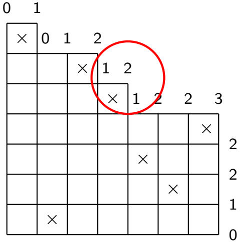

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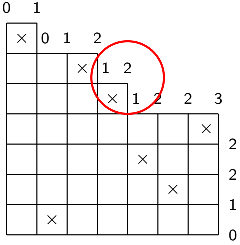

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→ The labels rounding any peak are of the form $a, a+1, a$.

→ We say such labeling have the **peak property**.

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Lemma (Bloom-Elizalde '13)

Our bijection $\Pi : \mathcal{R}_n(231) \to \mathcal{L}_n(231)$ *induces a bijection*

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Doing so we obtain Bóna's result:

$$\sum_{n\geq 0} |S_n(2314)| z^n = \sum_{n\geq 0} |\mathcal{L}_n^{\times}(312)| z^n = \frac{32z}{1+20z-8z^2-(1-8z)^{3/2}}.$$