A Brief Introduction to Model Checking

Model Checking

- A technique for verifying finite state concurrent systems;
  - a benefit on this restriction: largely automatic;
  - a problem to fight: state space explosion;
- A logic pointview: the system as a semantical model $\mathcal{M}$, and a property as a logical formula φ;
  - to check whether $\mathcal{M} \vdash \varphi$ (by *exhaustive search*);
  - possible approaches: model checking chooses to work on models directly;
- Reasonable efficiency, giving answers in seconds/minutes;
- Counter-examples provide insight to understand the failures.

Model Checking

- Pioneers' work: West and Zafiropulo (1977, 1978), Clarke and Emerson (1981), and Quielle and Sifakis (1981);
- Applications: electric circuits, communication protocols, digital controllers, system designs, ..., widely accepted in industry;
- A book: *Model Checking*, E.M. Clarke, O. Grumberg and D.A. Peled, MIT Press, 2000;
- Model checkers: FDR, Spin, Morϕ, νSMV, CADP, Uppaal, PRISM, HyTech, COSPAN, STeP, Kronos ...

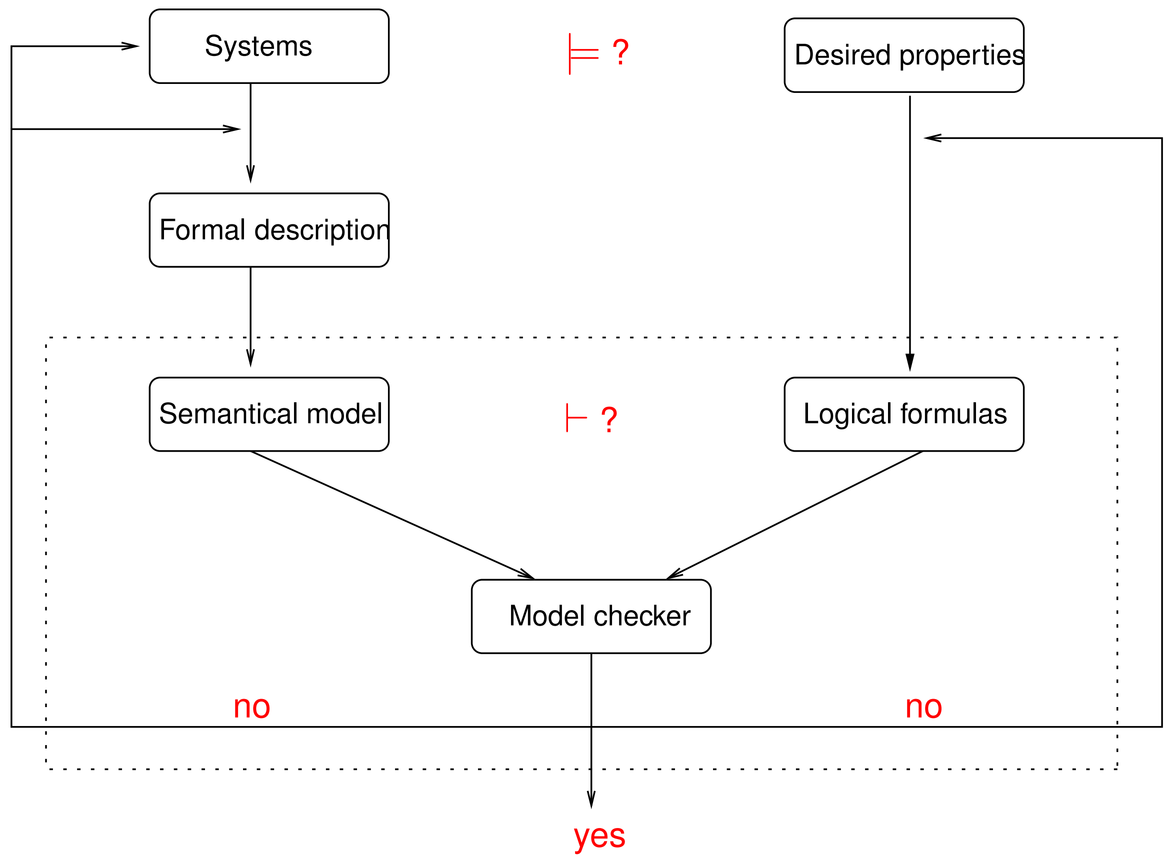
Model Checking

To apply it, we need the follows:

- Modeling languages: describe the systems, e.g. a process algebraic language μCRL;
  - semantics of the languages, e.g. LTS, Kripke structures, automata;
- Specification languages: formulate properties, e.g. LTL, CTL, regular alternation-free μ-calculus;
  - safety and liveness properties;
  - $[\mathsf{T}^{*}\cdot\mathsf{error}]\ \mathsf{F}$;
  - $[\ \mathsf{T}^{*}\cdot\mathsf{send}\cdot(\neg\mathsf{receive})^{*}]\ \langle(\neg\mathsf{receive})^{*}\cdot\mathsf{receive}\rangle\ \mathsf{T}$;
- Algorithms: verify properties.

Model Checking

The process:



Model Checking

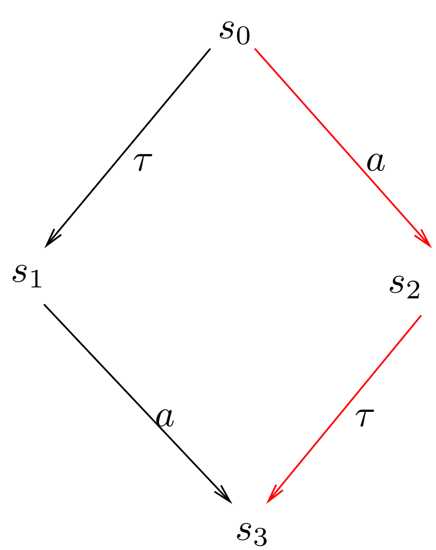
Research issues:

- Approaches to fight state space explosion;
- Expressiveness of logics;
- Efficiency of algorithms.

Model Checking

Partial order reduction:

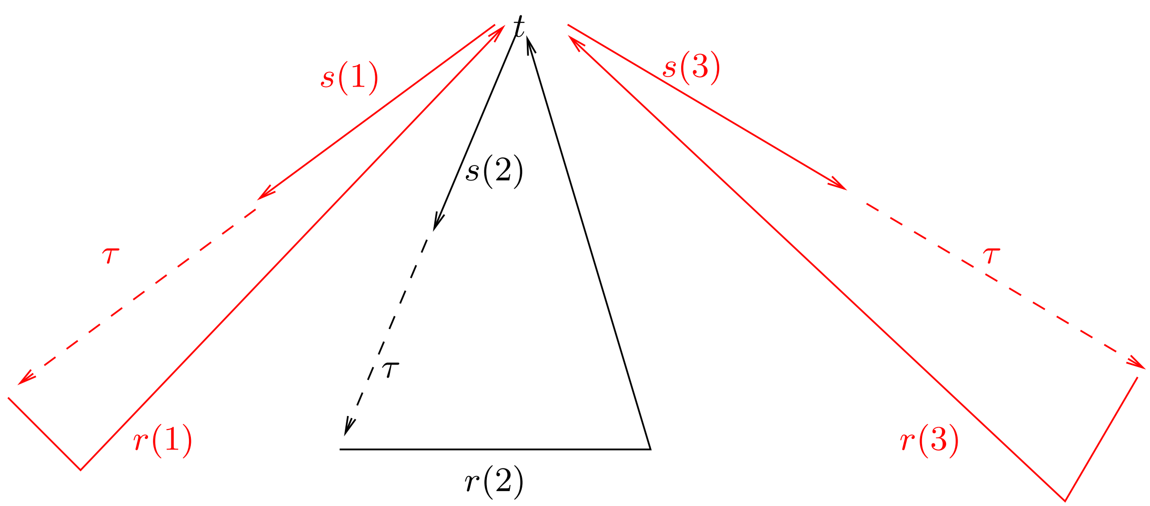
- Idea: fix a particular order of interleaving behaviors, while preserving properties of interest;
- CWI: τ-confluence reduction (preserving branching bisimulation).



Model Checking

Symmetry reduction (similar to data independence):

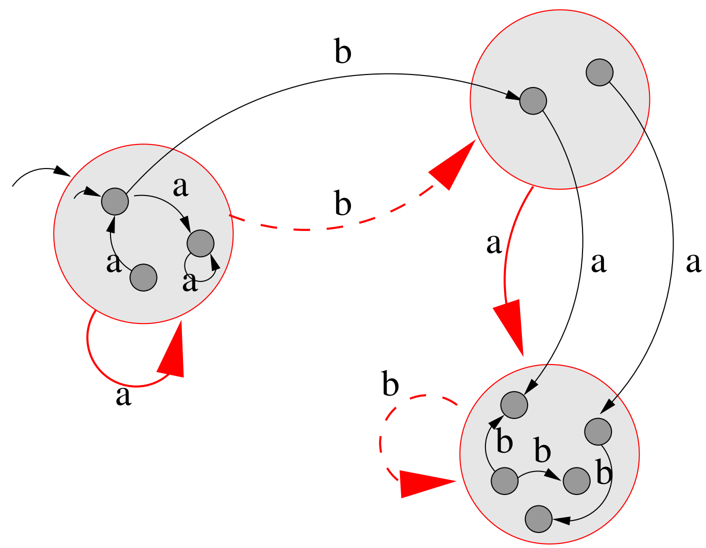
- Idea: construct a quotient structure, by exploiting automorphisms of the system's state space;
- CWI: symmetry reduction for LTSs.



Model Checking

Abstraction (on data):

- Idea: replace a semantical model by an abstract (simpler, finite) model, the abstraction needs to be safe;
- CWI: abstract interpretation for μCRL; patterns for uniform parallel processes, abstraction for liveness properties.



Model Checking

On-the-fly:

- Idea: not generate unnecessary state space, especially when a formula is false;
- CWI: interface with the model checker CADP.

Model Checking

Symbolic model checking (OBDDs):

- Kenneth L. McMillan, *PhD Thesis*, CMU, 1992;
- Idea: avoid explicit enumeration of set, by expressing set as a propositional formula;

  OBDDs as data structures to represent the state space;
- CWI: a checker for modal formulas for processes with data.

$S = [0, 2, 4, 5, 6, 7] \Rightarrow$

$S = [000, 010, 100, 101, 110, 111] \Rightarrow$

$S = \{s | s_3 = 0 \vee s_1 = 1\}$, $(s = s_1 s_2 s_3)$

Model Checking

Distributed and parallel model checking:

- Problem: the state space does not completely fit into the main memory of a computer;
- Idea: increase the computational power by building a cluster of stations;
- CWI: distributed state space generation and reduction w.r.t. strong and branching bisimulation.

Model Checking

More recent challenging issues:

- Timed, hybrid, probabilistic, mobile systems, e.g. Uppaal, Kronos, PRISM, ?;
- Software verification, e.g. Spin;
- Source code verification, e.g. Bandera;
- Infinite-state systems, e.g. regular model checkers Fast, Trex;
- Challenging case studies, e.g. NASA Mars exploration rover.

Simplifying Itai-Rodeh Leader Election for Anonymous Rings

Leader Election

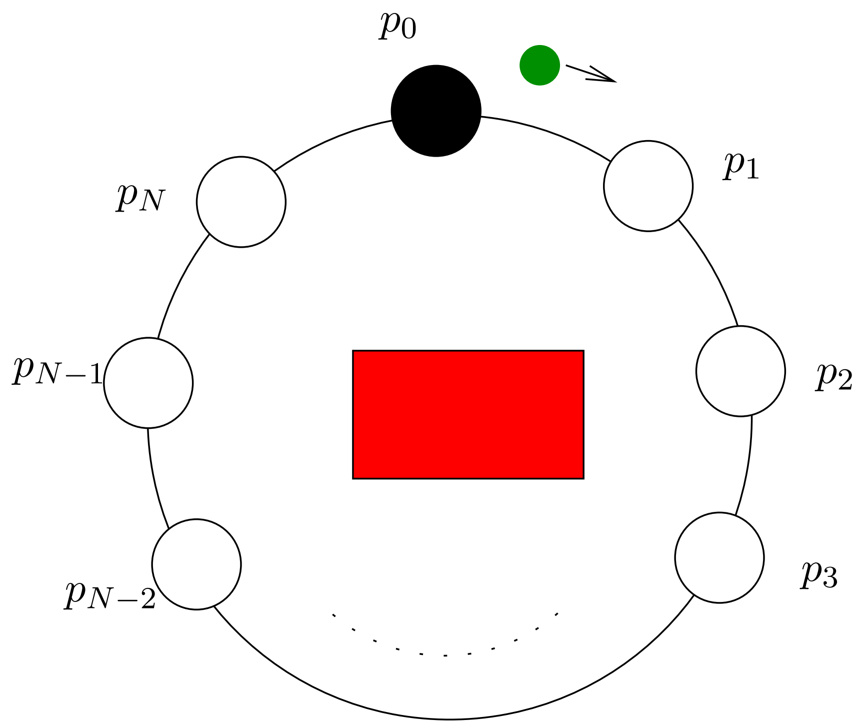


Figure 1: Mutual exclusion: token recovery.

Leader Election

Many algorithms:

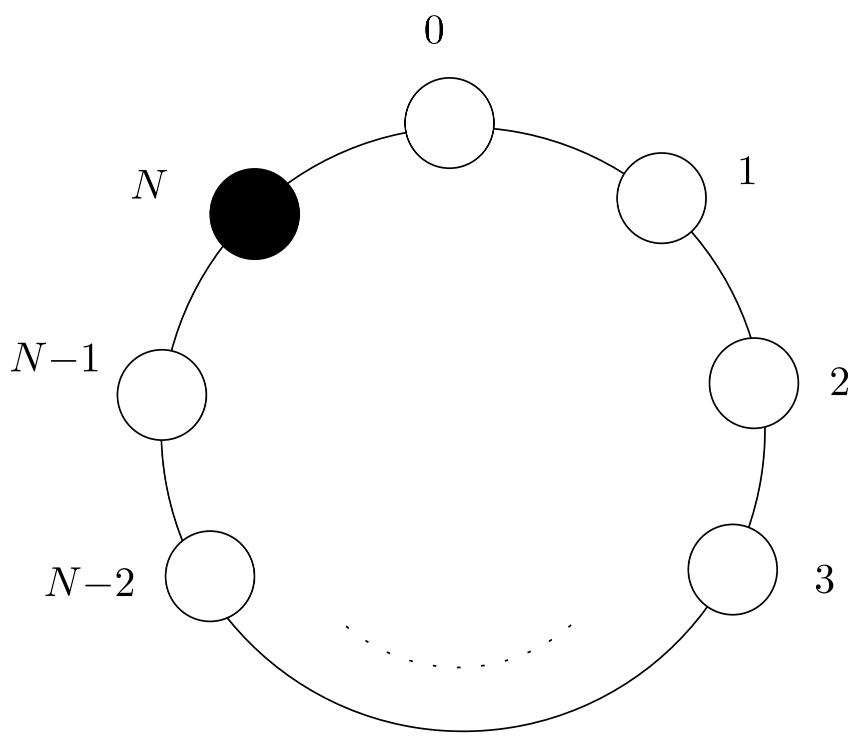
- Communication mechanism: *asynchronous vs. synchronous*;
- Process names: *unique identities vs. anonymous*;
- Network topology: *ring, tree, complete graph*;
- ...

Plan of the Talk

1. The Chang-Roberts algorithm and the Itai-Rodeh algorithm;
2. Algorithm $\mathcal{A}$: leader election without round numbers;
3. Algorithm $\mathcal{B}$: leader election without bits;
4. Performance analysis in PRISM;
5. Conclusions and future works.

The Chang-Roberts Algorithm

Processes have unique identity and send messages with identity; process with maximal identity is elected as the leader.



States of processes: $\{active, passive, leader\}$

Anonymous Rings

Some cases where processes cannot be distinguished by means of unique identities:

1. as the number of processes increases, it is difficult to keep all the identities of processes distinct;
2. identities cannot always be sent around the network, e.g. FireWire, the IEEE 1394 high performance serial bus.

The Itai-Rodeh Algorithm

Probabilistic method to break symmetry!

Assumption: processes have the knowledge of the ring size n.

Difficulties: each process selects a *random identity* from a finite set, so different processes may carry the same identity. Each process needs to

- recognize the message sent by its own – hop counter;
- realize name clashes – bit; and
- recognize old messages and start a new round – $round$ number.

The Itai-Rodeh Algorithm

- Initially, all processes are active, and each process p_i randomly selects its identity $id_i \in \{1, \ldots, k\}$ and sends the message $(id_i, 1, 1, true)$.
- Upon receipt of a message $(id, round, hop, bit)$, a passive process p_i ($state_i = passive$) passes on the message, increasing the counter hop by one; an active process p_i ($state_i = active$) behaves according to one of the following steps:

The Itai-Rodeh Algorithm

1. if $hop = n$ and $bit = true$, then p_i becomes the leader ($state'_i = leader$);
2. if $hop = n$ and $bit = false$, then p_i selects a new random identity $id'_i \in \{1, \ldots, k\}$, moves to the next round ($round'_i = round_i + 1$), and sends the message $(id'_i, round'_i, 1, true)$;
3. if $hop < n$ and $(round, id) = (round_i, id_i)$, then p_i passes on the message $(id, round, hop + 1, false)$;
4. if $(round, id) > (round_i, id_i)$, then p_i becomes passive ($state'_i = passive$) and passes on the message $(id, round, hop + 1, bit)$;
5. if $(round, id) < (round_i, id_i)$, then p_i purges the message.

The Itai-Rodeh Algorithm

Theorem 1 [Itai and Rodeh 1981] The Itai-Rodeh algorithm terminates with probability one, and upon termination a unique leader has been elected.

The Itai-Rodeh Algorithm

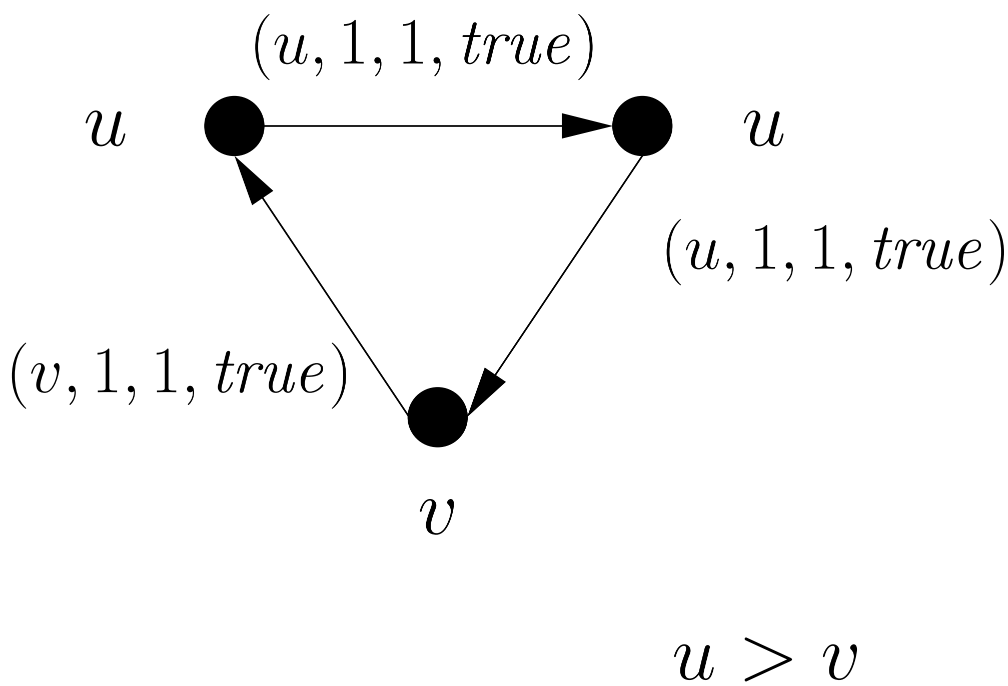


Figure 2: The Itai-Rodeh Algorithm: an example $n = 3$

The Itai-Rodeh Algorithm

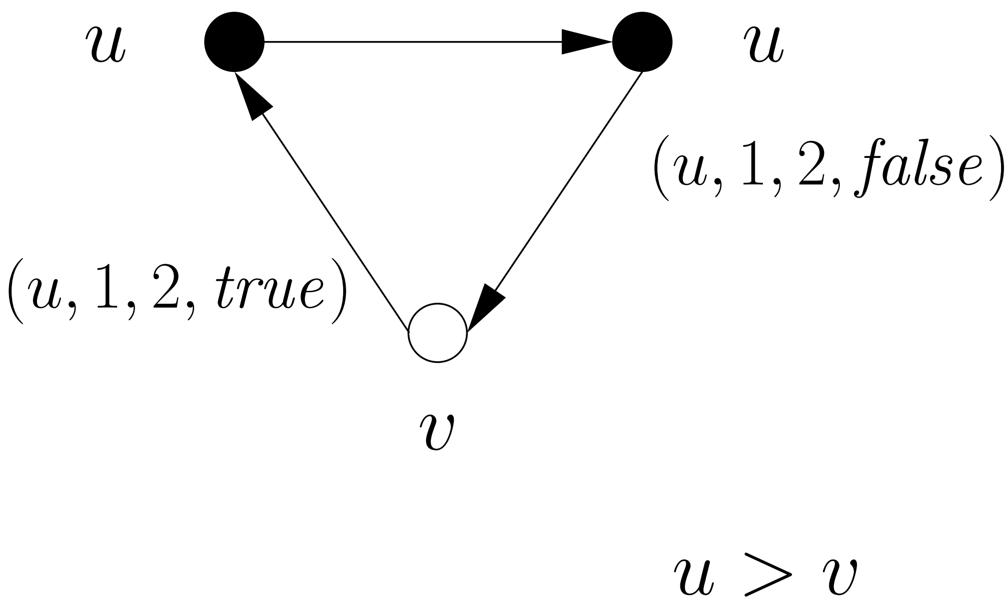


Figure 3: The Itai-Rodeh Algorithm: an example $n = 3$

The Itai-Rodeh Algorithm

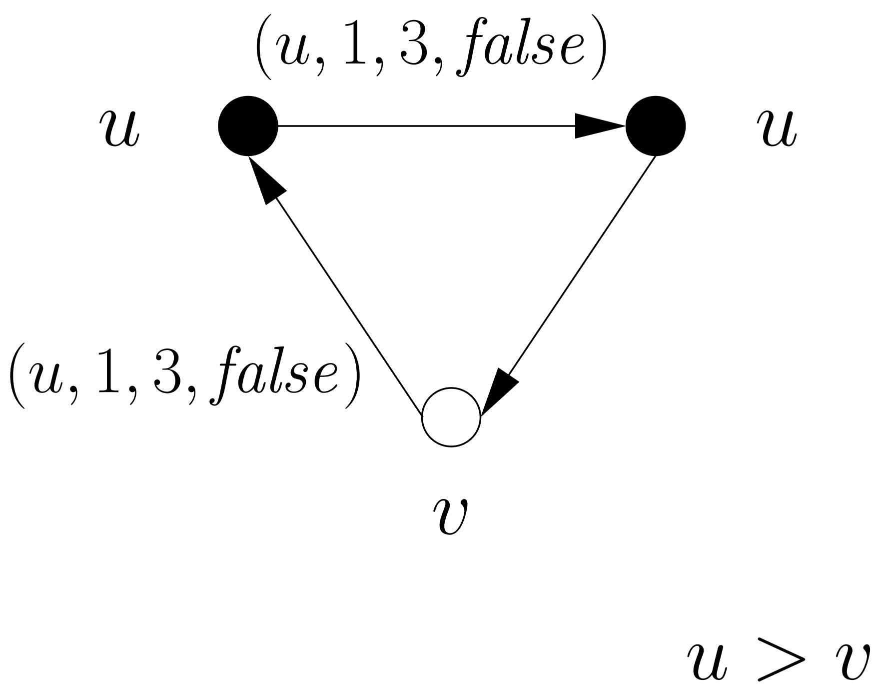


Figure 4: The Itai-Rodeh Algorithm: an example $n = 3$

The Itai-Rodeh Algorithm

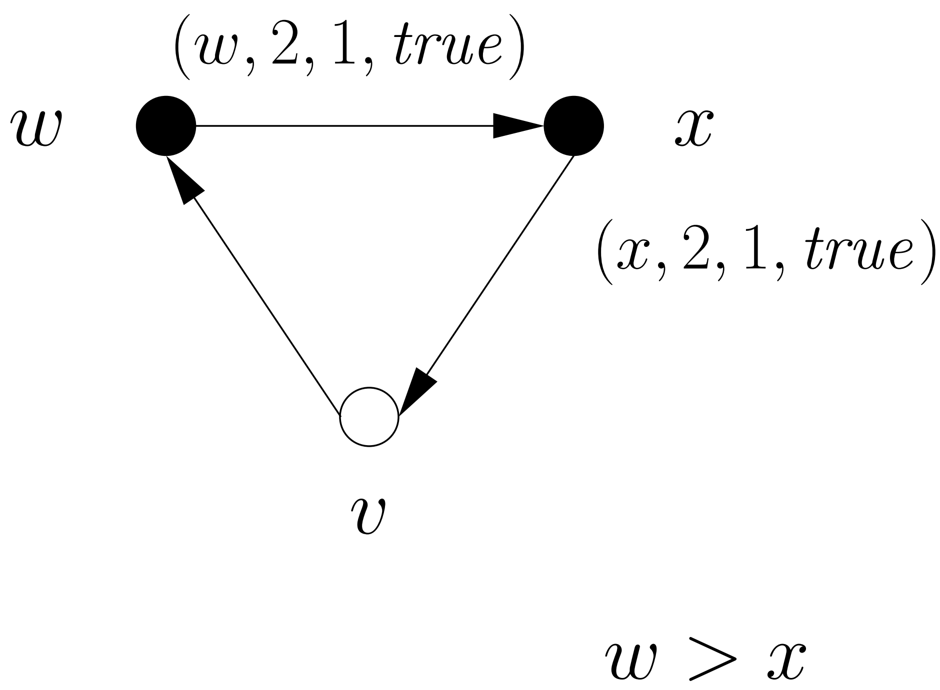


Figure 5: The Itai-Rodeh Algorithm: an example $n = 3$

The Itai-Rodeh Algorithm

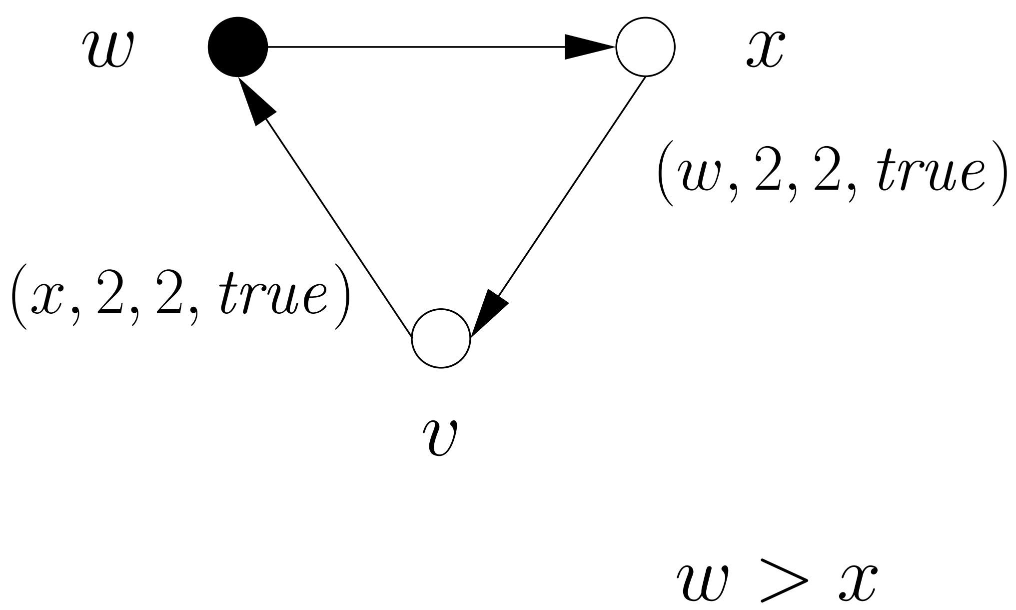
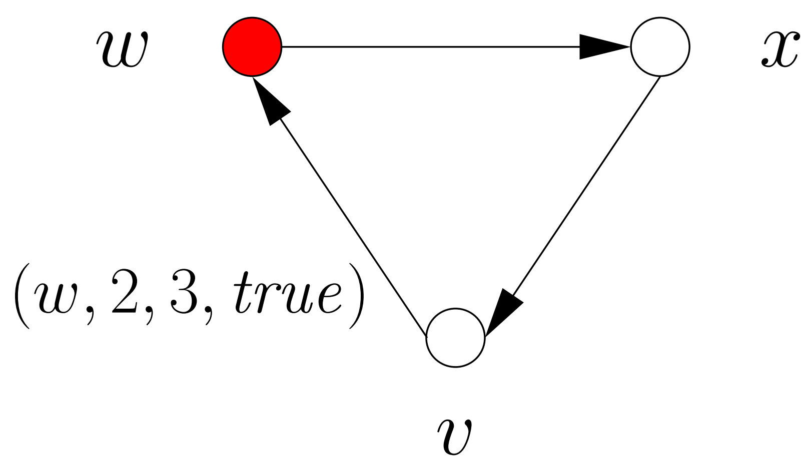


Figure 6: The Itai-Rodeh Algorithm: an example $n = 3$

The Itai-Rodeh Algorithm



$$w > x$$

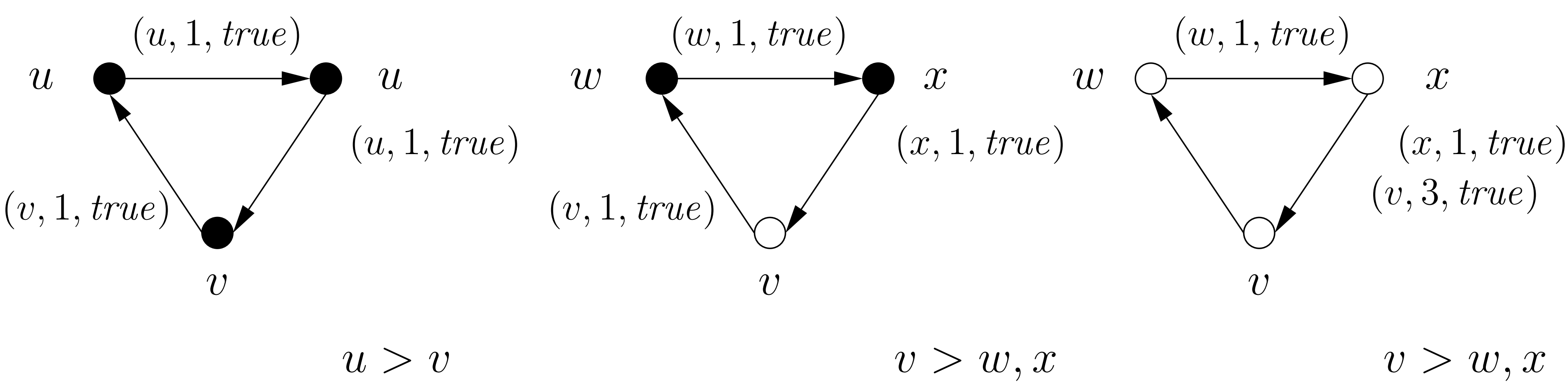
Figure 7: The Itai-Rodeh Algorithm: an example $n = 3$

Round Number are Essential!

Observations:

- Itai-Rodeh leader election has infinite state space, (due to round numbers).

Round numbers are essential if channels are not FIFO.



Leader Election without Round Numbers

Observations:

- if channels are FIFO, round numbers are redundant.

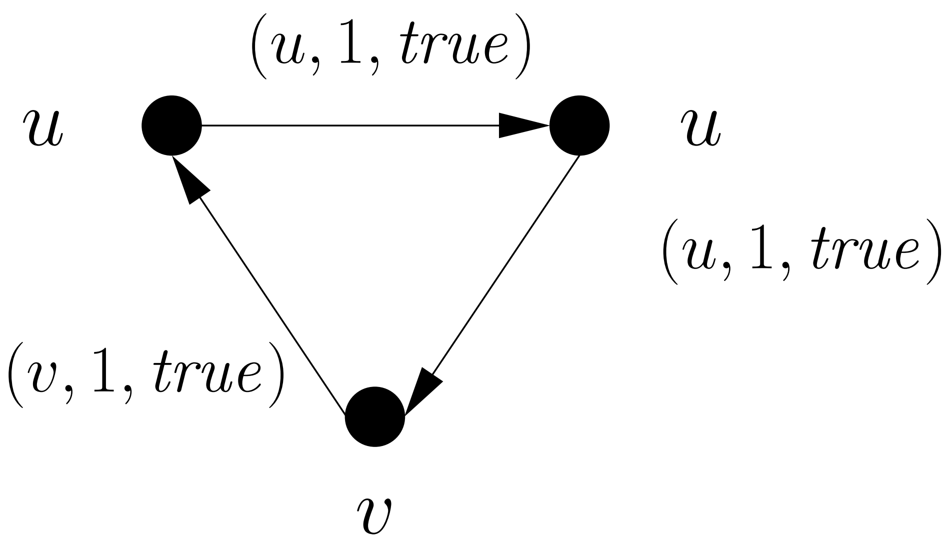
Proposition 2 Consider the Itai-Rodeh algorithm where all channels are FIFO. When an active process receives a message, then the round numbers of the process and the message are always the same.

Leader Election without Round Numbers

Algorithm $\mathcal{A}$: messages have the form of (id, hop, bit). Passive processes behave the same as before.

1. if $hop = n$ and $bit = true$, then p_i becomes the leader ($state'_i = leader$);
2. if $hop = n$ and $bit = false$, then p_i selects a new random identity $id'_i \in \{1, \ldots, k\}$ and sends the message $(id'_i, 1, true)$;
3. if $hop < n$ and $id = id_i$, then p_i passes on the message $(id, hop + 1, false)$;
4. if $id > id_i$, then p_i becomes passive ($state'_i = passive$) and passes on the message $(id, hop + 1, bit)$;
5. if $id < id$, then p_i purges the message.

Leader Election without Round Numbers



$$u > v$$

Figure 8: Algorithm $\mathcal{A}$: an example $n = 3$ (step 1)

Leader Election without Round Numbers

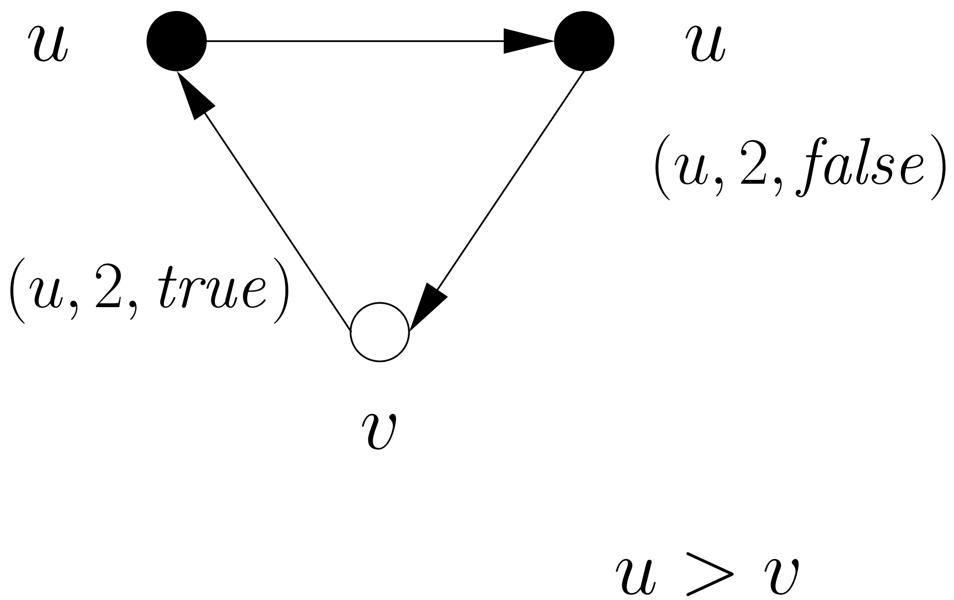


Figure 9: Algorithm $\mathcal{A}$: an example $n = 3$ (step 2)

Leader Election without Round Numbers

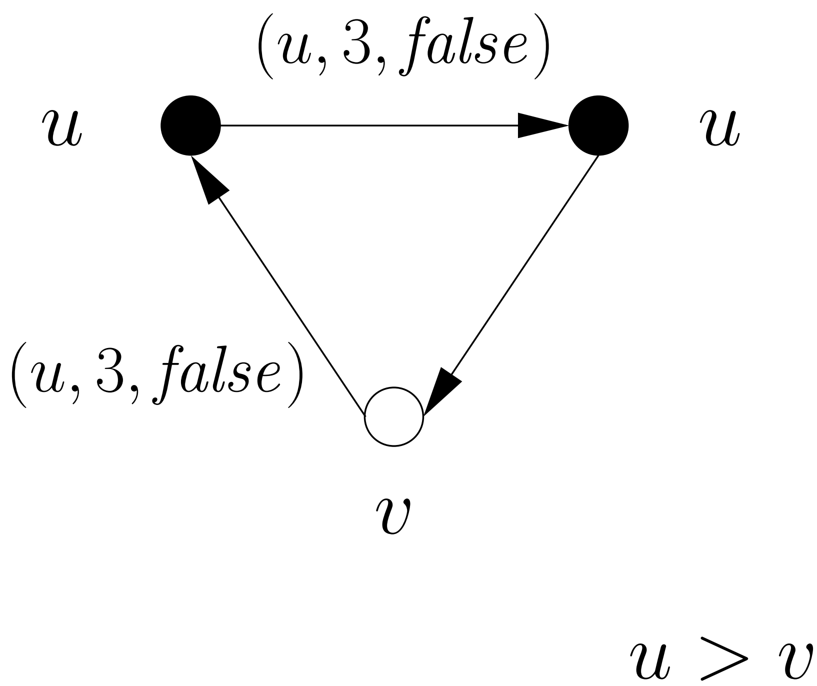
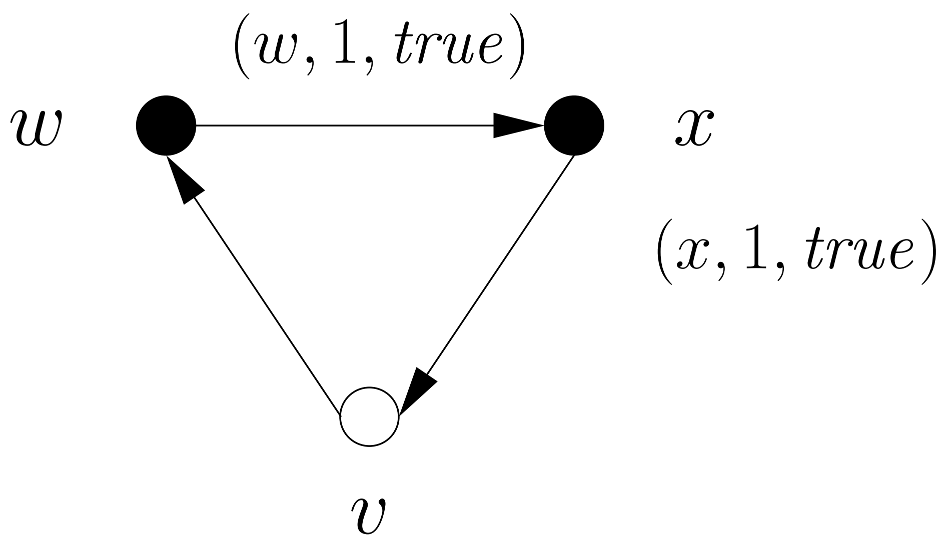


Figure 10: Algorithm $\mathcal{A}$: an example $n = 3$ (step 3)

Leader Election without Round Numbers



$$w > x$$

Figure 11: Algorithm $\mathcal{A}$: an example $n = 3$ (step 4)

Leader Election without Round Numbers

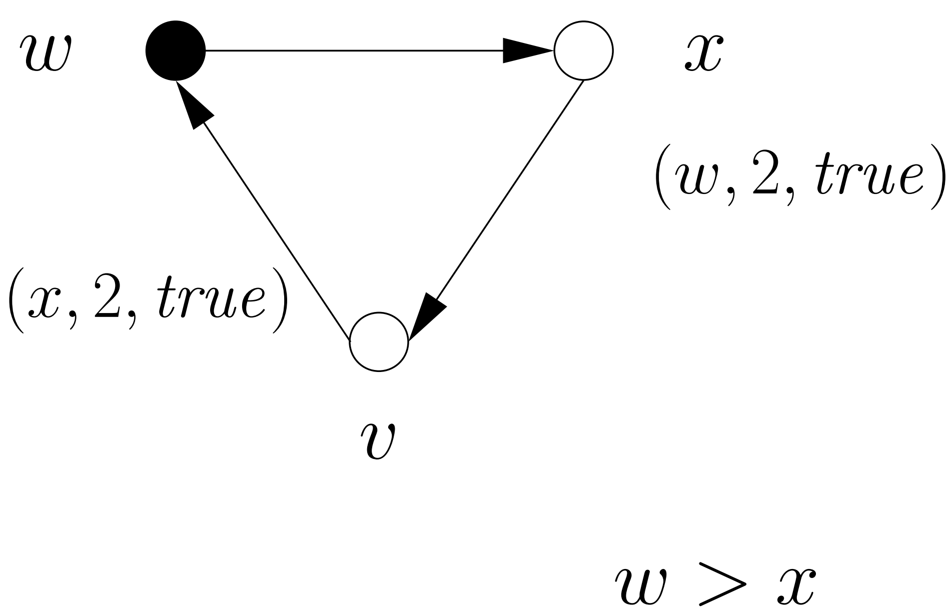
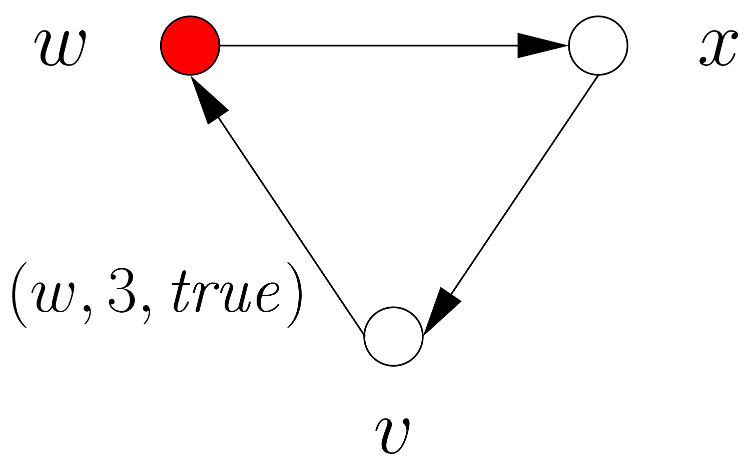


Figure 12: Algorithm $\mathcal{A}$: an example $n = 3$ (step 5)

Leader Election without Round Numbers



$w > x$

Figure 13: Algorithm $\mathcal{A}$: an example $n = 3$ (step 6)

Leader Election without Round Numbers

	Processes	Identities	Channel size	FIFO	States	Transitions
Ex.1	2	2	2	yes	127	216
Ex.2	3	3	3	yes	5,467	12,360
Ex.3	4	3	4	yes	99,329	283,872

Table 1: Model checking result for Algorithm $\mathcal{A}$ with FIFO channels in PRISM

Theorem 3 Let channels be FIFO. Then Algorithm $\mathcal{A}$ terminates with probability one, and upon termination exactly one leader is elected.

Proof. Reuse the proof of Itai and Rodeh and by Proposition 2. $\boxtimes$

Leader Election without Bits

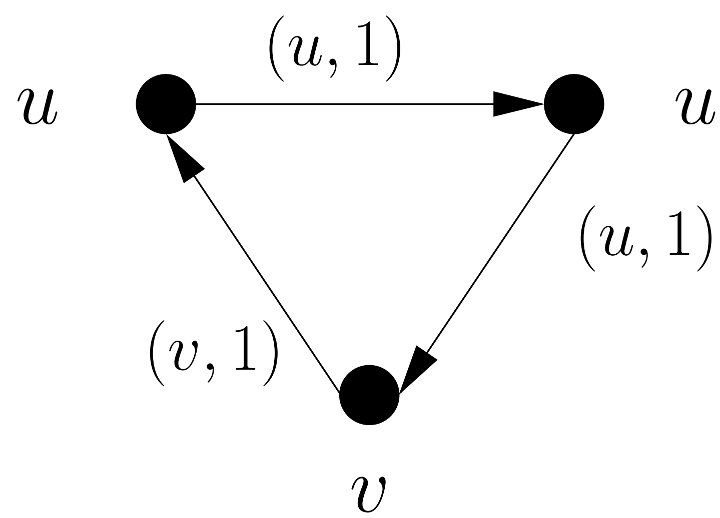
Observation:

- an active process p_i detects a name clash, it is not necessary for p_i to wait for its own message to return.

Algorithm $\mathcal{B}$: messages have the form of (id, hop).

1. if $hop = n$ and $id = id_i$, then p_i becomes the leader ($state_i' = leader$);
2. if $hop < n$ and $id = id_i$, then p_i selects a new random identity $id'_i \in \{1, \ldots, k\}$ and sends the message $(id'_i, 1)$;
3. if $id > id_i$, then p_i becomes passive ($state_i' = passive$) and passes on the message $(id, hop + 1)$;
4. if $id < id_i$, then p_i purges the message.

Leader Election without Bits



$$u > v$$

Figure 14: Algorithm $\mathcal{B}$: an example $n = 3$ (step 1)

Leader Election without Bits

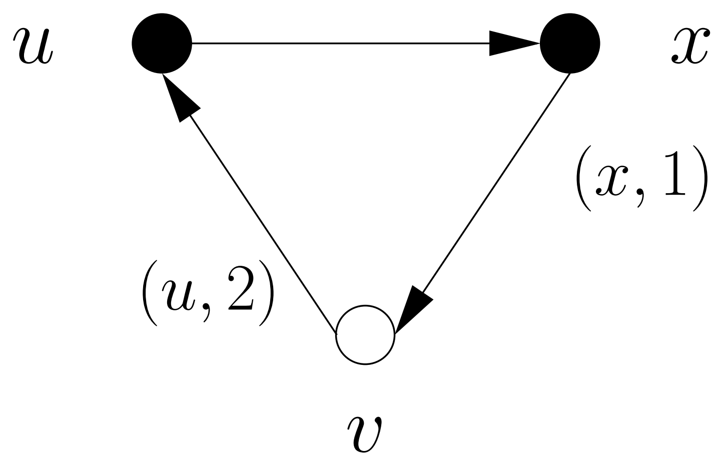
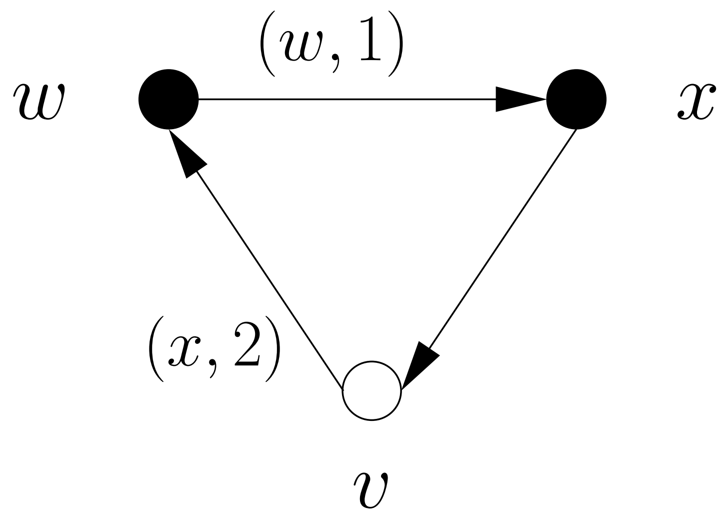


Figure 15: Algorithm $\mathcal{B}$: an example $n = 3$ (step 2)

Leader Election without Bits



$w > x$

Figure 16: Algorithm $\mathcal{B}$: an example $n = 3$ (step 3)

Leader Election without Bits

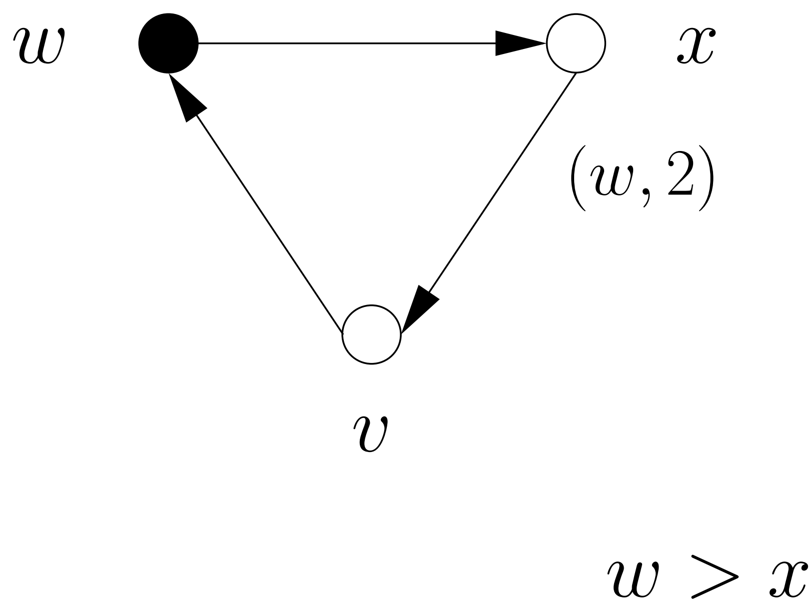
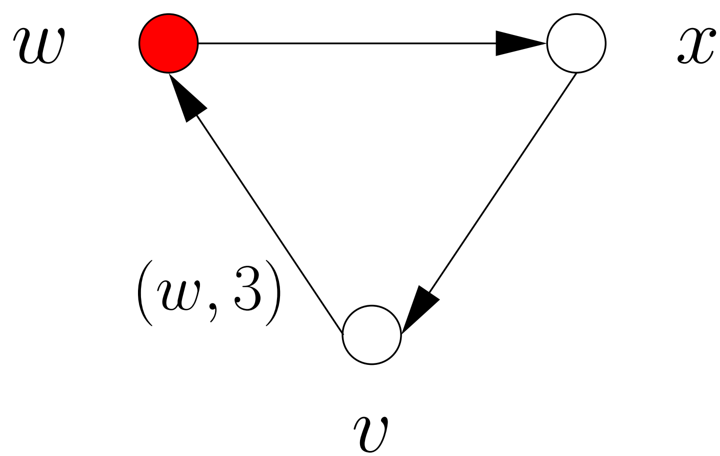


Figure 17: Algorithm $\mathcal{B}$: an example $n = 3$ (step 4)

Leader Election without Bits



$w > x$

Figure 18: Algorithm $\mathcal{B}$: an example $n = 3$ (step 5)

Leader Election without Bits

	Processes	Identities	Channel size	FIFO	States	Transitions
Ex.1	2	2	2	yes	97	168
Ex.2	3	3	3	yes	6,019	14,115
Ex.3	4	3	4	yes	176,068	521,452
Ex.4	4	4	4	yes	537,467	1,615,408
Ex.5	5	2	5	yes	752,047	2,626,405

Table 2: Model checking result for Algorithm $\mathcal{B}$ with FIFO channels

Leader Election without Bits

Theorem 4 Let channels be FIFO. Then Algorithm $\mathcal{B}$ terminates with probability one, and upon termination exactly one leader is elected.

Proof is not easy!

Performance Analysis in PRISM

The probability that Algorithms $\mathcal{A}$ and $\mathcal{B}$ terminate within a given number of transitions.

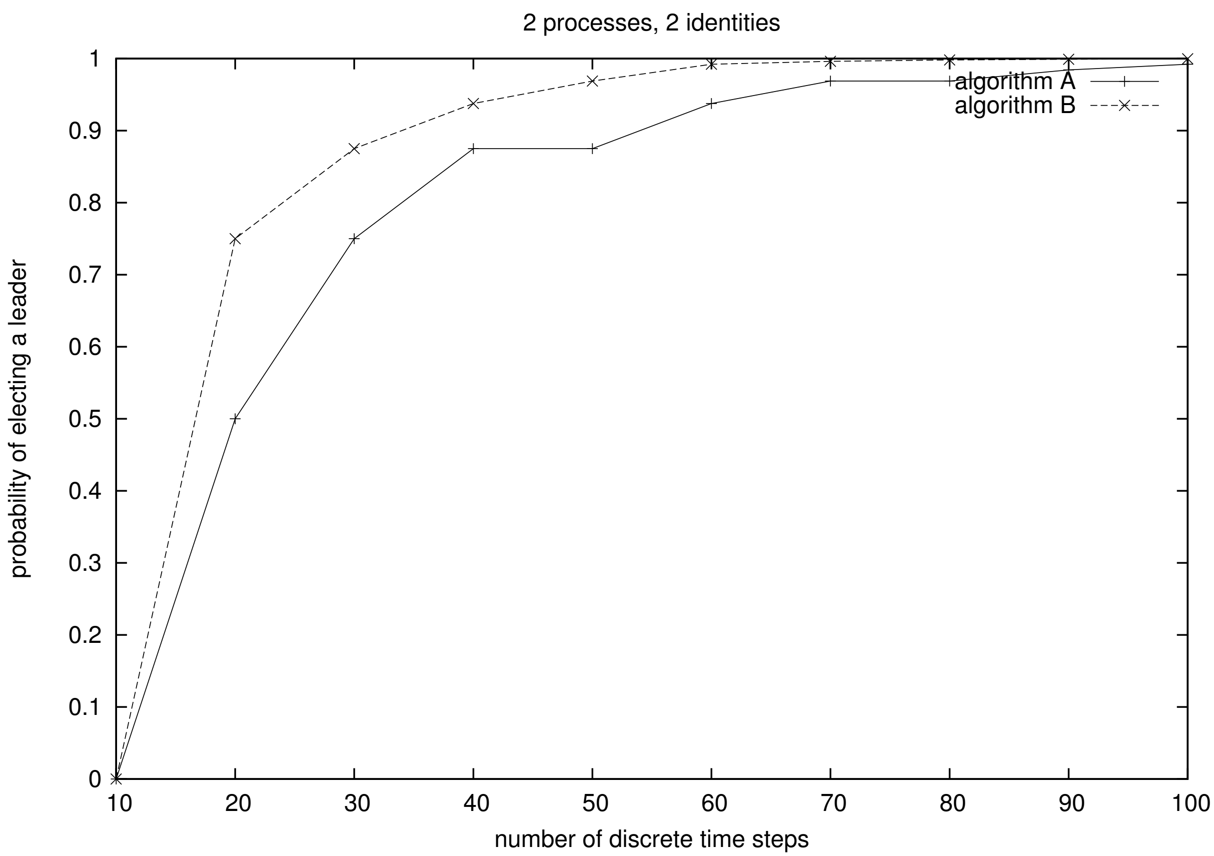


Figure 19: The probability of electing a leader with deadlines.

Performance Analysis in PRISM

The probability that Algorithms $\mathcal{A}$ and $\mathcal{B}$ terminate within a given number of transitions.

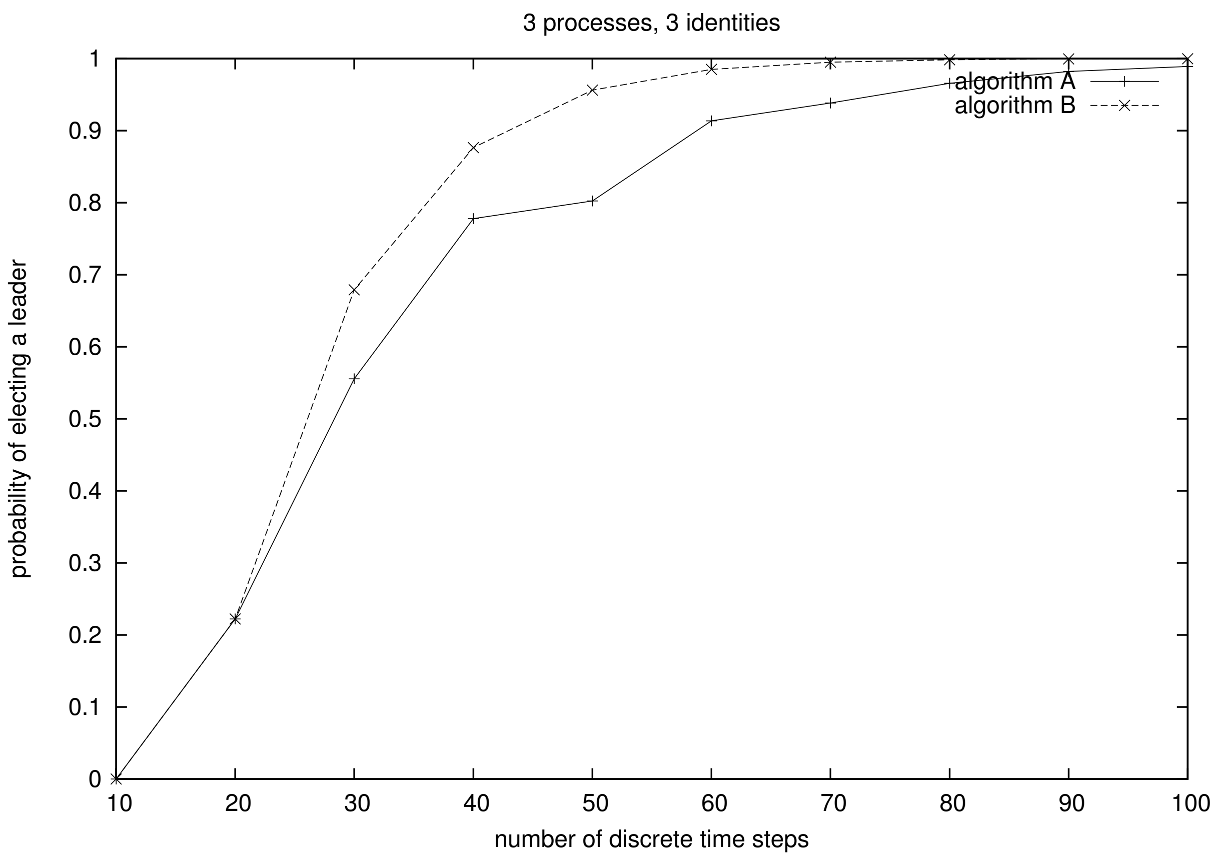


Figure 20: The probability of electing a leader with deadlines.

Performance Analysis in PRISM

The expected number of steps before a unique leader is elected for each algorithm.

	Processes	Identities	Channel size	Steps ($\mathcal{A}$)	Steps ($\mathcal{B}$)
Ex.1	2	2	2	25.0	19.0
Ex.2	3	3	3	33.6	29.3
Ex.3	4	3	4	52.5	46.0

Conclusions and Future Works

- We developed two new leader election algorithms for anonymous rings;
- Model checking and performance analysis of both algorithms in PRISM;
- We gave a manual correctness proof for each algorithm;
- When $k = 2$, both algorithms $\mathcal{A}$ and $\mathcal{B}$ are correct even if channels are not FIFO;
- We developed two more probabilistic leader election algorithms, based on the Dolev-Klawe-Rodeh algorithm;
- We are going to check the proofs in PVS.

The Proof of Theorem 4

Definition 5 The processes and messages *between* a process p and a message m are the ones that are encountered when traveling in the ring from p to m.

Lemma 6 Let active process p have identity id_p and message m have identity id_m. If $id_p \neq id_m$, then there is an active process or message between p and m with an identity $\geq \min\{id_p, id_m\}$.

Proof. We apply induction on execution sequences. $\boxtimes$

The Proof of Theorem 4

Definition 7 An active process p is *related to* a message m if they have the same identity id, and all active processes and messages between p and m have an identity smaller than id.

Lemma 8 Let active process p be related to message m. Let ξ be the maximum of all identities of active processes and messages between p and m ($\xi = 0$ if there are none).

1. Between p and m, there is an equal number of active processes and of messages with identity ξ; and
2. if p is not the originator of m, then there is an active process or message between p and m.

Proof. We apply induction on execution sequences. $\boxtimes$

The Proof of Theorem 4

Definition 9 We say that an active process or message is *maximal* if its identity is maximal among the active processes or messages in the ring, respectively. In the following proposition we write ξ_π and ξ_μ for the identity of maximal active processes and messages, respectively. We write $\#_\pi$ and $\#_\mu$ for the number of maximal active processes and messages, respectively.

Proposition 10 Until a leader is elected, there exist active processes and messages in the ring, and $\xi_\pi = \xi_\mu$ and $\#_\pi = \#_\mu$.

Proof. We apply induction on execution sequences. $\boxtimes$

Finally, by induction on execution sequences, and use Proposition 10, we can prove Theorem 4.