Template-Based Static Analysis of Hybrid Systems meets Strategy Iteration

Thomas Martin Gawlitza

VERIMAG, Grenoble, France

This presentation is based on the work of

Assalé Adjé, Alexandru Costan, Thao Dang, Stéphane Gaubert, Thomas Martin Gawlitza, Éric Goubault, Matthieu Martel, Sylvie Putot, Helmut Seidl, Ankur Taly, and Sarah Zennou

NSV, July 14th, 2011

Template-based

Unbounded Time Verification

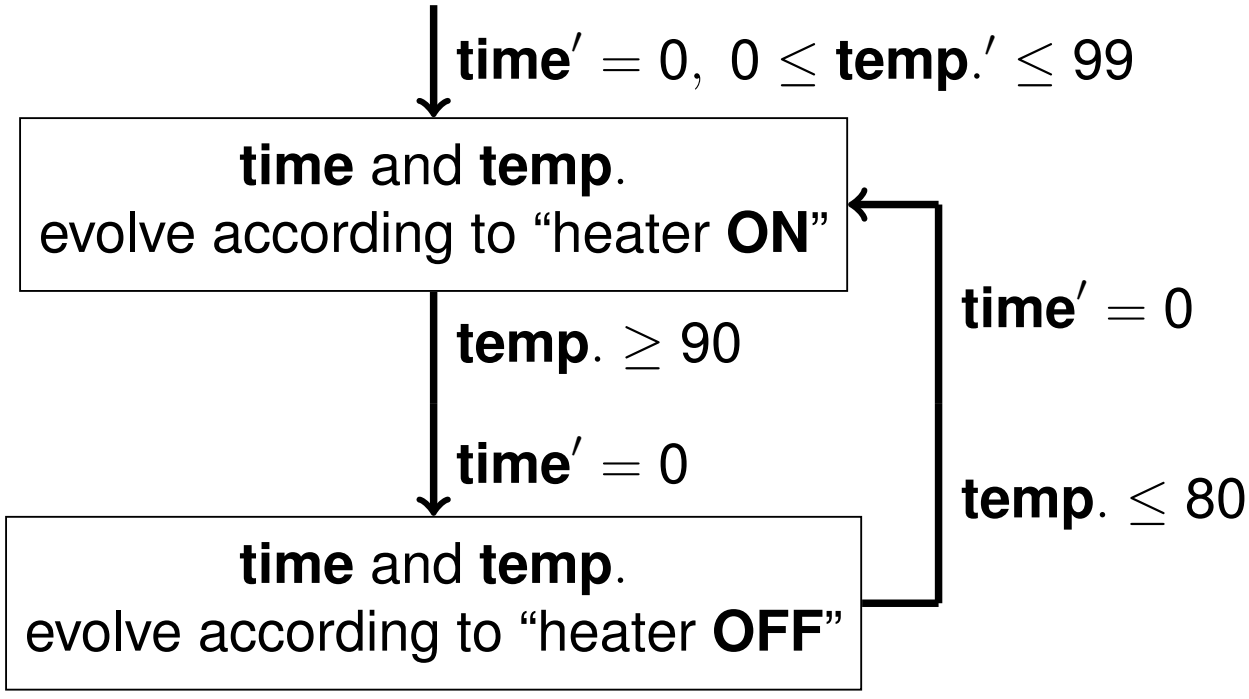
of

Hybrid Systems

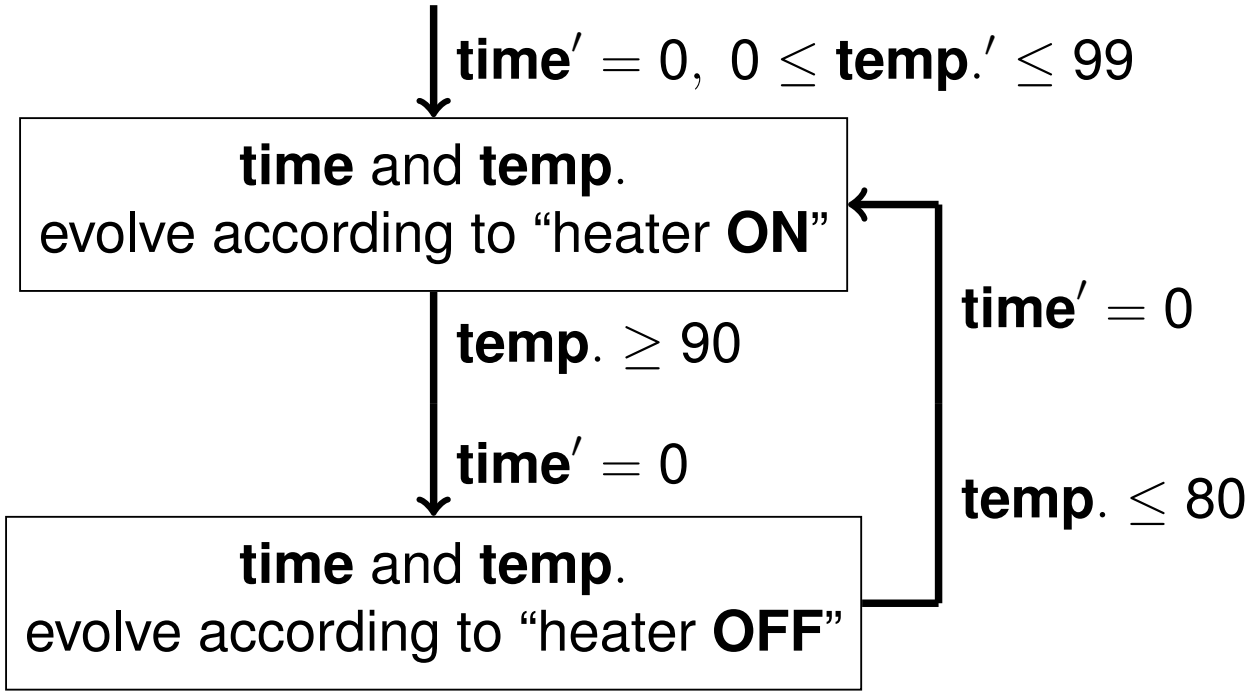
Hybrid System?

Hybrid System?

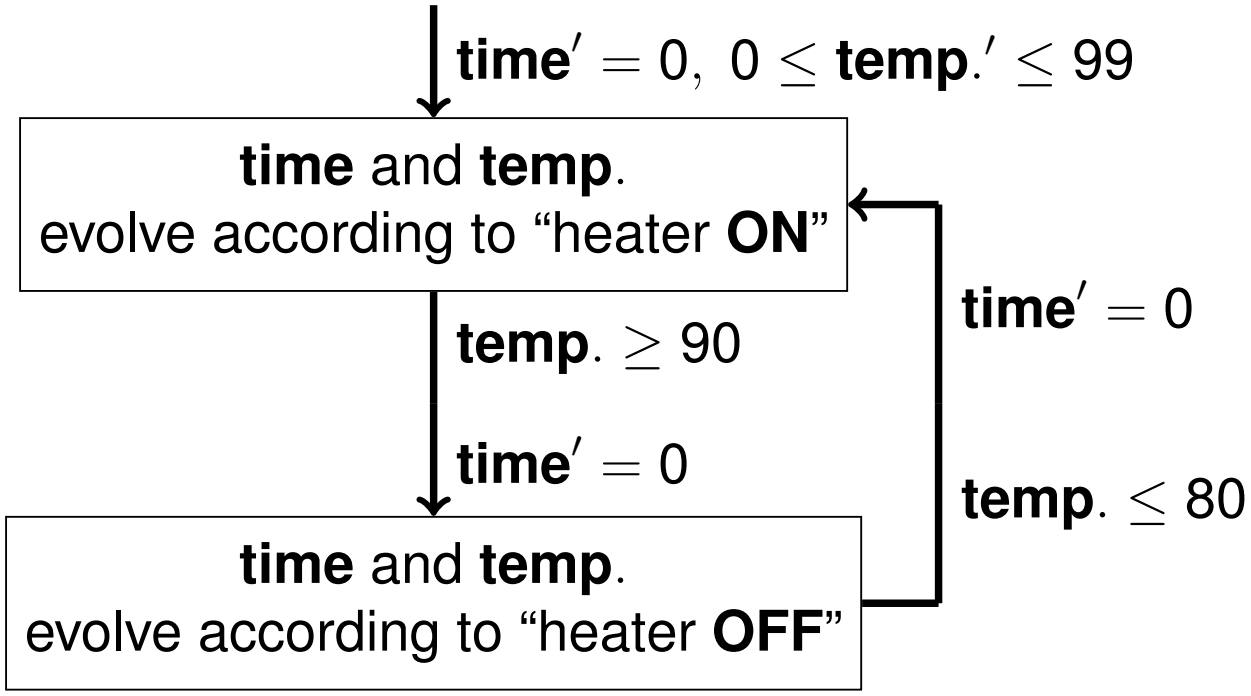
Hybrid System?

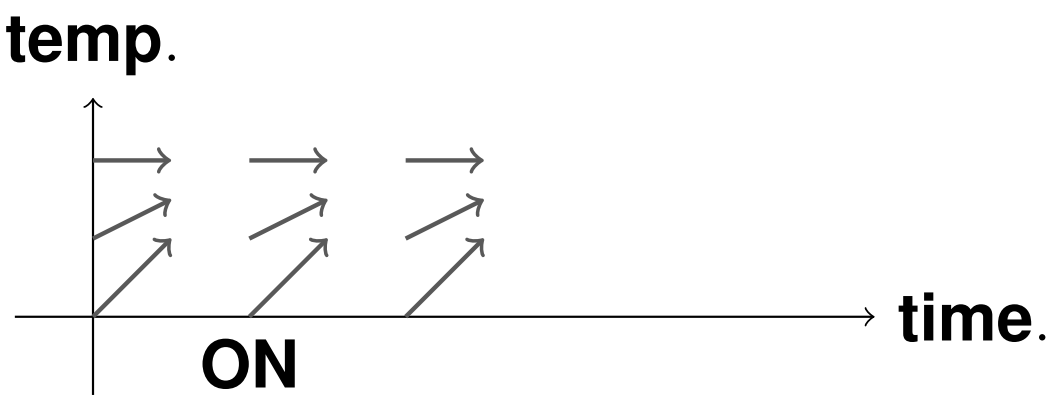


Hybrid System?

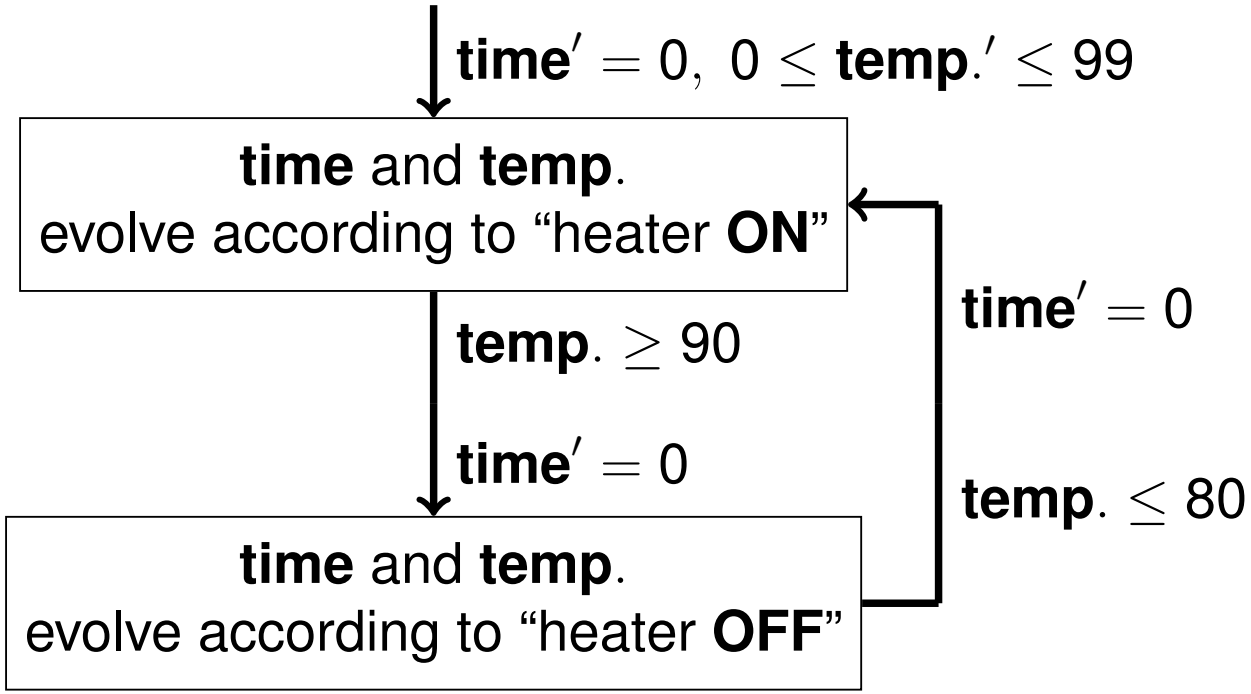


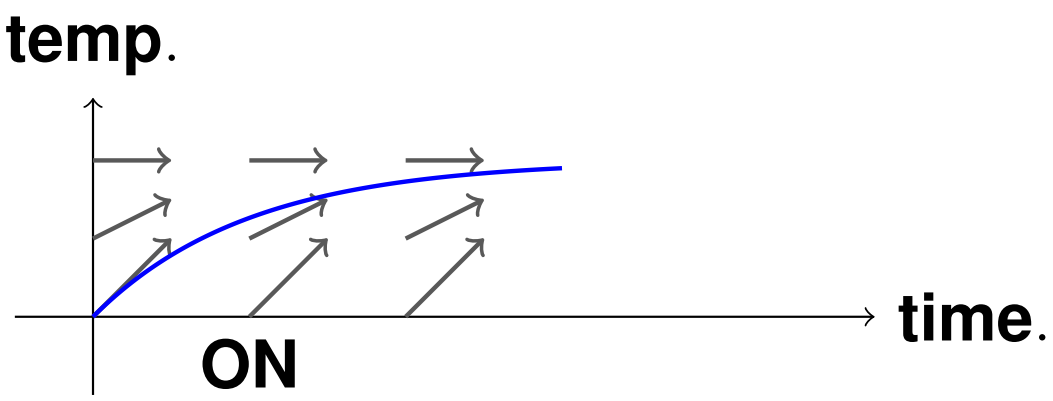
Hybrid System?



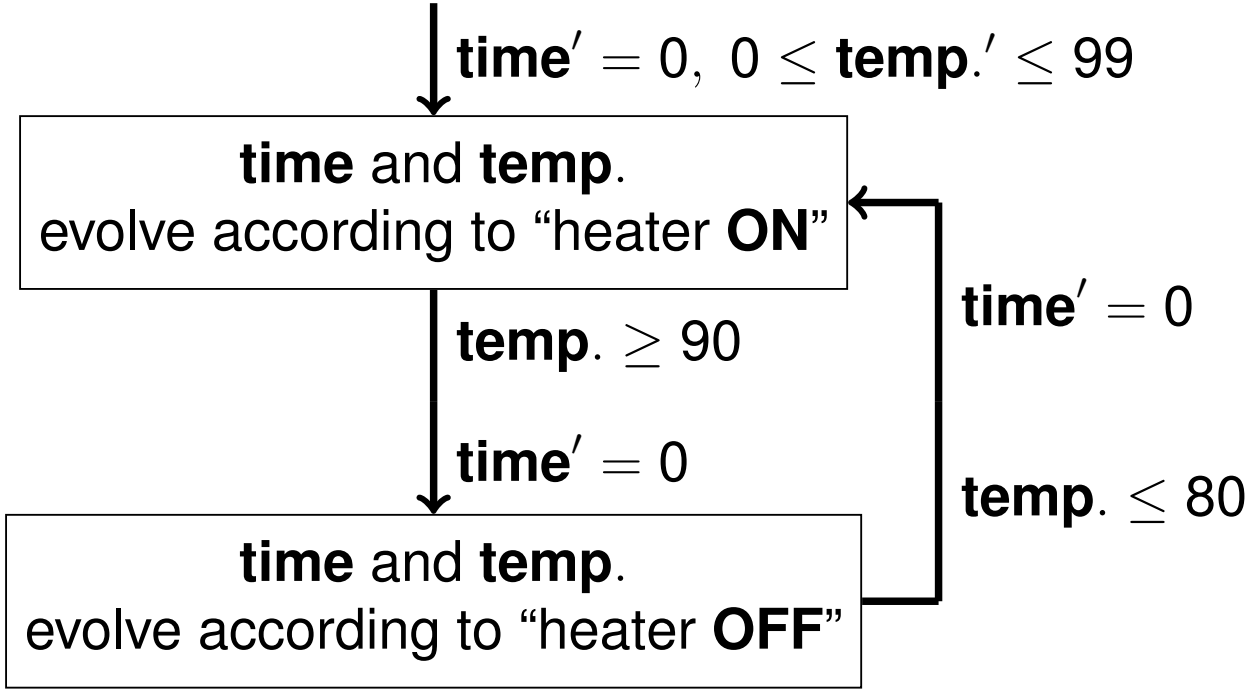


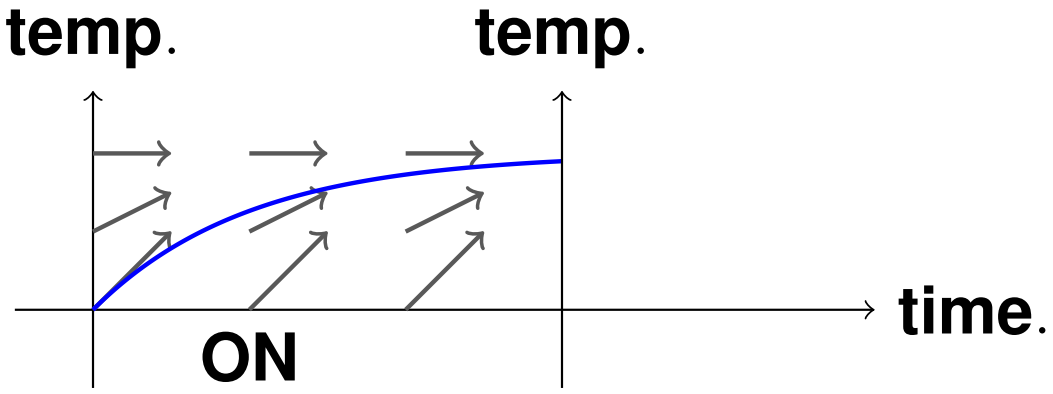
Hybrid System?



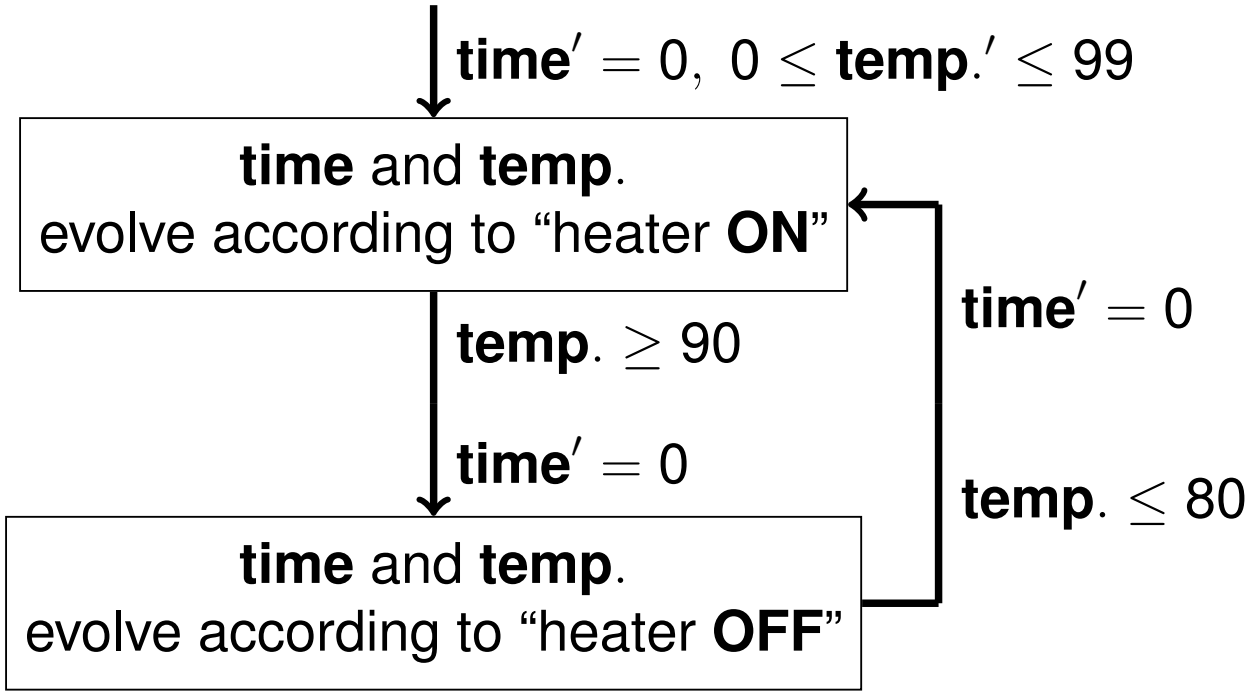


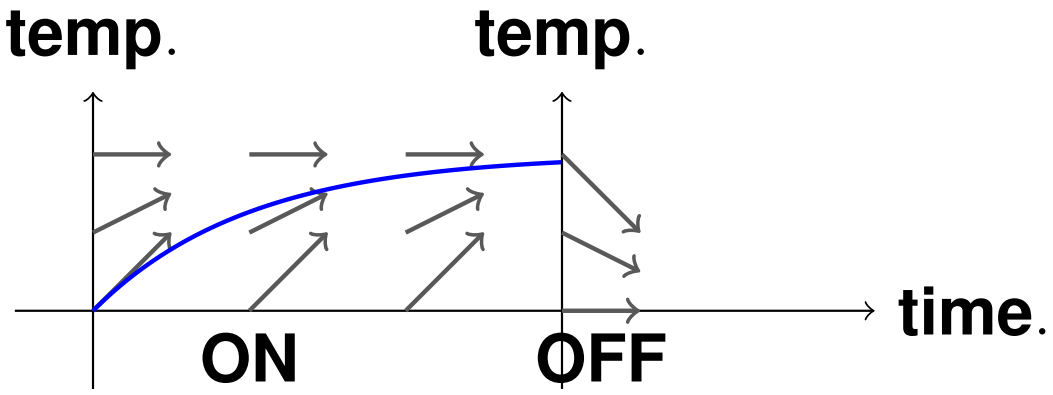
Hybrid System?



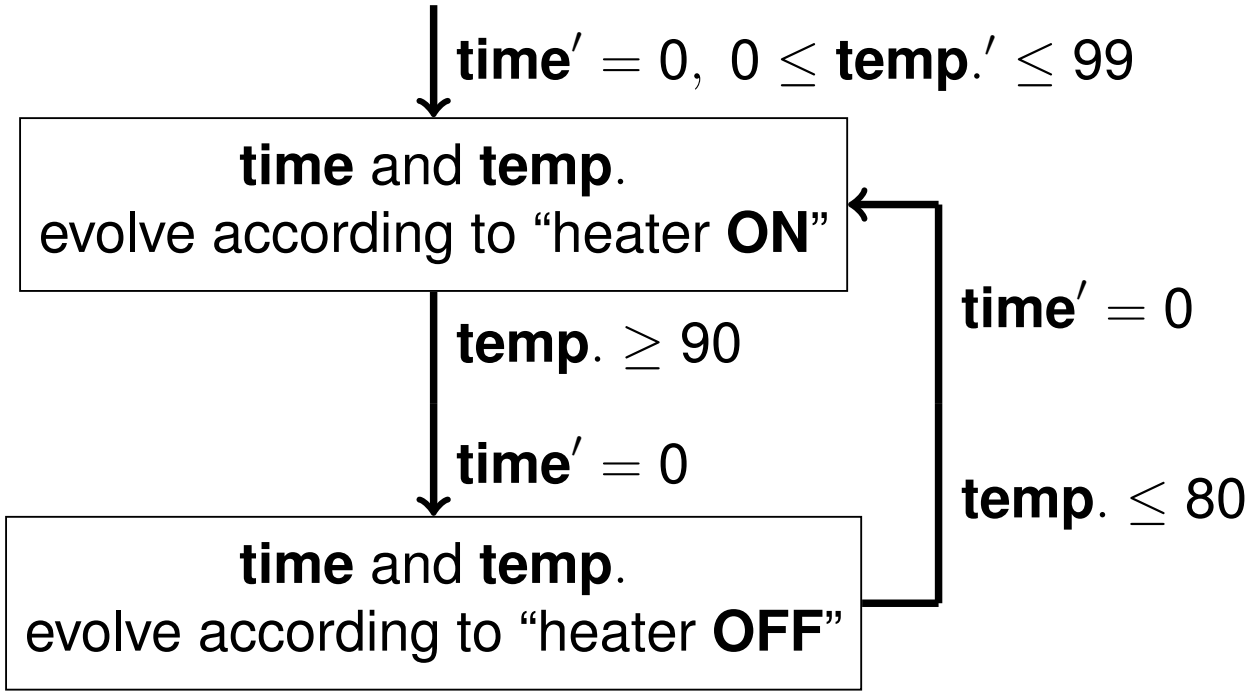


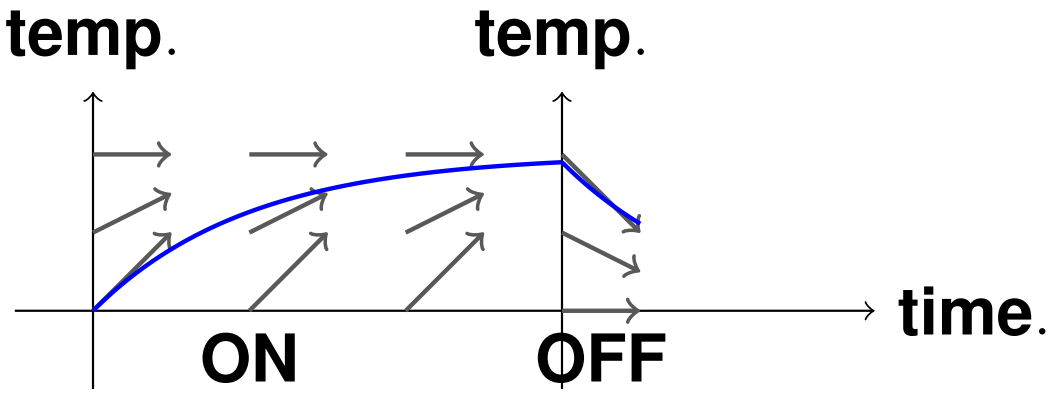
Hybrid System?



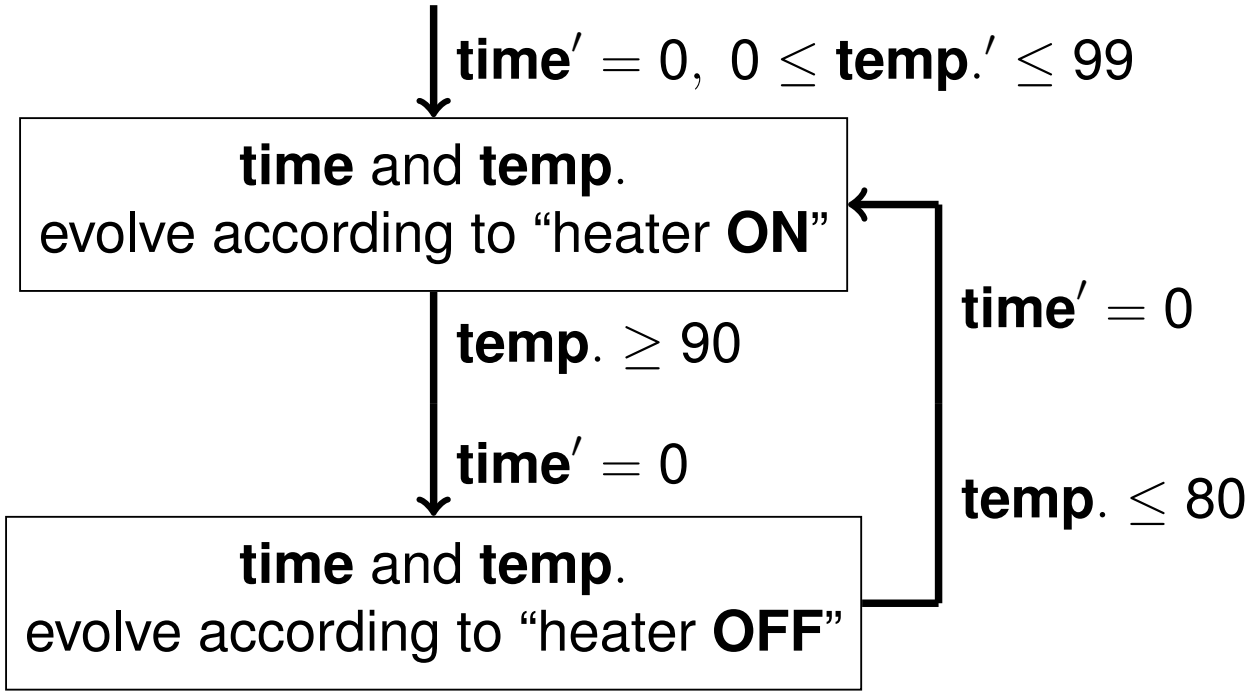


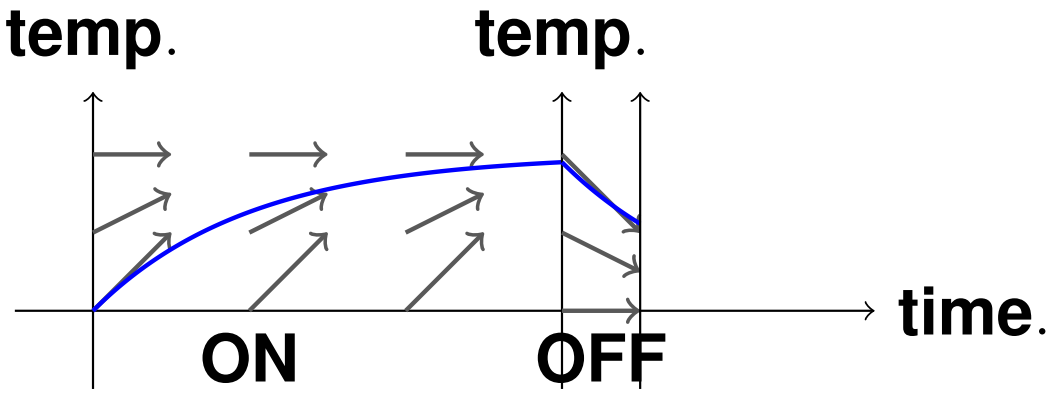
Hybrid System?



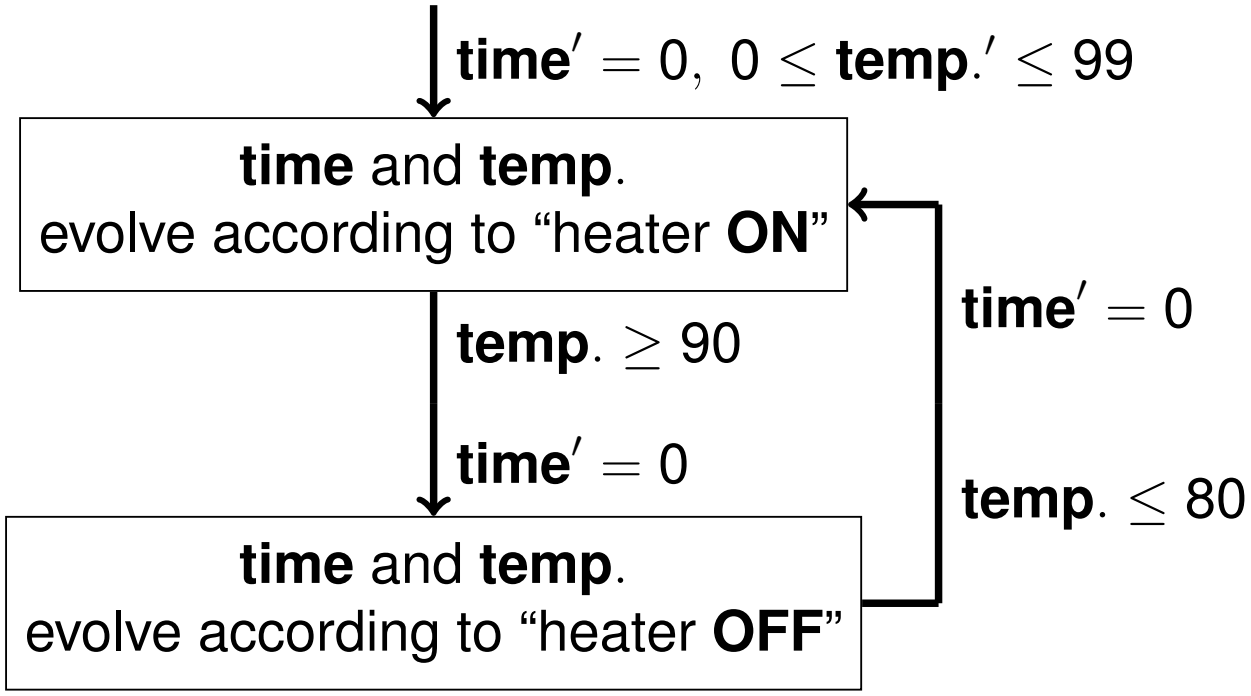


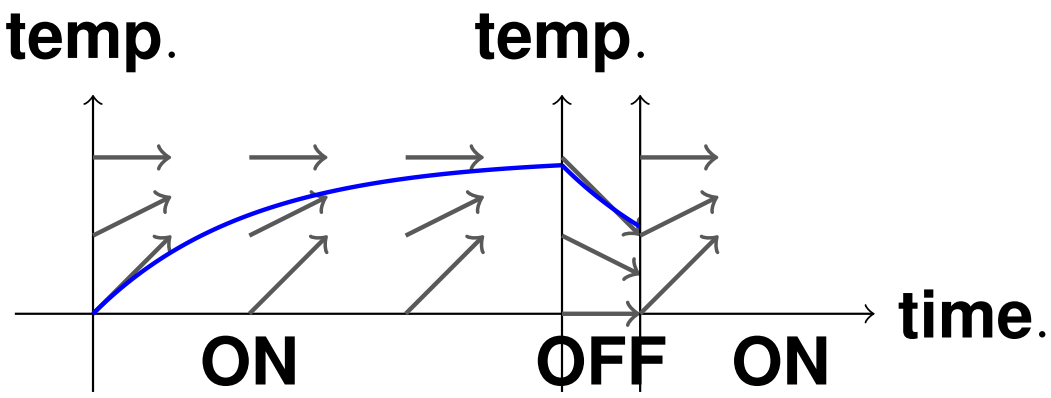
Hybrid System?



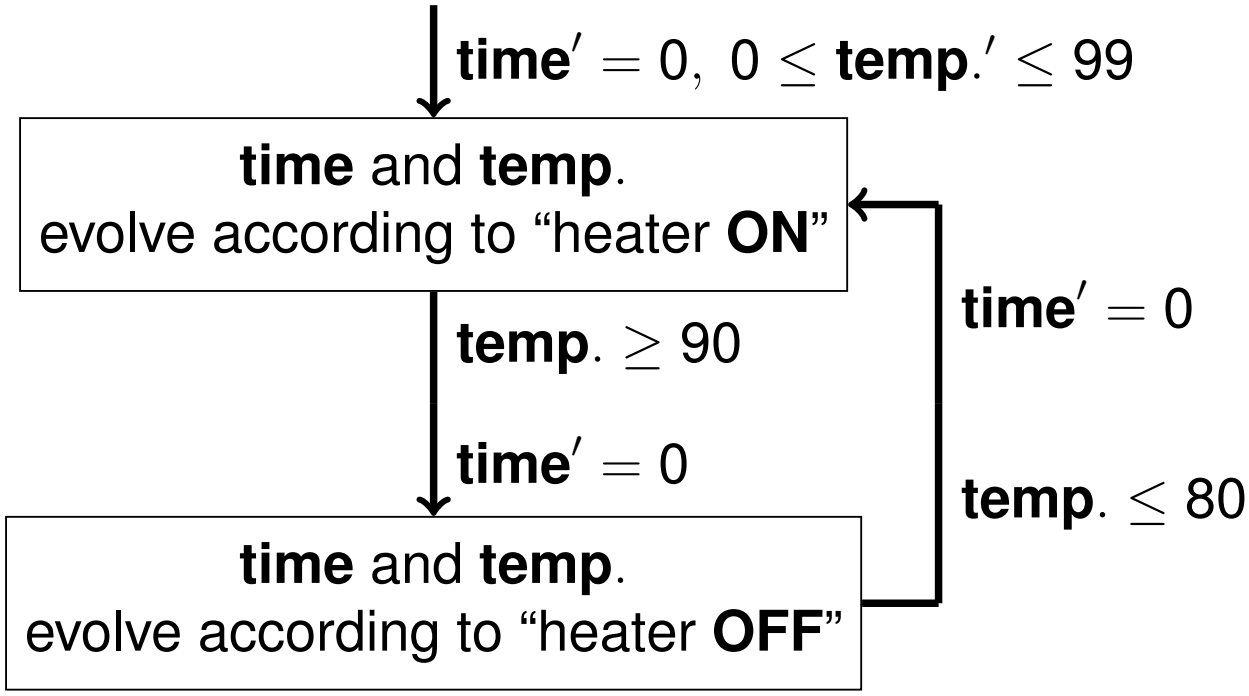


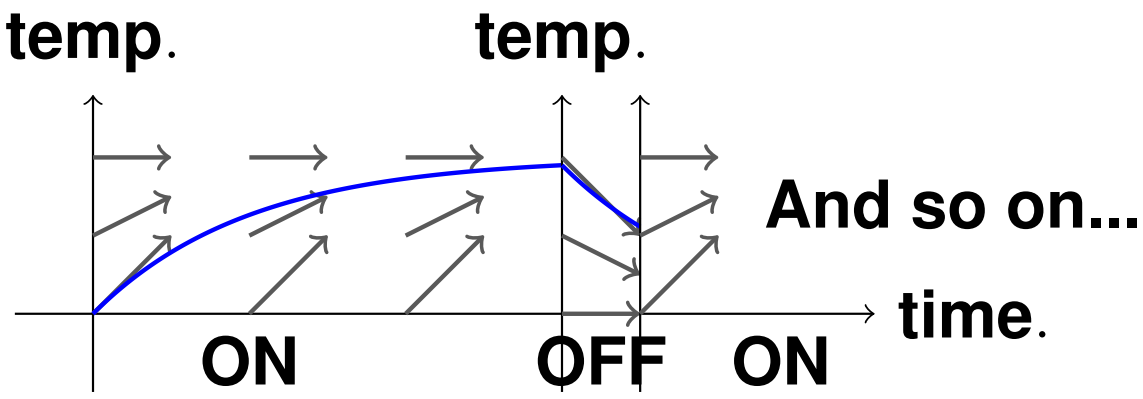
Hybrid System?





Hybrid System?





Unbounded Time Verification?

Unbounded Time Verification?

Does the system stay in a **safe** region **forever**?

Unbounded Time Verification?

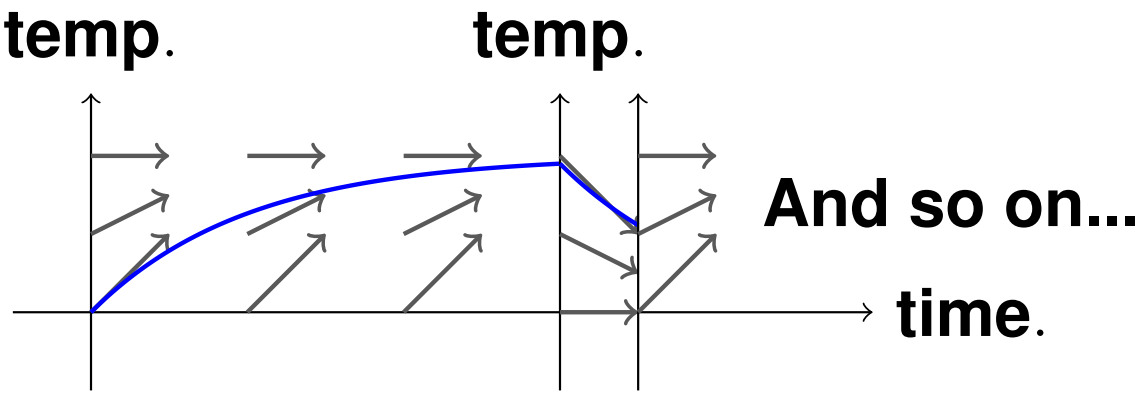
Does the system stay in a **safe** region **forever**?

Example: **Grandma-Stays-Alive-Property**

Unbounded Time Verification?

Does the system stay in a **safe** region **forever**?

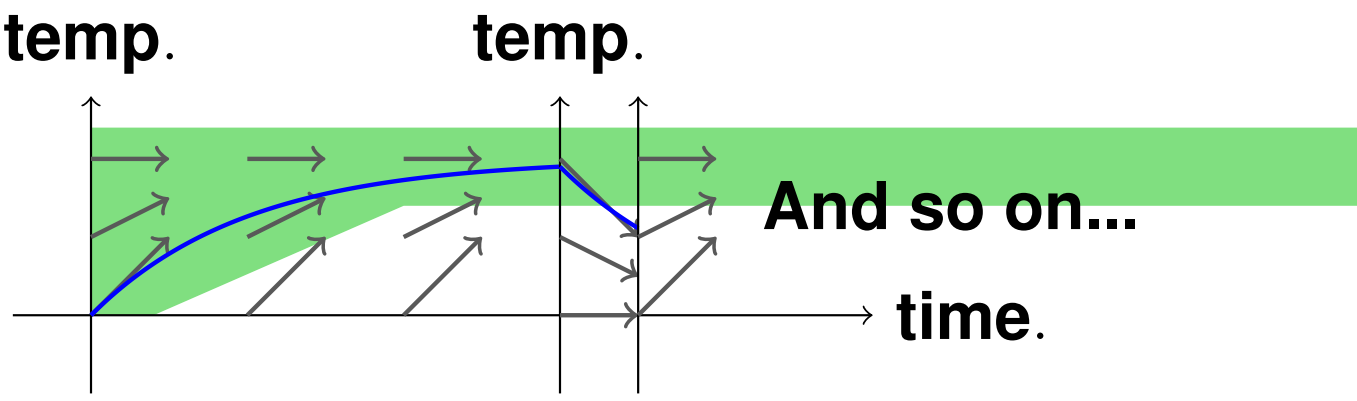
Example: **Grandma-Stays-Alive-Property**



Unbounded Time Verification?

Does the system stay in a **safe** region **forever**?

Example: **Grandma-Stays-Alive-Property**



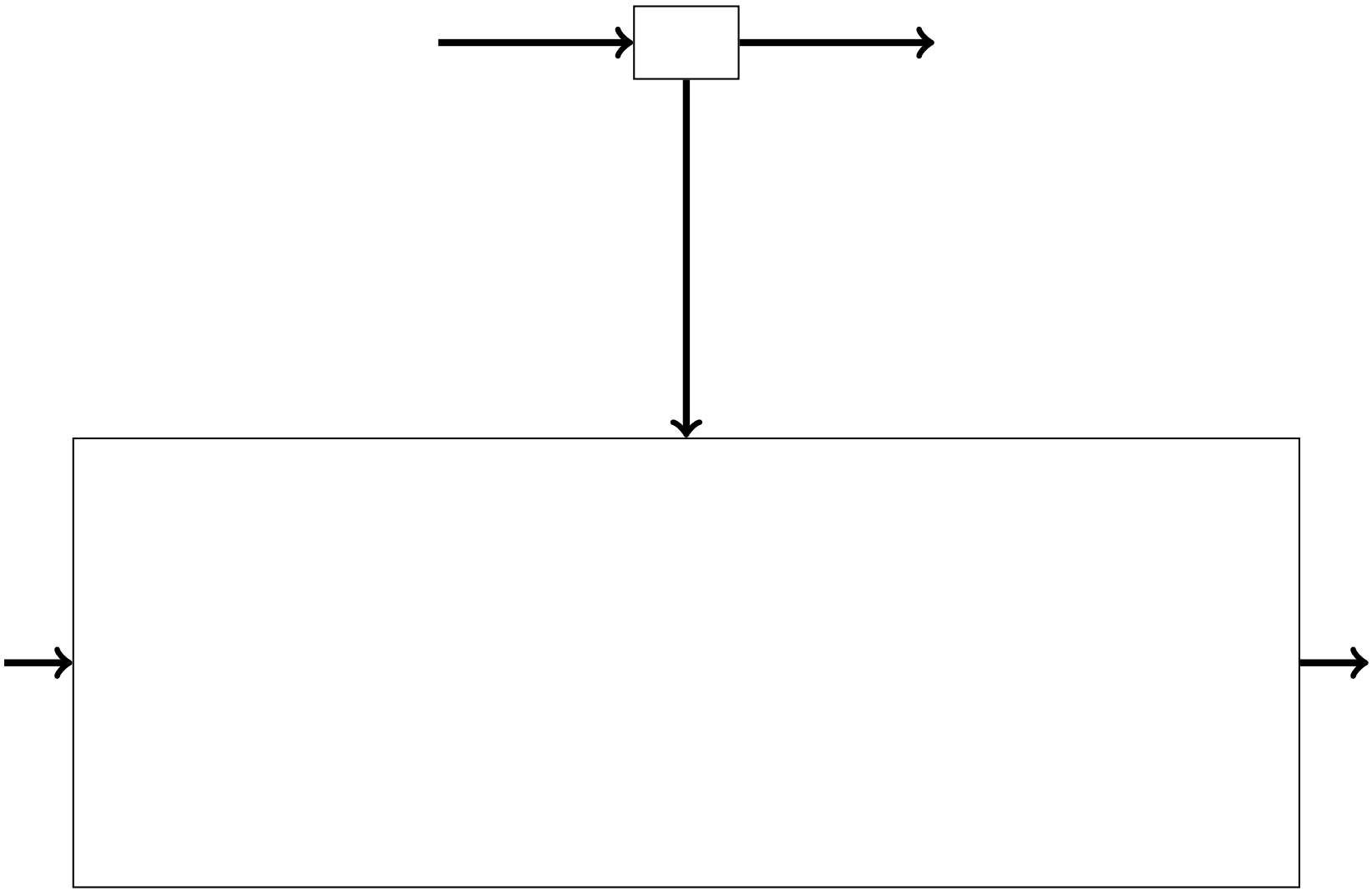
Here:

Affine

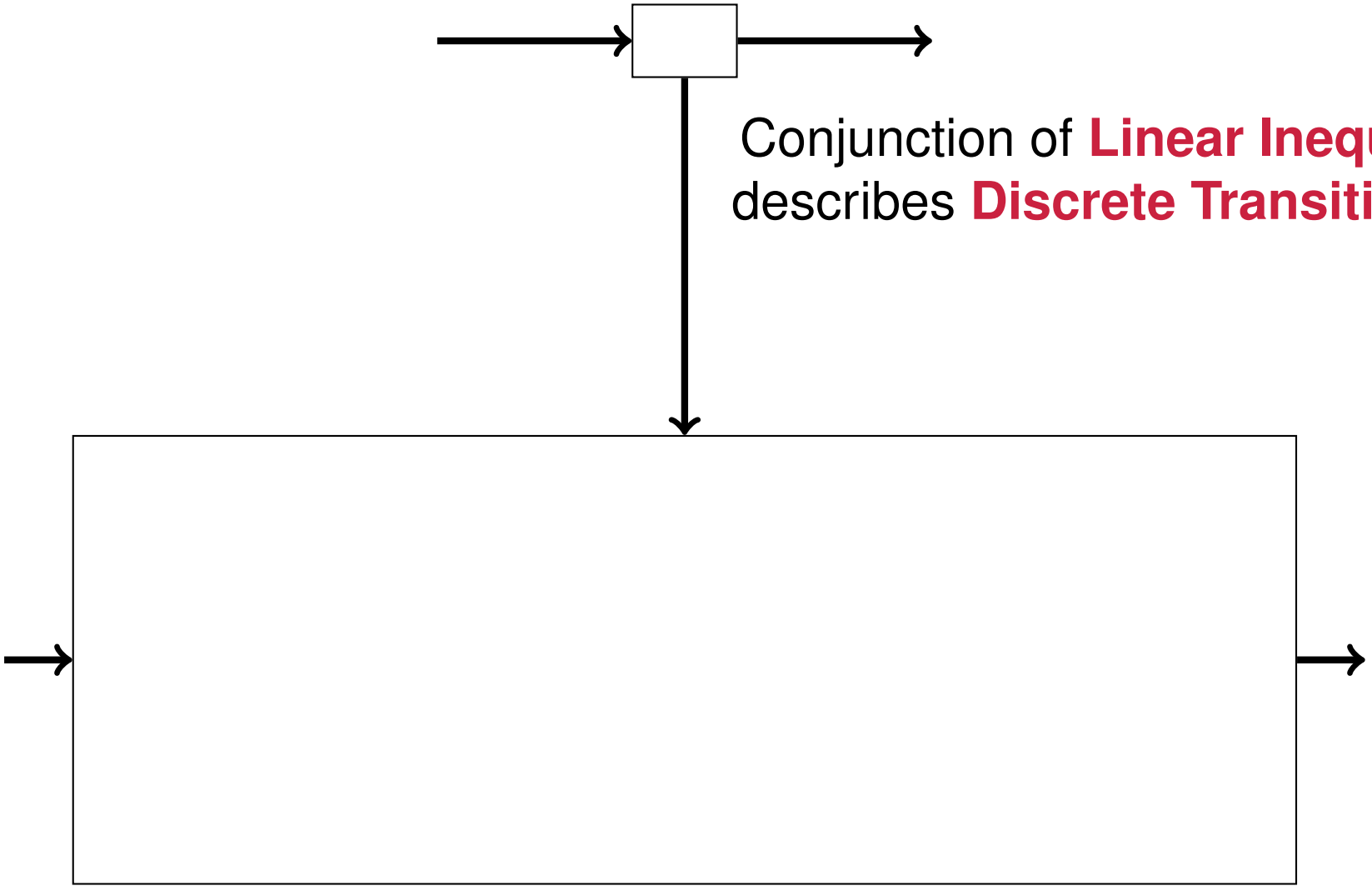
Hybrid Automata

(with Uncertainty)

Affine?

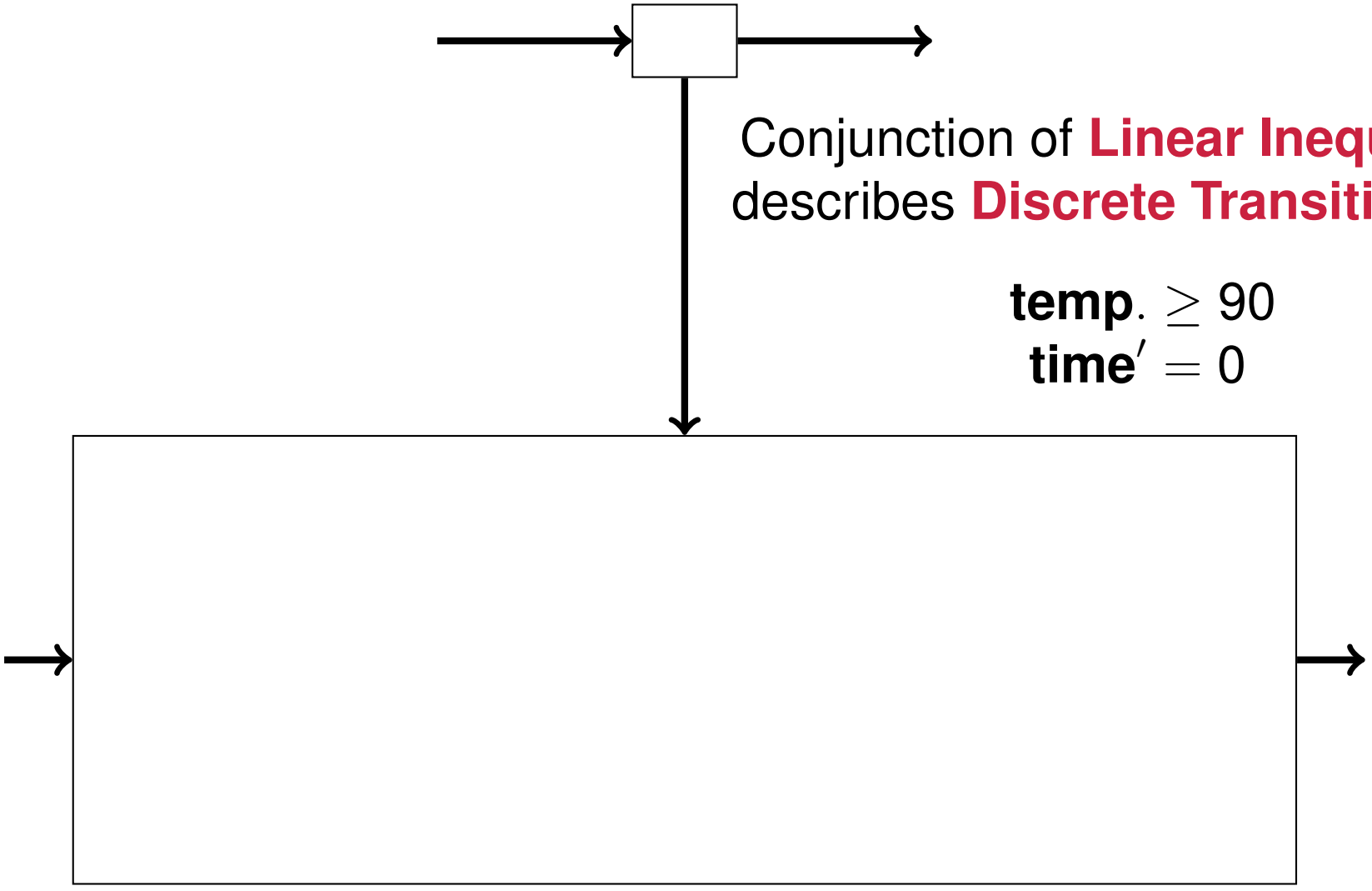


Affine?



Conjunction of **Linear Inequalities**
describes **Discrete Transition**

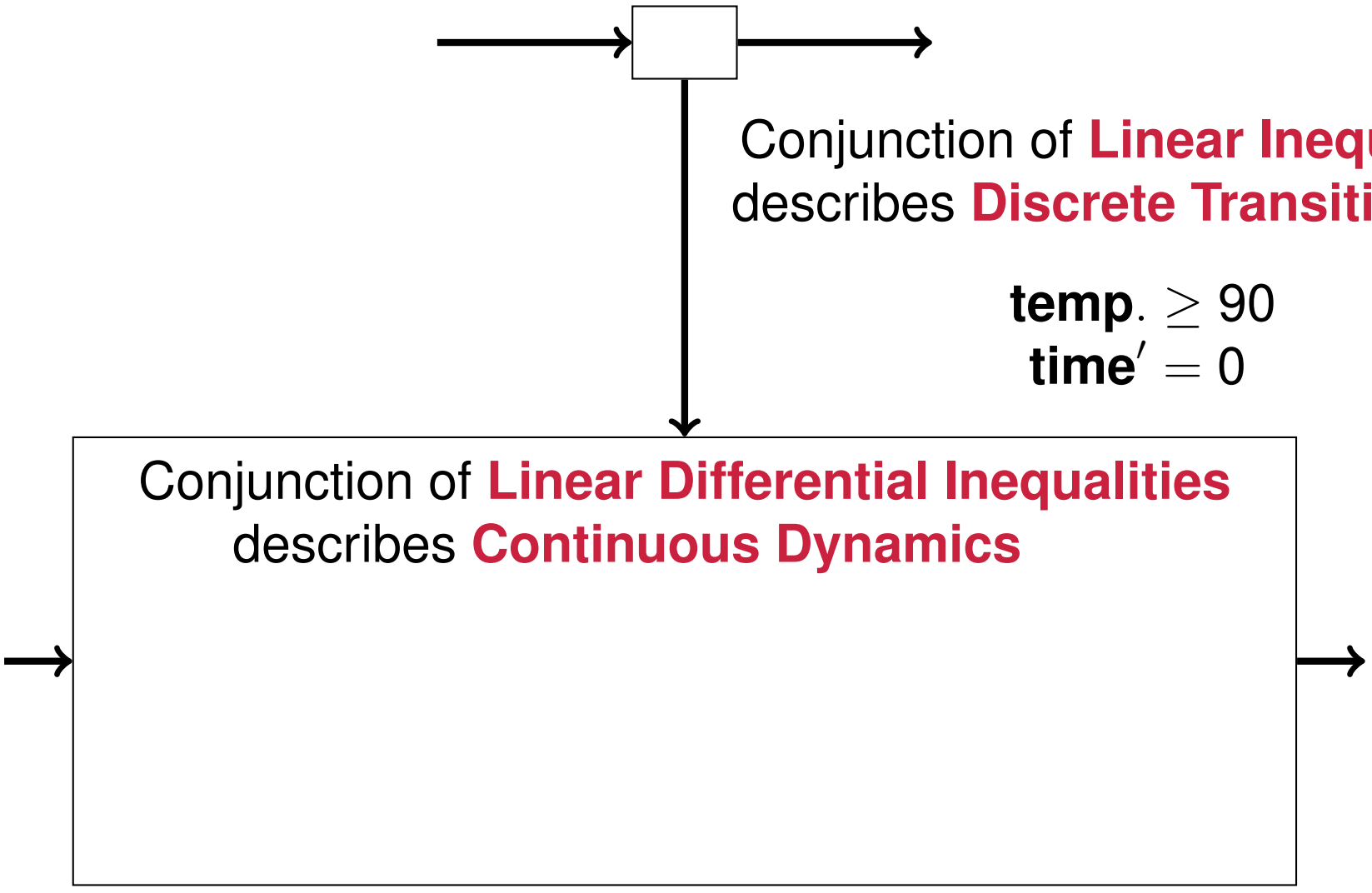
Affine?



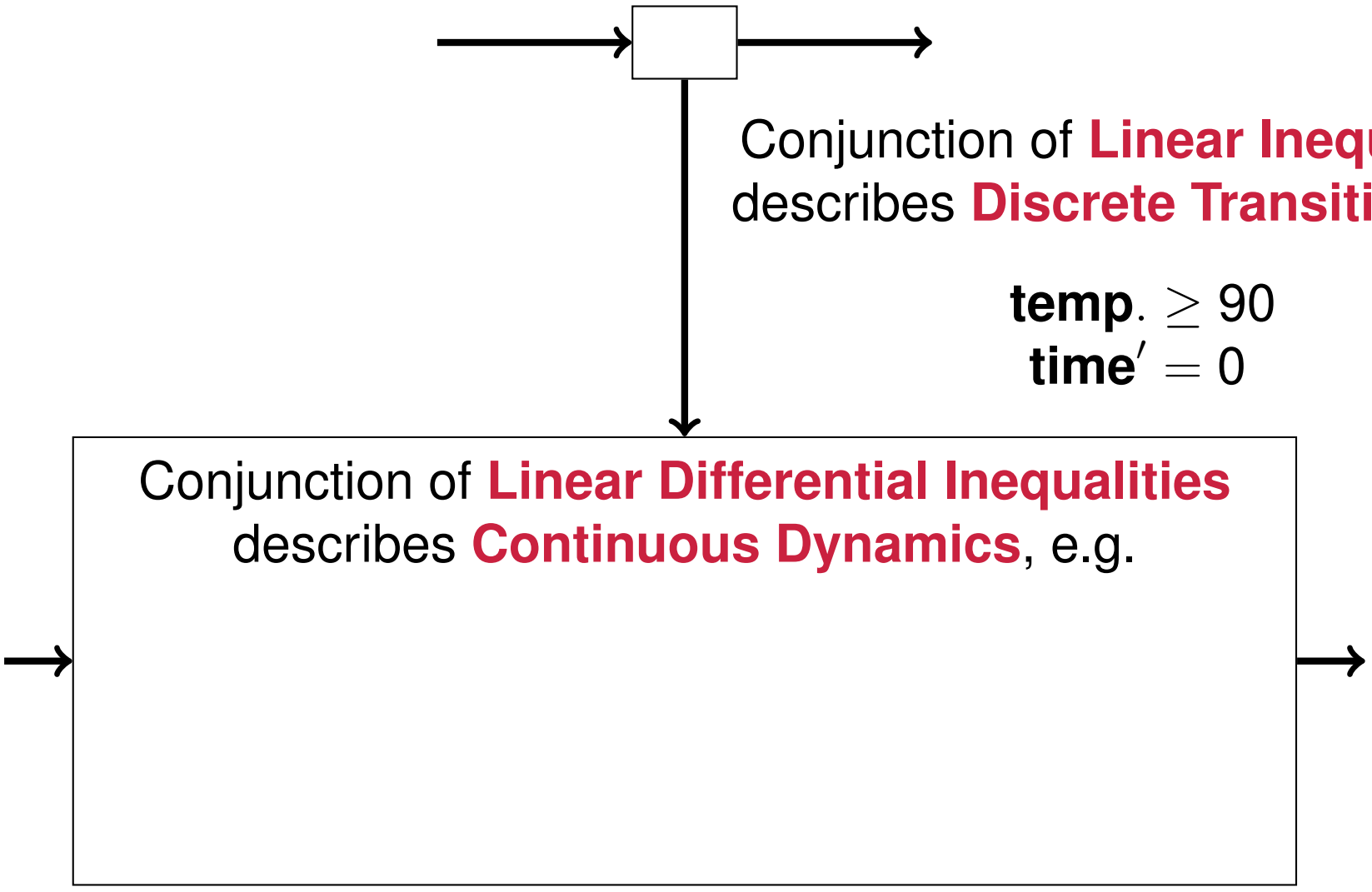
Conjunction of **Linear Inequalities** describes **Discrete Transition**, e.g.

$$\mathbf{temp}. \geq 90$$
$$\mathbf{time}' = 0$$

Affine?


Conjunction of Linear Inequalities
describes Discrete Transition, e.g.
temp. ≥ 90
time′ = 0
Conjunction of Linear Differential Inequalities
describes Continuous Dynamics

Affine?


Conjunction of Linear Inequalities
describes Discrete Transition, e.g.
temp. ≥ 90
time′ = 0
Conjunction of Linear Differential Inequalities
describes Continuous Dynamics, e.g.

Affine?

Conjunction of **Linear Inequalities** describes **Discrete Transition**, e.g.

$$\mathbf{temp}. \geq 90$$
$$\mathbf{time}' = 0$$

Conjunction of **Linear Differential Inequalities** describes **Continuous Dynamics**, e.g.

$$\dot{\mathbf{time}}(t) = 1 \qquad \text{for all } t \in \mathbb{R}_{\geq 0}$$
$$\dot{\mathbf{temp}}.(t) = \tfrac{1}{100} \cdot (100 - \mathbf{temp}.(t)) \qquad \text{for all } t \in \mathbb{R}_{\geq 0}$$
$$\mathbf{temp}.(t) \leq 90$$

Affine?

Conjunction of **Linear Inequalities** describes **Discrete Transition**, e.g.

$$\begin{aligned} \textbf{temp}. &\geq 90 \\ \textbf{time}' &= 0 \end{aligned}$$

Conjunction of **Linear Differential Inequalities** describes **Continuous Dynamics**, e.g.

$$\begin{aligned} \dot{\textbf{time}} &= 1 \\ \dot{\textbf{temp}}. &= \tfrac{1}{100} \cdot (100 - \textbf{temp}.) \\ \textbf{temp}. &\leq 90 \end{aligned}$$

Affine?

Conjunction of **Linear Inequalities** describes **Discrete Transition**, e.g.

$$\mathbf{temp}. \geq 90$$
$$\mathbf{time}' = 0$$

Conjunction of **Linear Differential Inequalities** describes **Continuous Dynamics**, e.g.

$$\begin{aligned} \dot{\mathbf{time}} &= 1 \\ \dot{\mathbf{temp}}. &= \tfrac{1}{100} \cdot (100 - \mathbf{temp}.) + [-\tfrac{1}{10}, \tfrac{1}{10}]\mathbf{temp}. \\ \mathbf{temp}. &\leq 90 \end{aligned}$$

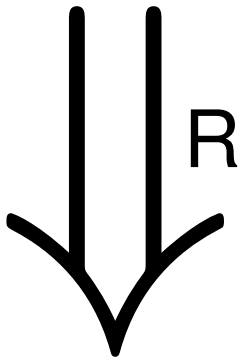
The Annoying Truth:

The Annoying Truth:

Affine Hybrid Automata are **Turing Complete**

The Annoying Truth:

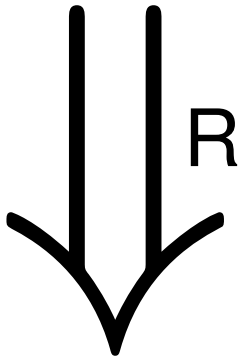
Affine Hybrid Automata are **Turing Complete**



Rice's Theorem

The Annoying Truth:

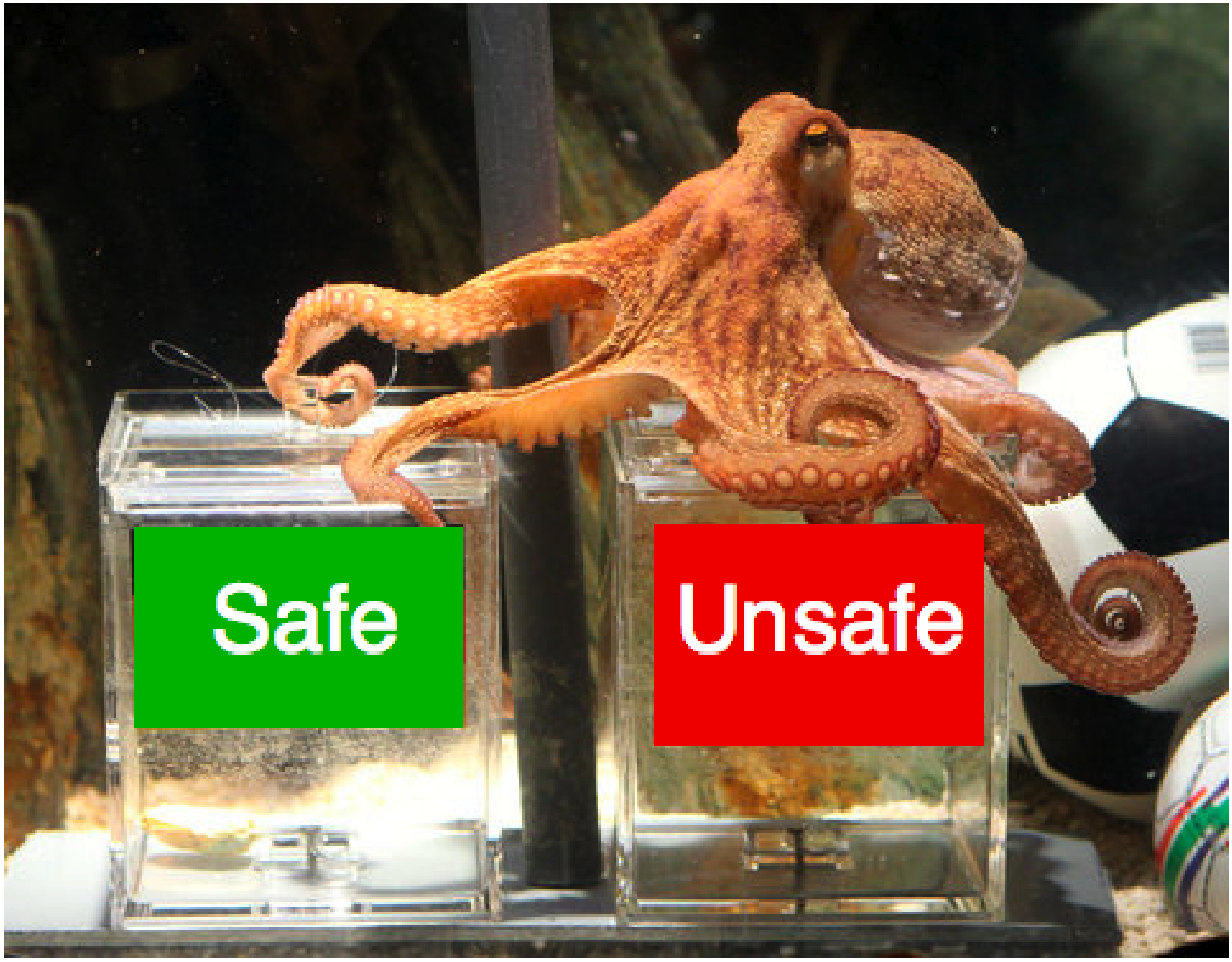
Affine Hybrid Automata are **Turing Complete**



Rice's Theorem

Checking/Verifying
Non-Trivial Properties
is
Undecidable

Octopus-based Verifier



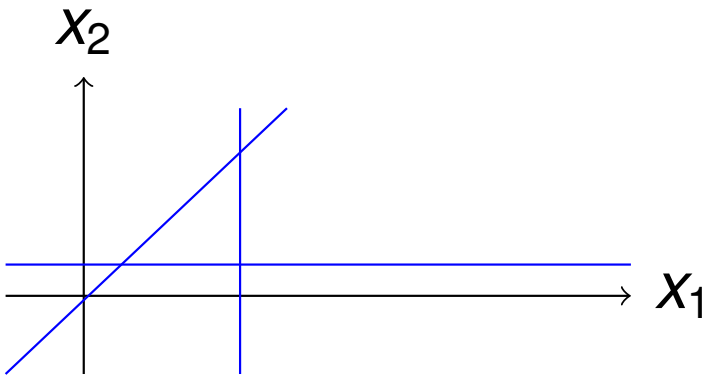
The author decided to not show this slide, since the contained humor might be misunderstood as an offense.

Outline

Abstract Domain: Template Polyhedra [Sriram Sank...]

Abstract Domain: **Template Polyhedra** [Sriram Sank...]

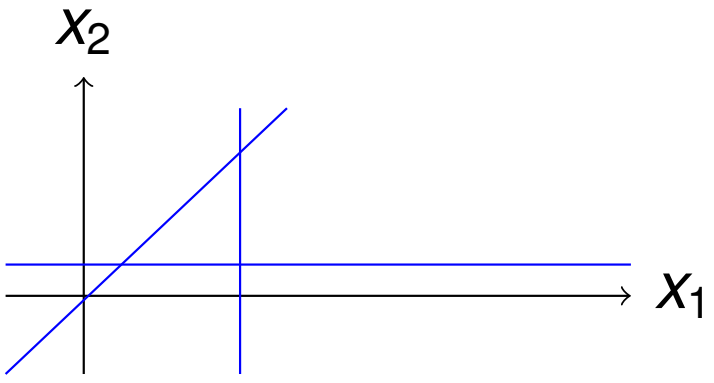
Fixed set of **linear templates**:



$$\simeq \; x_1, -x_1, x_2, -x_2, x_1 - x_2, x_2 - x_1$$

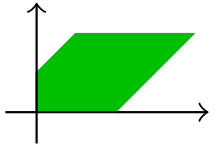
Abstract Domain: **Template Polyhedra** [Sriram Sank...]

Fixed set of **linear templates**:



$$\simeq \; x_1, -x_1, x_2, -x_2, x_1 - x_2, x_2 - x_1$$

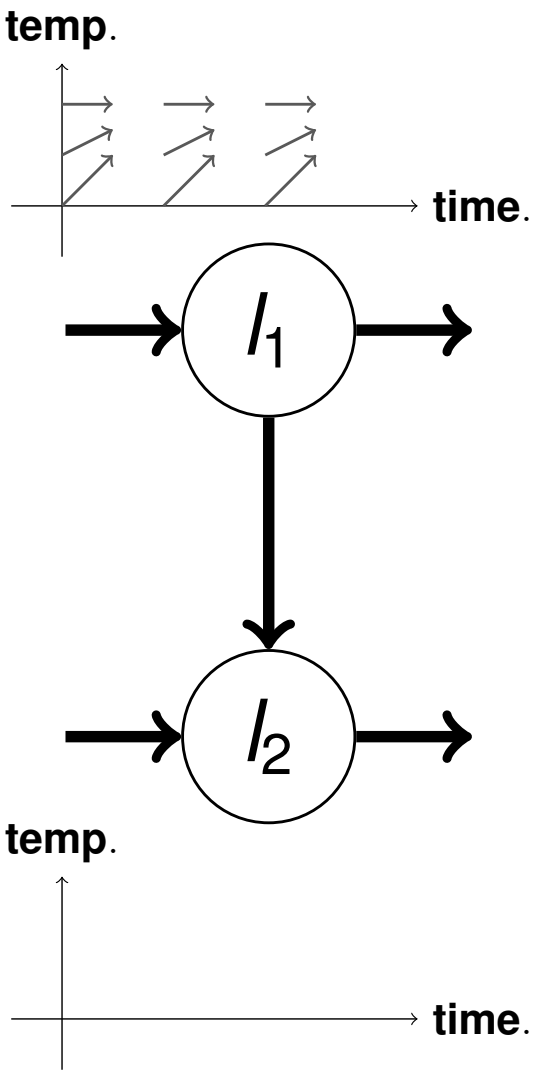
A template polyhedron:



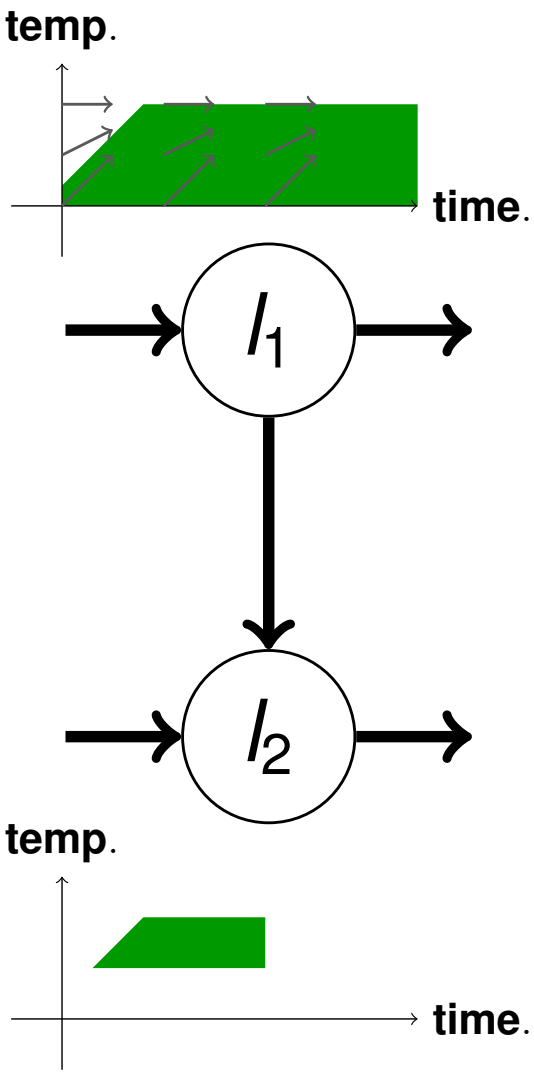
$$= \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mid \begin{array}{r} x_1 \leq \infty \\ -x_1 \leq 0 \\ x_2 \leq 0.5 \\ -x_2 \leq 0 \\ x_1 - x_2 \leq 0.5 \\ x_2 - x_1 \leq 0.25 \end{array} \right\} = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mid \begin{array}{r} x_1 \leq 1 \\ -x_1 \leq 0 \\ x_2 \leq 0.5 \\ -x_2 \leq 0 \\ x_1 - x_2 \leq 0.5 \\ x_2 - x_1 \leq 0.25 \end{array} \right\}$$

Abstract Interpretation?

Abstract Interpretation?

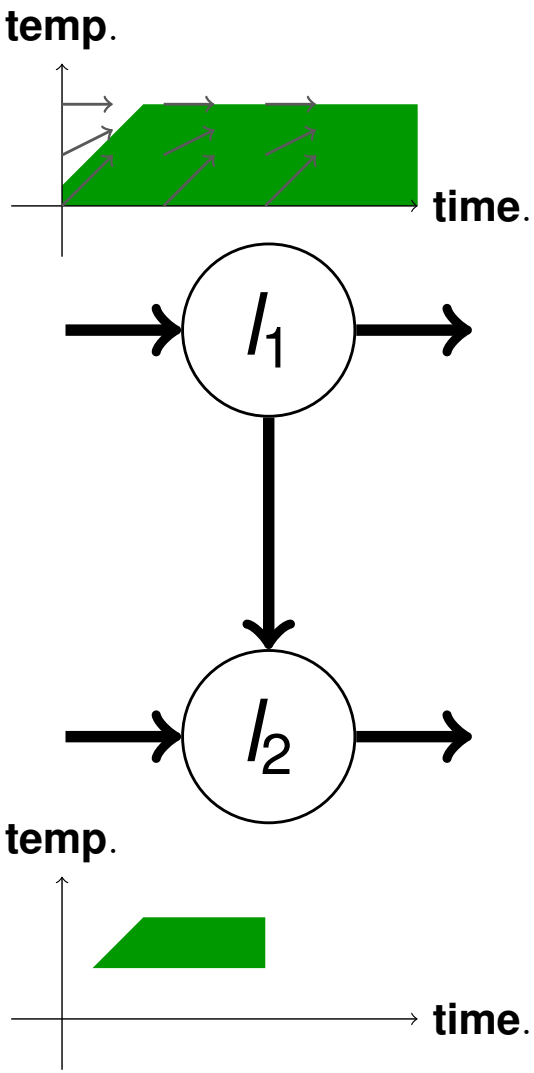


Abstract Interpretation?

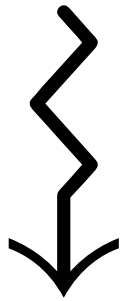


Compute the
Smallest Template Polyhedra
that are
invariant

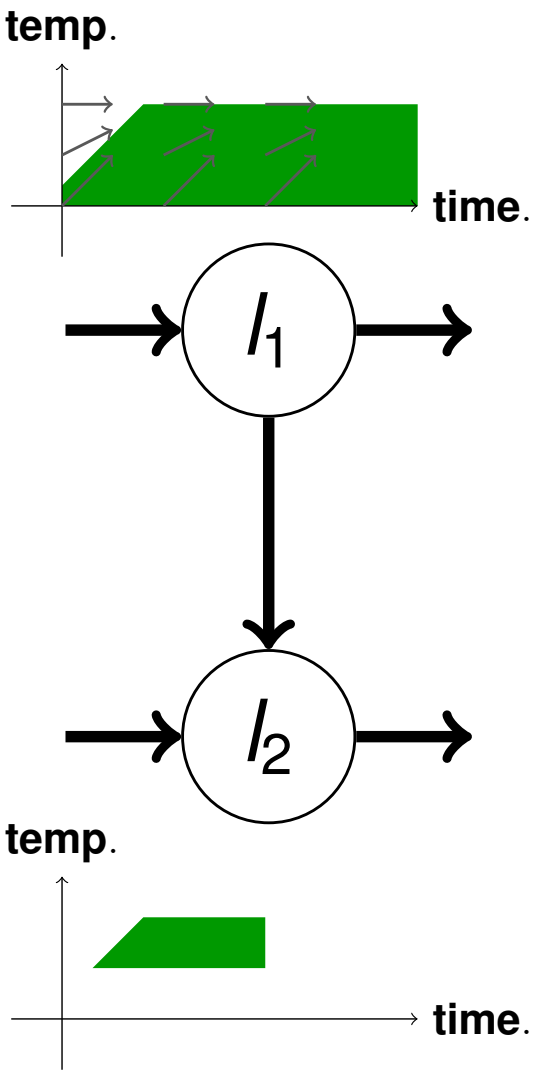
Abstract Interpretation?



Compute the
Smallest Template Polyhedra
that are
invariant



Abstract Interpretation?



Compute the
Smallest Template Polyhedra
that are
invariant

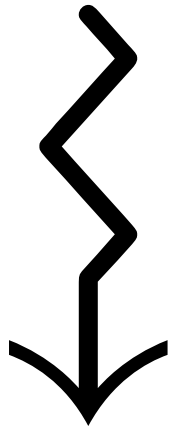
Safe Over-approximation
of the set of
Reachable States

Difficult?

Difficult?

$$\mathbf{P} \overset{?}{\ni} \text{Parity Games} \leq_{\mathbf{P}} \text{Mean Payoff Games} \begin{cases} \leq_{\mathbf{P}} \text{Our Problem} \\ \in \mathbf{NP} \cap \mathbf{coNP} \end{cases}$$

Our Problem



Mathematical Optimization Problem

Assumption: The linear templates are $t_1, \ldots, t_m$

Assumption: The linear templates are $t_1, \ldots, t_m$

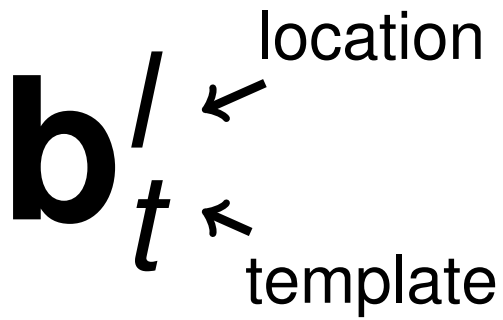
Variables: $\mathbf{b}_t^{\prime}$ $\in \overline{\mathbb{R}} := \mathbb{R} \cup \{\pm\infty\}$

Assumption: The linear templates are $t_1, \ldots, t_m$

Variables: $\mathbf{b}_t^l$ ← location $\in \overline{\mathbb{R}} := \mathbb{R} \cup \{\pm\infty\}$

Assumption: The linear templates are $t_1, \ldots, t_m$

Variables: $\mathbf{b}_t^l \in \overline{\mathbb{R}} := \mathbb{R} \cup \{\pm\infty\}$

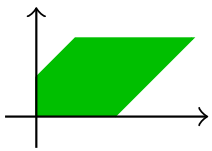


Assumption: The linear templates are $t_1, \ldots, t_m$

Variables: $\mathbf{b}_t^l \in \overline{\mathbb{R}} := \mathbb{R} \cup \{\pm\infty\}$

(l: location, t: template)

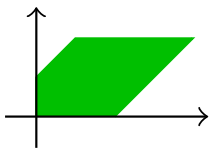
Meaning: **Value** of $\mathbf{b}_t^l$ $=$ **upper bound** on t at l, i.e.,

$(\mathbf{b}_{t_1}^l, \ldots, \mathbf{b}_{t_m}^l)$ represents  at l

Assumption: The linear templates are $t_1, \ldots, t_m$

Variables: $\mathbf{b}_t^l$ (l: location, t: template) $\in \overline{\mathbb{R}} := \mathbb{R} \cup \{\pm\infty\}$

Meaning: **Value** of $\mathbf{b}_t^l$ $=$ **upper bound** on t at l, i.e.,

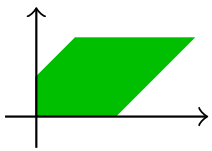
$(\mathbf{b}_{t_1}^l, \ldots, \mathbf{b}_{t_m}^l)$ represents  at l

Compute: **Minimal Solution** of:

Assumption: The linear templates are $t_1, \ldots, t_m$

Variables: $\mathbf{b}_t^l$ (l: location, t: template) $\in \overline{\mathbb{R}} := \mathbb{R} \cup \{\pm\infty\}$

Meaning: **Value** of $\mathbf{b}_t^l$ $=$ **upper bound** on t at l, i.e.,

$(\mathbf{b}_{t_1}^l, \ldots, \mathbf{b}_{t_m}^l)$ represents  at l

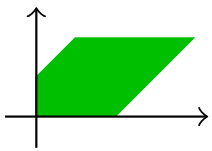
Compute: **Minimal Solution** of:

for all discrete transitions (l_1, stmt, l_2)

Assumption: The linear templates are $t_1, \ldots, t_m$

Variables: $\mathbf{b}_t^l$ (l: location, t: template) $\in \overline{\mathbb{R}} := \mathbb{R} \cup \{\pm\infty\}$

Meaning: **Value** of $\mathbf{b}_t^l$ $=$ **upper bound** on t at l, i.e.,

$(\mathbf{b}_{t_1}^l, \ldots, \mathbf{b}_{t_m}^l)$ represents  at l

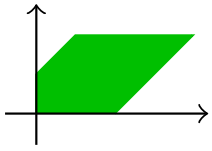
Compute: **Minimal Solution** of:

$$\mathbf{b}_t^{l_2} \geq [\![\text{stmt}]\!]_t^\sharp (\mathbf{b}_{t_1}^{l_1}, \ldots, \mathbf{b}_{t_m}^{l_1}) \quad \text{for all discrete transitions } (l_1, \text{stmt}, l_2)$$

Assumption: The linear templates are $t_1, \ldots, t_m$

Variables: $\mathbf{b}_t^l$ (l: location, t: template) $\in \overline{\mathbb{R}} := \mathbb{R} \cup \{\pm\infty\}$

Meaning: **Value** of $\mathbf{b}_t^l$ $=$ **upper bound** on t at l, i.e.,

$(\mathbf{b}_{t_1}^l, \ldots, \mathbf{b}_{t_m}^l)$ represents  at l

Compute: **Minimal Solution** of:

$$\mathbf{b}_t^{l_2} \geq [\![\text{stmt}]\!]_t^\sharp (\mathbf{b}_{t_1}^{l_1}, \ldots, \mathbf{b}_{t_m}^{l_1})$$

for all discrete transitions (l_1, stmt, l_2)

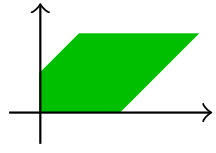
for all locations l

with Diff.-Inequalities D

Assumption: The linear templates are $t_1, \ldots, t_m$

Variables: $\mathbf{b}_t^l \in \overline{\mathbb{R}} := \mathbb{R} \cup \{\pm\infty\}$ (location l, template t)

Meaning: **Value** of $\mathbf{b}_t^l$ $=$ **upper bound** on t at l, i.e.,

$(\mathbf{b}_{t_1}^l, \ldots, \mathbf{b}_{t_m}^l)$ represents  at l

Compute: **Minimal Solution** of:

$\mathbf{b}_t^{l_2} \geq [\![\text{stmt}]\!]_t^\sharp (\mathbf{b}_{t_1}^{l_1}, \ldots, \mathbf{b}_{t_m}^{l_1})$ for all discrete transitions (l_1, stmt, l_2)

$\mathbf{b}_t^l \geq \text{BlowUp}_t^D(\mathbf{b}_{t_1}^l, \ldots, \mathbf{b}_{t_m}^l)$ for all locations l with Diff.-Inequalities D

$$[\![\text{stmt}]\!]^{\sharp}_{t}(d) \quad \textbf{?}$$

$$[\![\text{stmt}]\!]^{\sharp}_{t}(d) \quad \textbf{?}$$

Convention: $T := \begin{pmatrix} t_1 \\ \vdots \\ t_m \end{pmatrix}$, e.g. $T := \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$ for $\begin{matrix} 2x_1 - x_2 \\ x_2 \end{matrix}$

$$[\![\text{stmt}]\!]^{\sharp}_{t}(d) \quad \textbf{?}$$

Convention: $T := \begin{pmatrix} t_1 \\ \vdots \\ t_m \end{pmatrix}$, e.g. $T := \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$ for $\begin{matrix} 2x_1 - x_2 \\ x_2 \end{matrix}$

Concretization: $\gamma(d) = \{x \in \mathbb{R}^n \mid Tx \leq d\}$ $\forall d \in \overline{\mathbb{R}}^m$

$$[\![\text{stmt}]\!]_t^\sharp(d) \quad ?$$

Convention: $T := \begin{pmatrix} t_1 \\ \vdots \\ t_m \end{pmatrix}$, e.g. $T := \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$ for $\begin{matrix} 2x_1 - x_2 \\ x_2 \end{matrix}$

Concretization: $\gamma(d) = \{x \in \mathbb{R}^n \mid Tx \leq d\}$ $\quad \forall d \in \overline{\mathbb{R}}^m$

Abstraction: $\alpha(X) = \textbf{min}\ \{d \in \overline{\mathbb{R}}^m \mid \gamma(d) \supseteq X\}$ $\quad \forall X \subseteq \mathbb{R}^n$

$$[\![\text{stmt}]\!]_t^\sharp(d) \quad ?$$

Convention: $T := \begin{pmatrix} t_1 \\ \vdots \\ t_m \end{pmatrix}$, e.g. $T := \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$ for $\begin{matrix} 2x_1 - x_2 \\ x_2 \end{matrix}$

Concretization: $\gamma(d) = \{x \in \mathbb{R}^n \mid Tx \leq d\}$ $\quad \forall d \in \overline{\mathbb{R}}^m$

Abstraction: $\alpha(X) = \textbf{min}\ \{d \in \overline{\mathbb{R}}^m \mid \gamma(d) \supseteq X\}$ $\quad \forall X \subseteq \mathbb{R}^n$

Then:

$$[\![\text{stmt}]\!]^\sharp := \alpha \circ [\![\text{stmt}]\!] \circ \gamma$$

$$([\![\text{stmt}]\!]_{t_1}^\sharp, \ldots, [\![\text{stmt}]\!]_{t_m}^\sharp) := [\![\text{stmt}]\!]^\sharp$$

$$[\![\text{stmt}]\!]^{\sharp}_{t}(d) \quad \textbf{?}$$

$$[\![\text{stmt}]\!]^{\sharp}_{t}(d) \quad \textbf{?}$$

Example: stmt $=$ $x' = Ax + b$

$$[\![\text{stmt}]\!]^{\sharp}_{t}(d) \quad \textbf{?}$$

Example: stmt $=$ $x' = Ax + b$

$$[\![x'=Ax+b]\!]^{\sharp}_{t_i}(d)$$

Then:

$$[\![\text{stmt}]\!]^{\sharp}_{t}(d) \quad \textbf{?}$$

Example: stmt $=$ $x' = Ax + b$

$$[\![x' = Ax + b]\!]^{\sharp}_{t_i}(d) = \sup\{t_i(Ax + b) \mid Tx \leq d\}$$

Then:

$$[\![\text{stmt}]\!]^{\sharp}_{t}(d) \quad \textbf{?}$$

Example: stmt $=$ $x' = Ax + b$

$$[\![x'{=}Ax{+}b]\!]^{\sharp}_{t_i}(d) = \sup\{t_i(Ax+b) \mid Tx \leq d\}$$

Then:
$$= t_i b + \sup\{t_i Ax \mid Tx \leq d\}$$

$[\![\text{stmt}]\!]^\sharp_t(d)$?

Example: stmt $=$ $x' = Ax + b$

$$[\![x'=Ax+b]\!]^\sharp_{t_i}(d) = \sup\{t_i(Ax+b) \mid Tx \leq d\}$$

Then:

$$= t_i b + \sup\{t_i Ax \mid Tx \leq d\}$$

$$= t_i b + \inf\{d^\top y \mid yT = t_i A,\ y \geq 0\}$$

$[\![\text{stmt}]\!]^\sharp_t(d)$?

Example: stmt $=$ $x' = Ax + b$

Then:

$$[\![x'=Ax+b]\!]^\sharp_{t_i}(d) = \sup\{t_i(Ax+b) \mid Tx \leq d\}$$
$$= t_i b + \sup\{t_i Ax \mid Tx \leq d\}$$
$$= t_i b + \inf\{d^\top y \mid yT = t_i A,\ y \geq 0\}$$

Observation:

$$[\![x' = Ax + b]\!]^\sharp_{t_i}$$
$$\simeq$$

point-wise min of
finitely many **monotone** and **affine** operators.

$$\mathrm{BlowUp}_t^D(d) \quad \textbf{?}$$

$\mathrm{BlowUp}_t^D(d)$?

Example: ...

Convention: ...

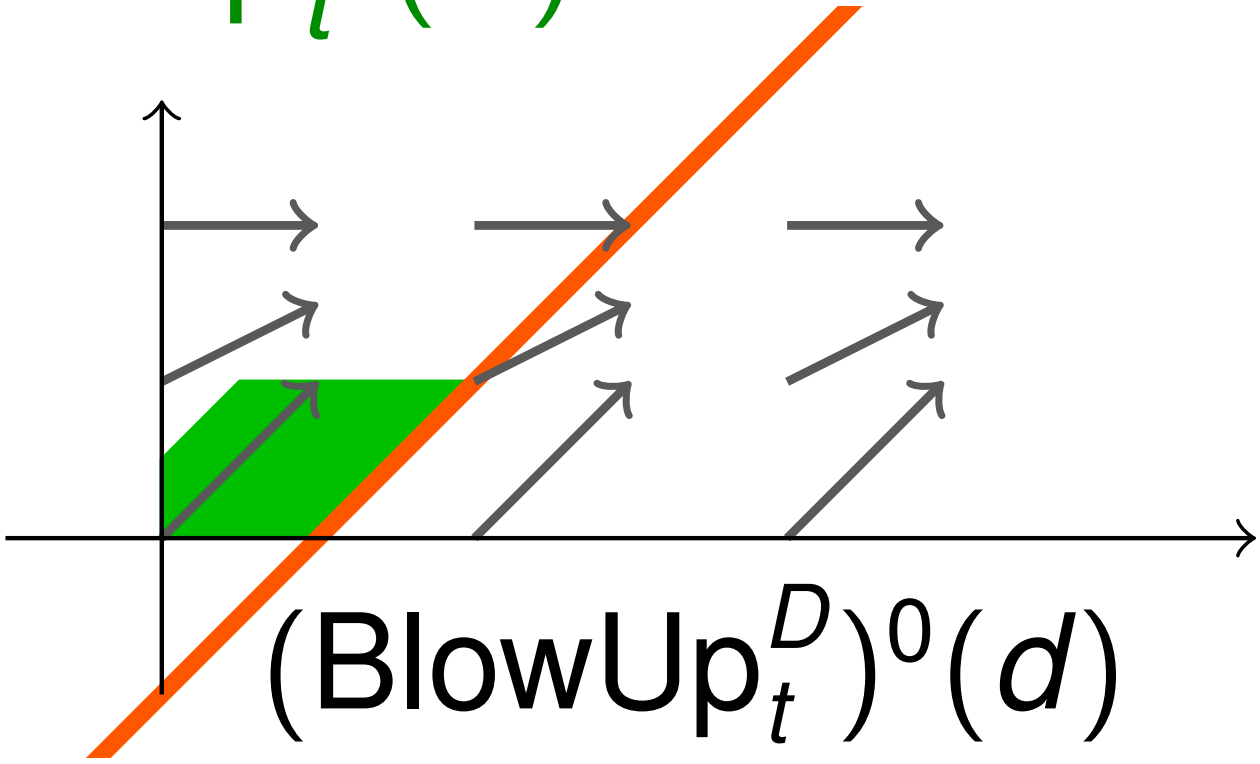
Then: ...

$\mathrm{BlowUp}_t^D(d)$?

Example: ...

Convention: ...

Then: ...

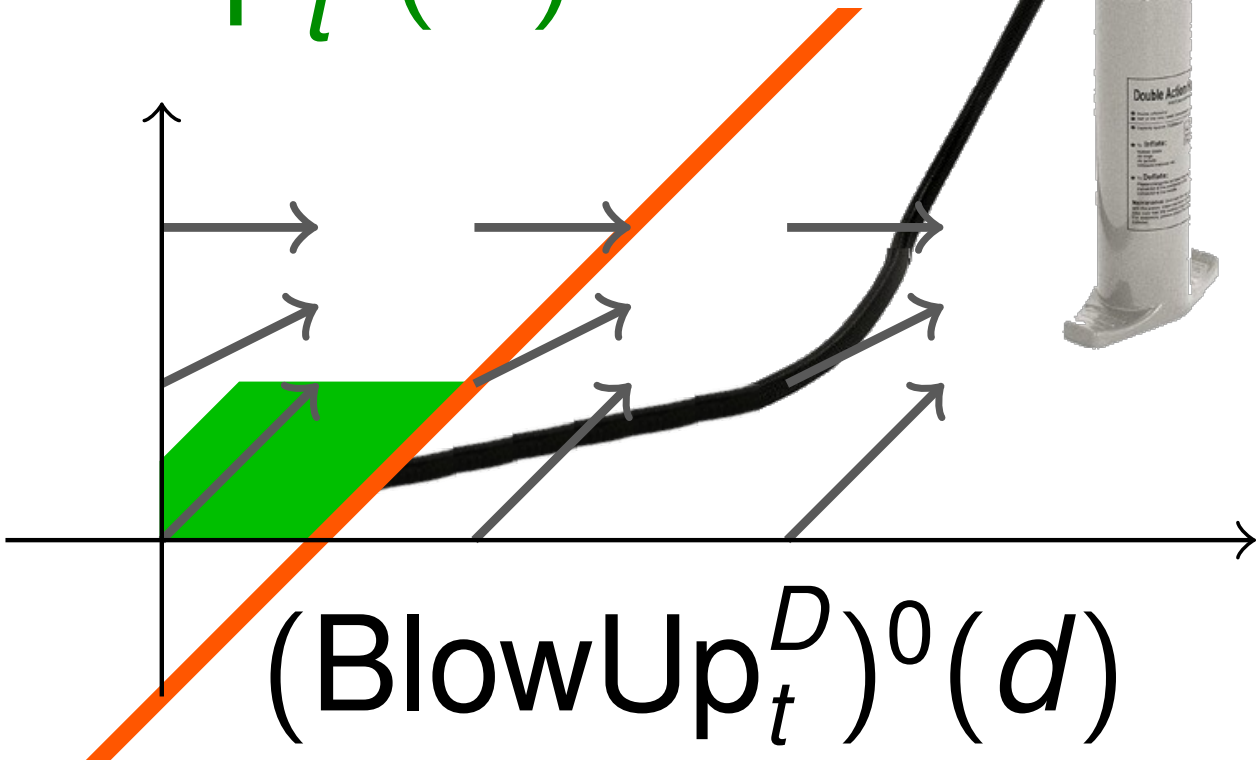


$\mathrm{BlowUp}_t^D(d)$?

Example: ...

Convention: ...

Then: ...

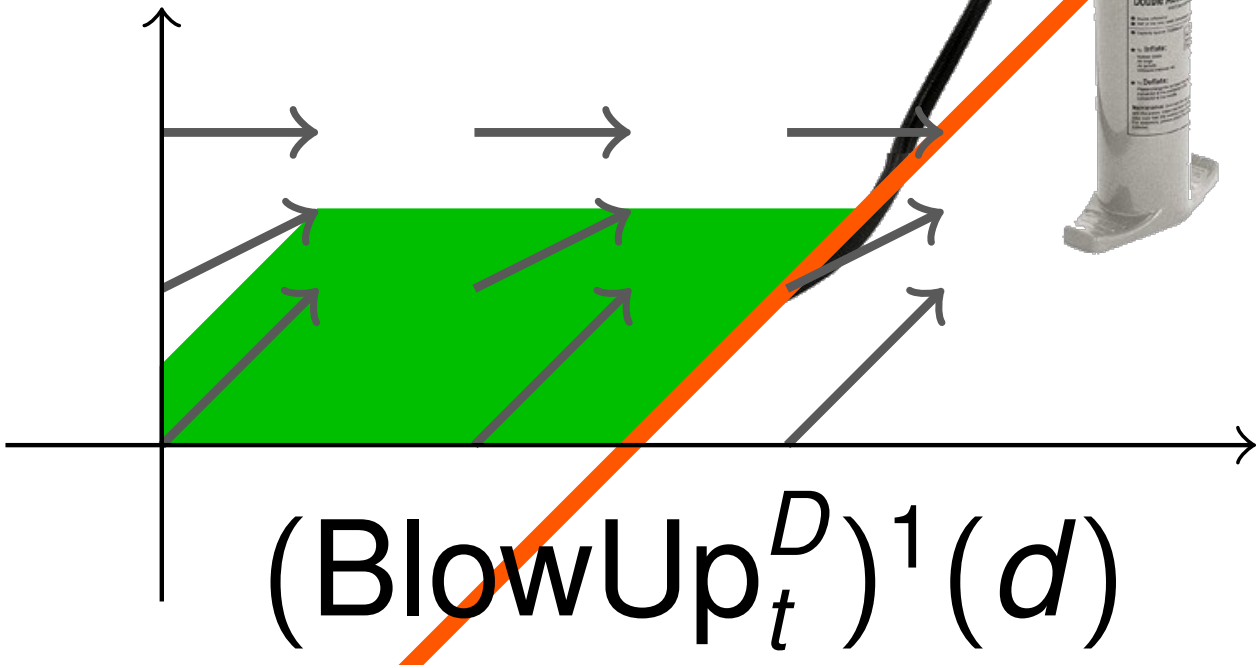


$\mathrm{BlowUp}_t^D(d)$?

Example: ...

Convention: ...

Then: ...

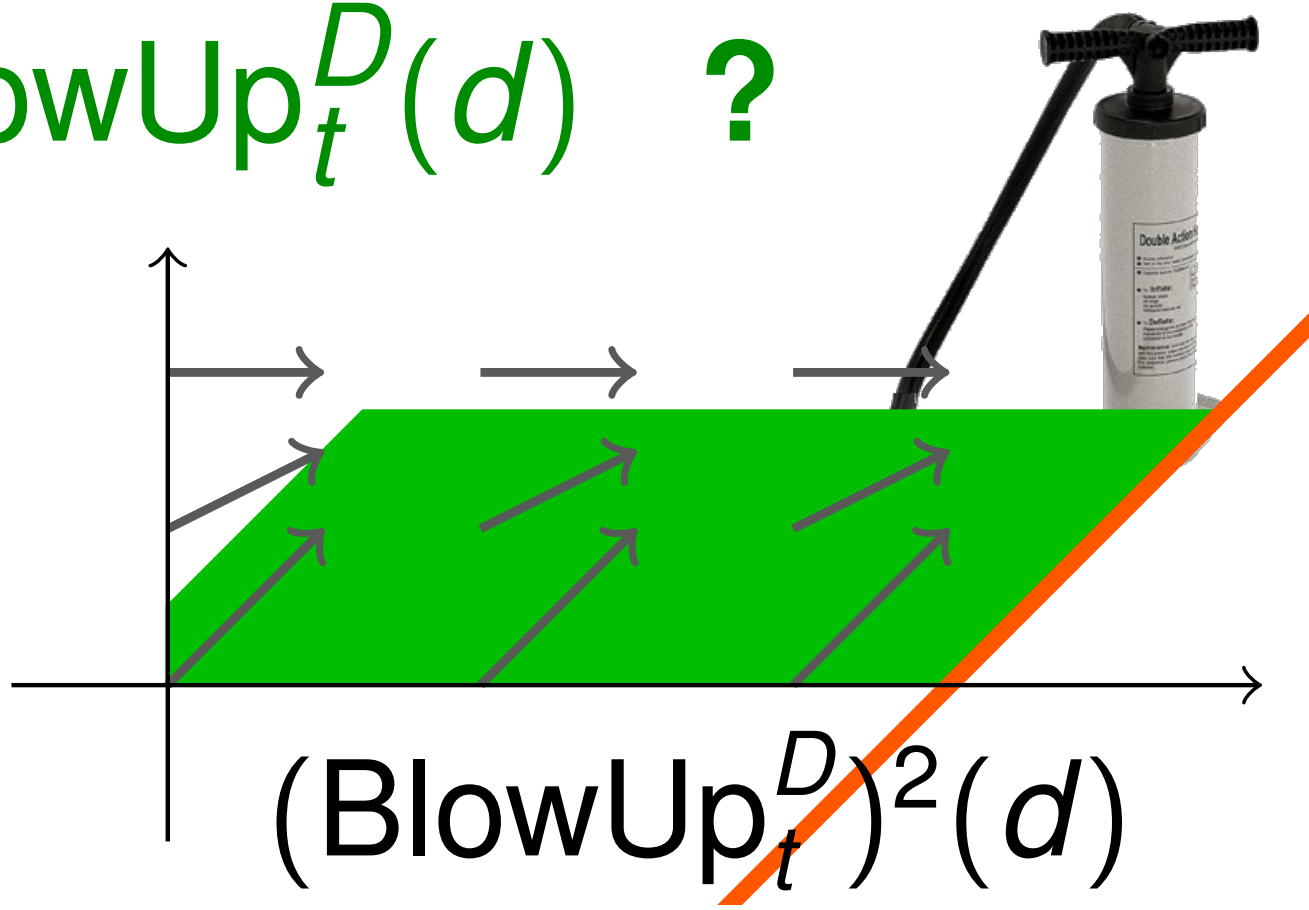


$\mathrm{BlowUp}_t^D(d)$?

Example: ...

Convention: ...

Then: ...

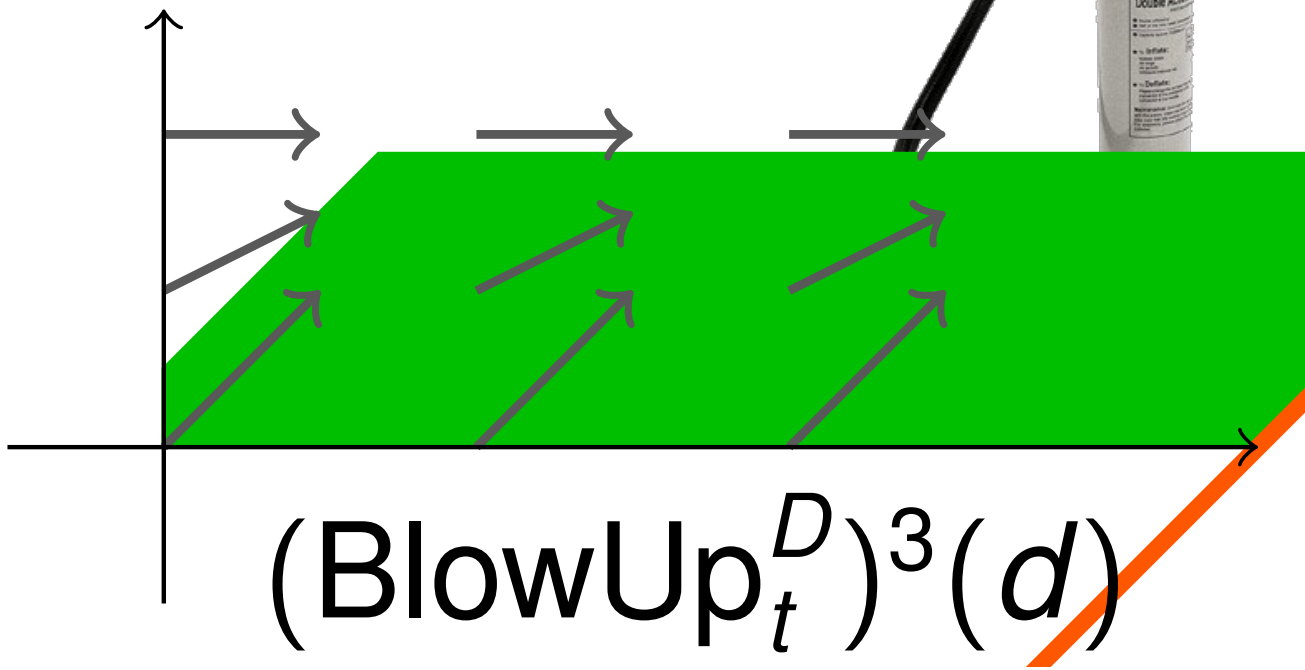


$\mathrm{BlowUp}_t^D(d)$?

Example: ...

Convention: ...

Then: ...



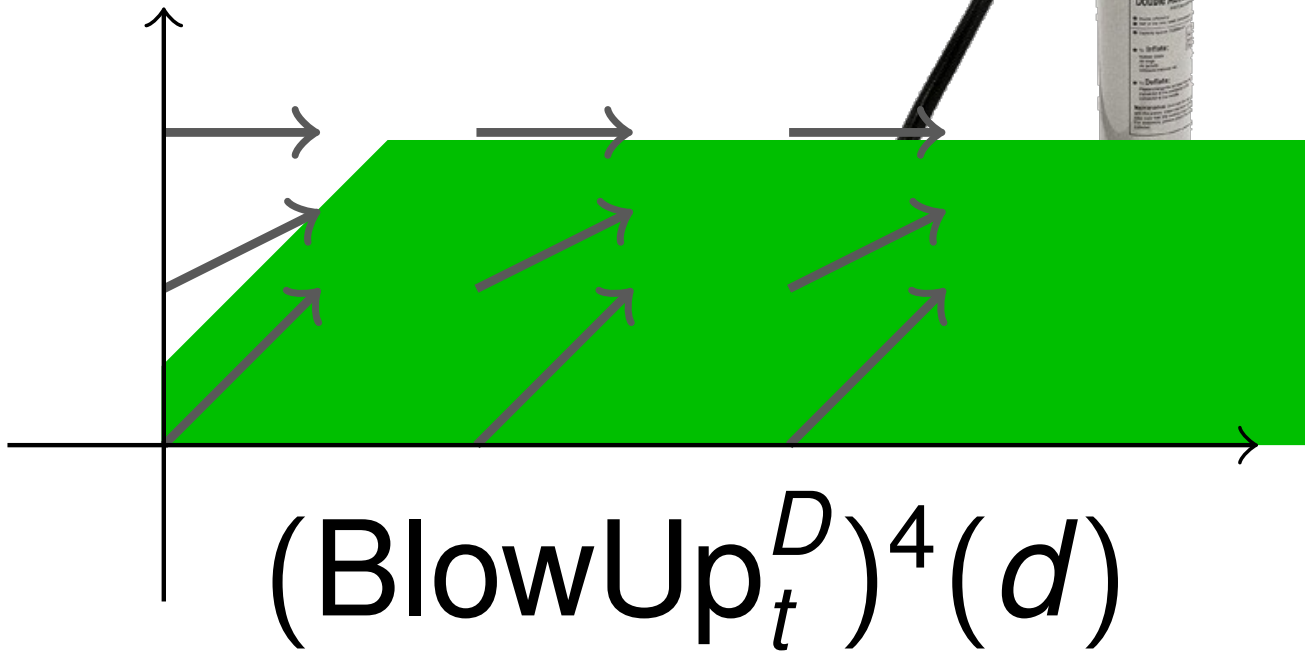
$(\mathrm{BlowUp}_t^D)^3(d)$

$\mathrm{BlowUp}_t^D(d)$?

Example: ...

Convention: ...

Then: ...



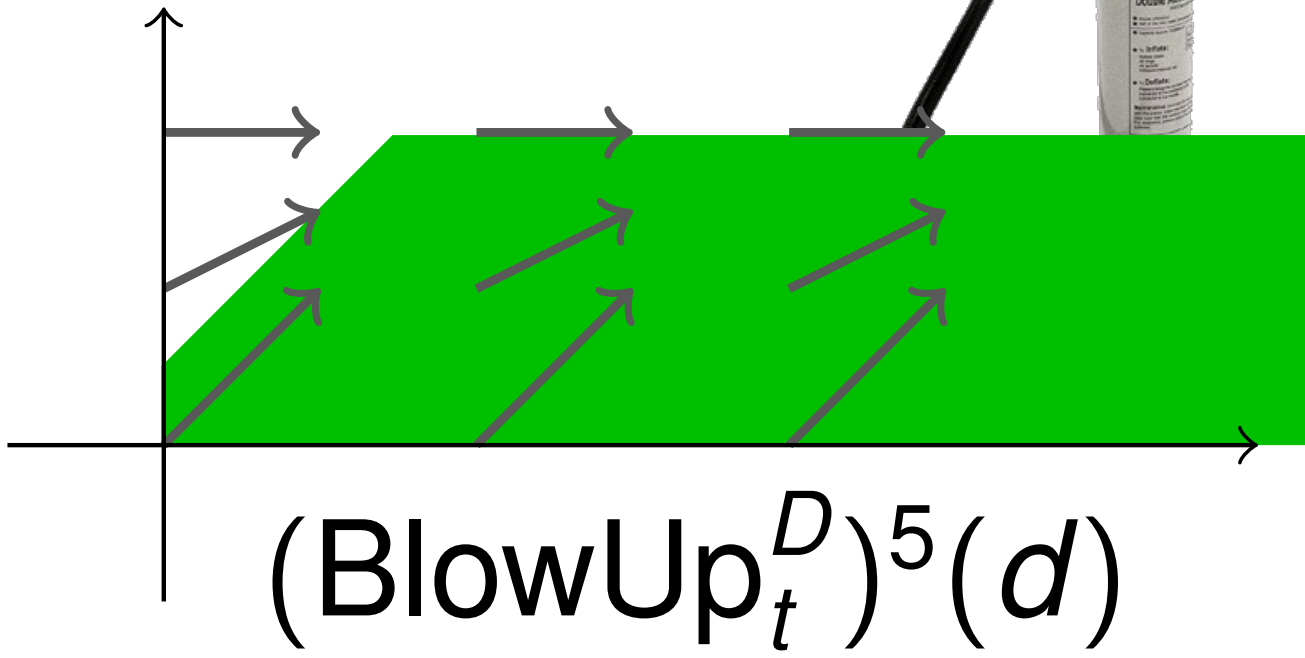
$(\mathrm{BlowUp}_t^D)^4(d)$

$\mathrm{BlowUp}_t^D(d)$?

Example: ...

Convention: ...

Then: ...



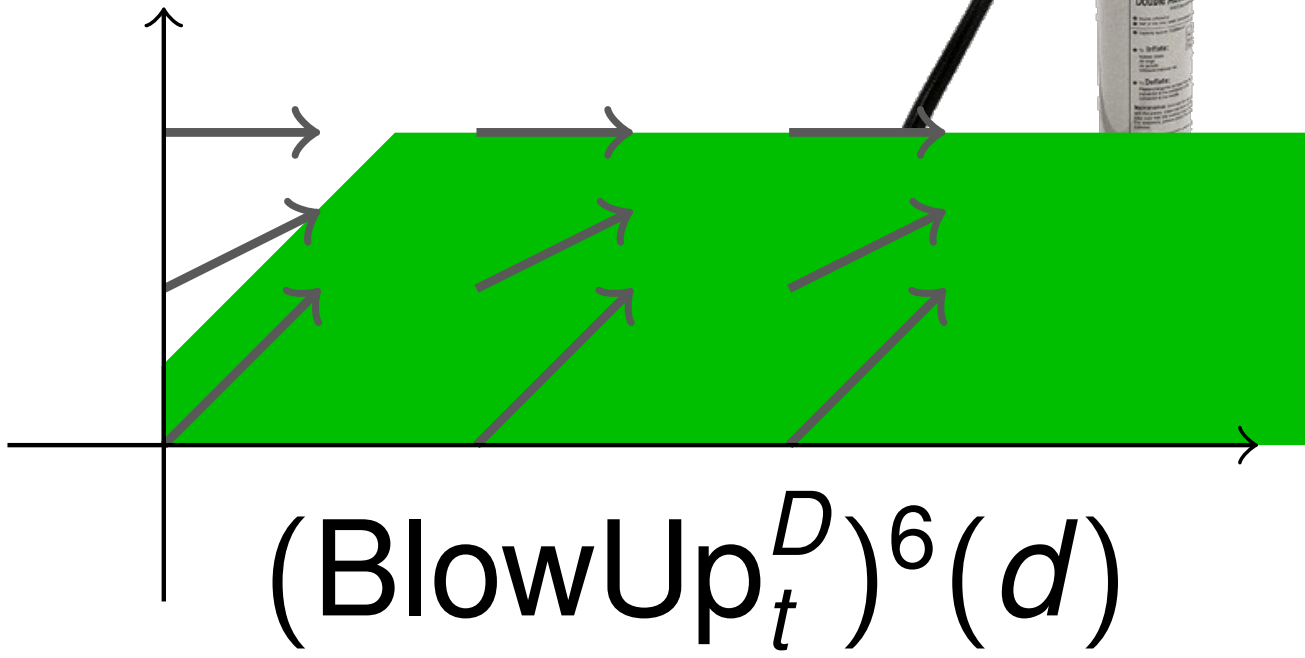
$(\mathrm{BlowUp}_t^D)^5(d)$

$\mathrm{BlowUp}_t^D(d)$?

Example: ...

Convention: ...

Then: ...



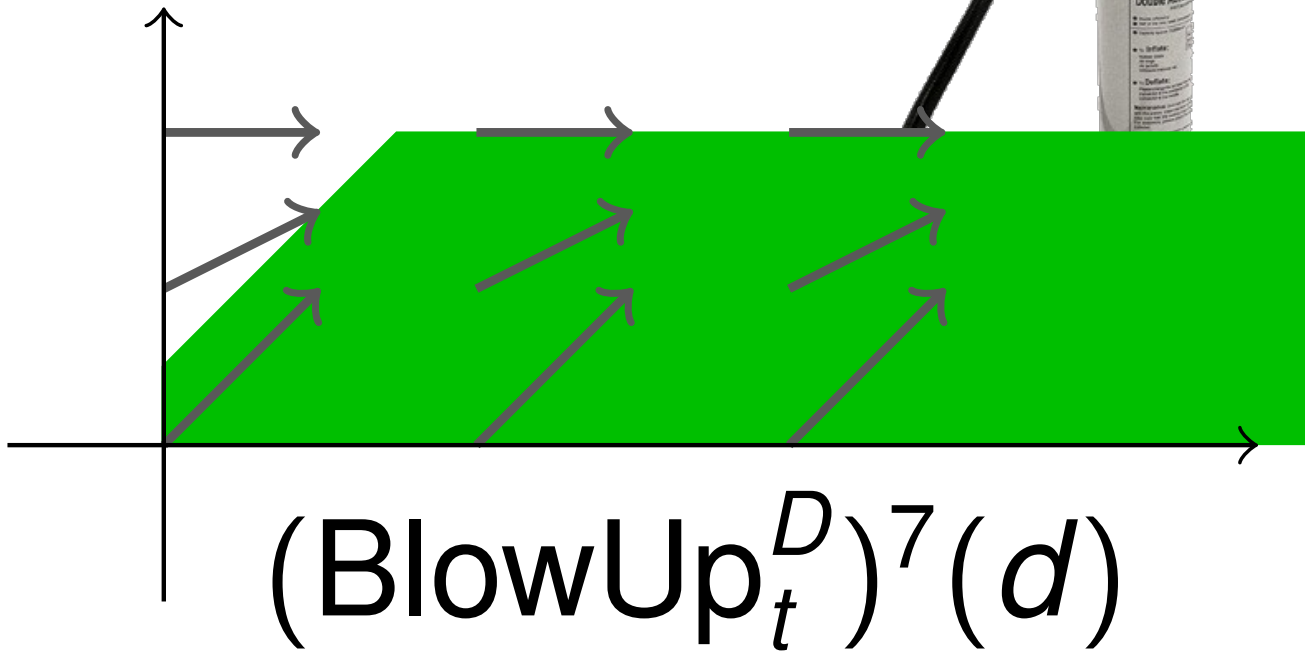
$(\mathrm{BlowUp}_t^D)^6(d)$

$\mathrm{BlowUp}_t^D(d)$?

Example: ...

Convention: ...

Then: ...



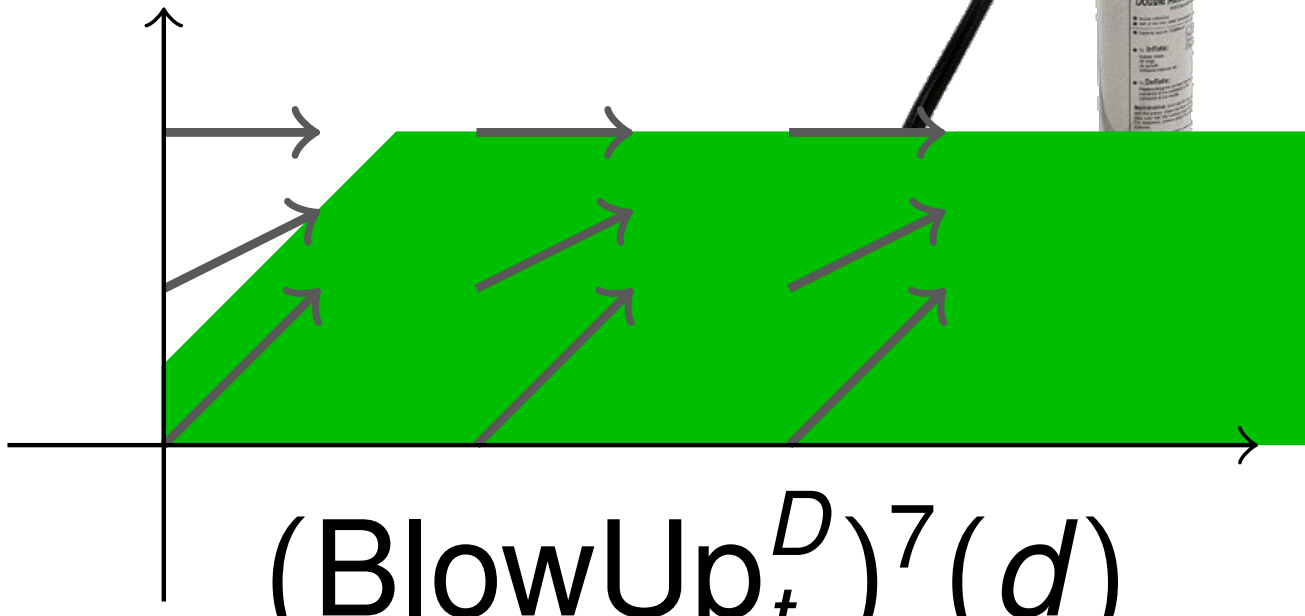
$(\mathrm{BlowUp}_t^D)^7(d)$

$\mathrm{BlowUp}_t^D(d)$?

Example: ...

Convention: ...

Then: ...



$(\mathrm{BlowUp}_t^D)^7(d)$

Observation:

BlowUp_t^D

$\simeq$

point-wise min of
finitely many **monotone** and **affine** operators.

$[\![\text{stmt}]\!]_t^\sharp : \overline{\mathbb{R}}^n \to \overline{\mathbb{R}}$ and $\text{BlowUp}_t^D : \overline{\mathbb{R}}^n \to \overline{\mathbb{R}}$

are

Point-wise min

of

finitely many

monotone and **affine** operators!

$[\![\text{stmt}]\!]_t^\sharp : \overline{\mathbb{R}}^n \to \overline{\mathbb{R}}$ and $\text{BlowUp}_t^D : \overline{\mathbb{R}}^n \to \overline{\mathbb{R}}$

are

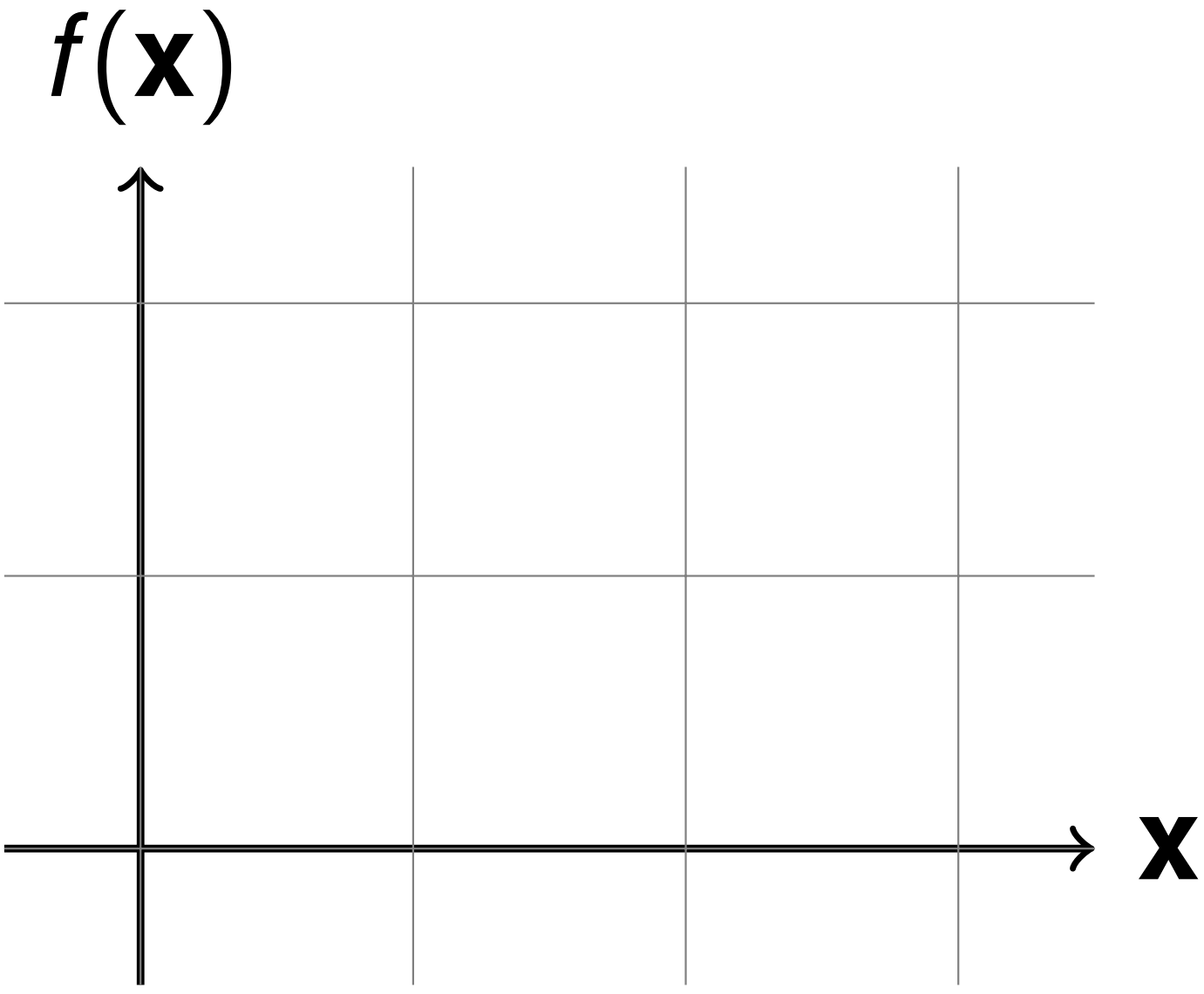
Point-wise min

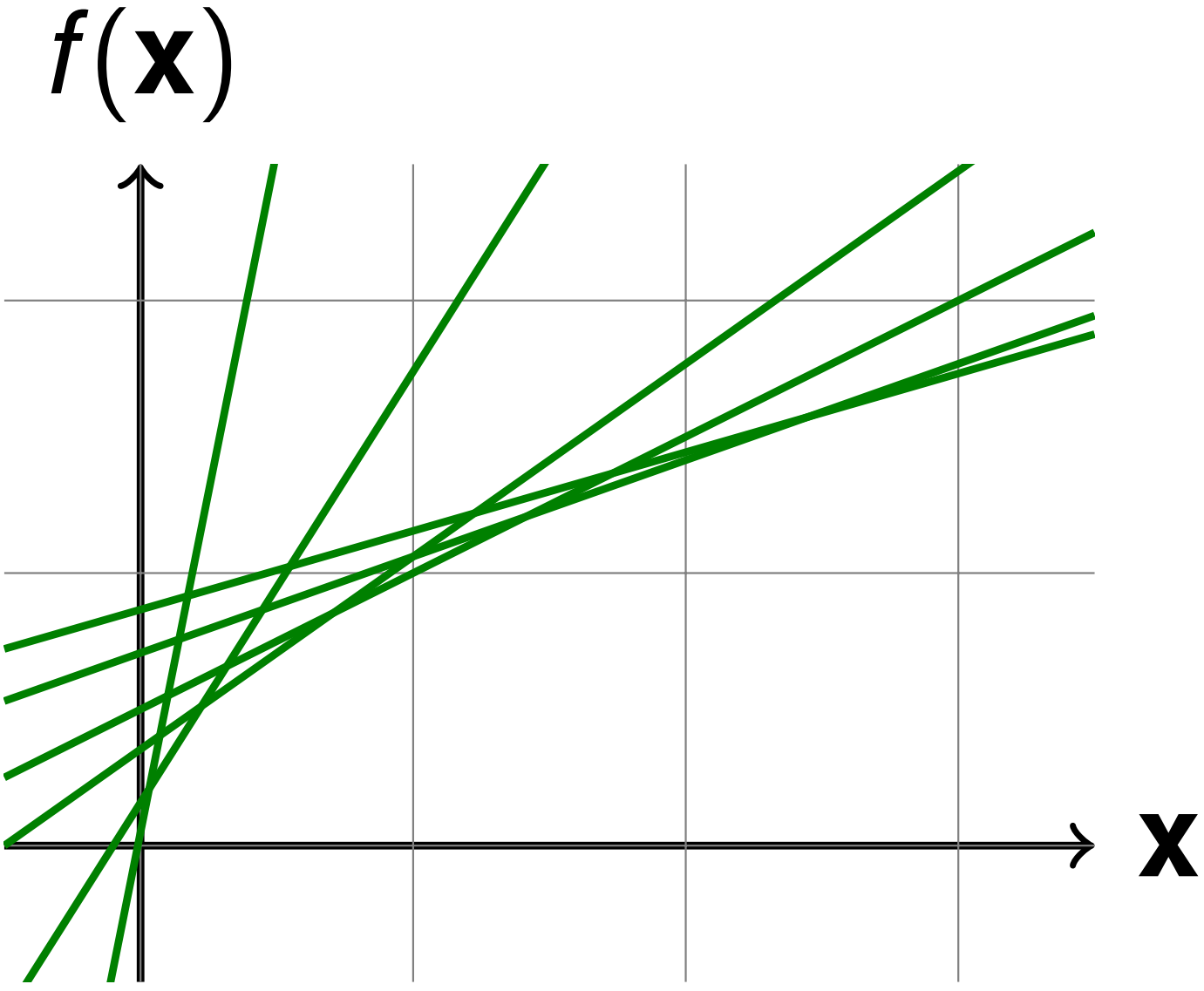
of

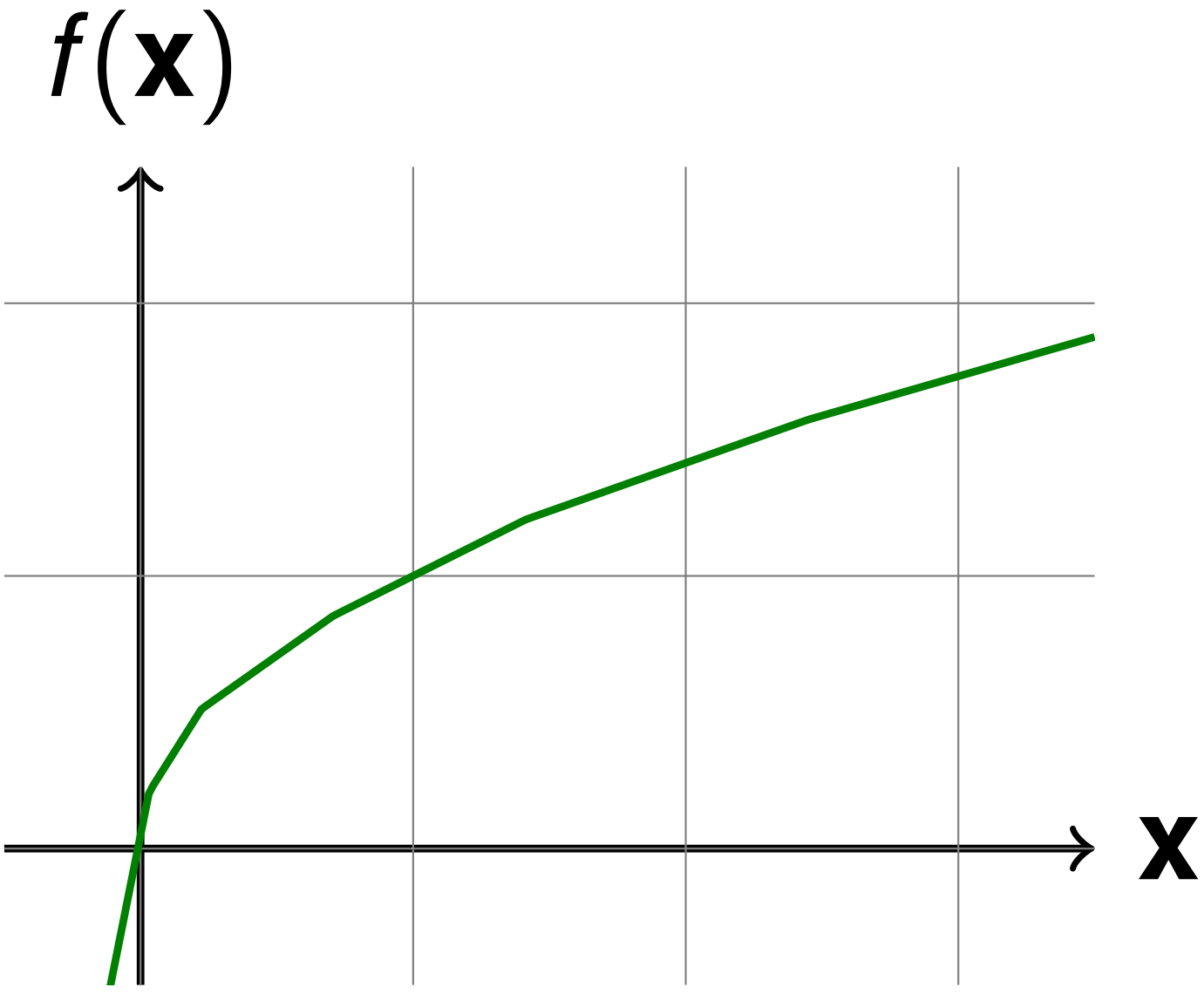
finitely many

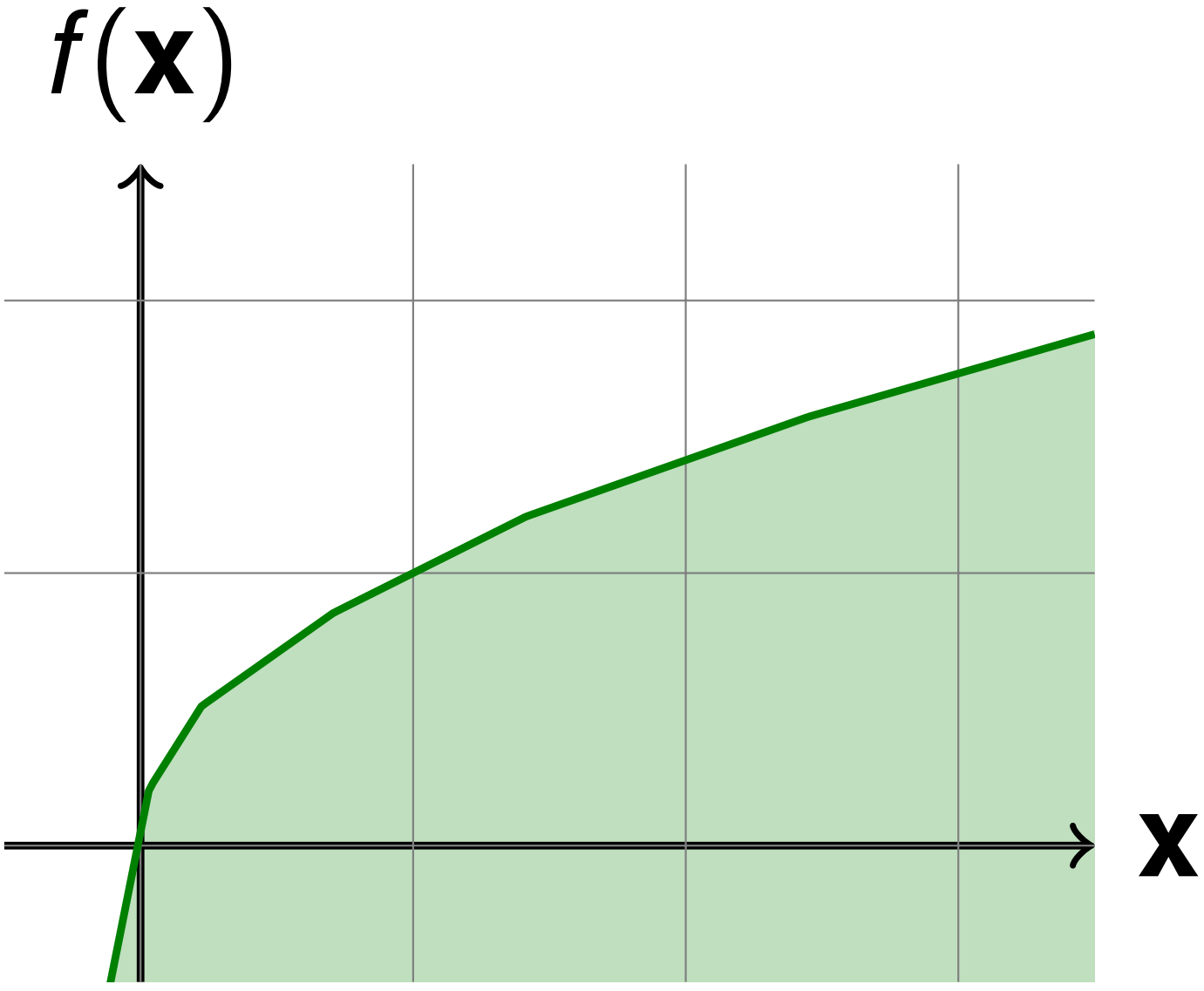
monotone and **affine** operators!

So What?

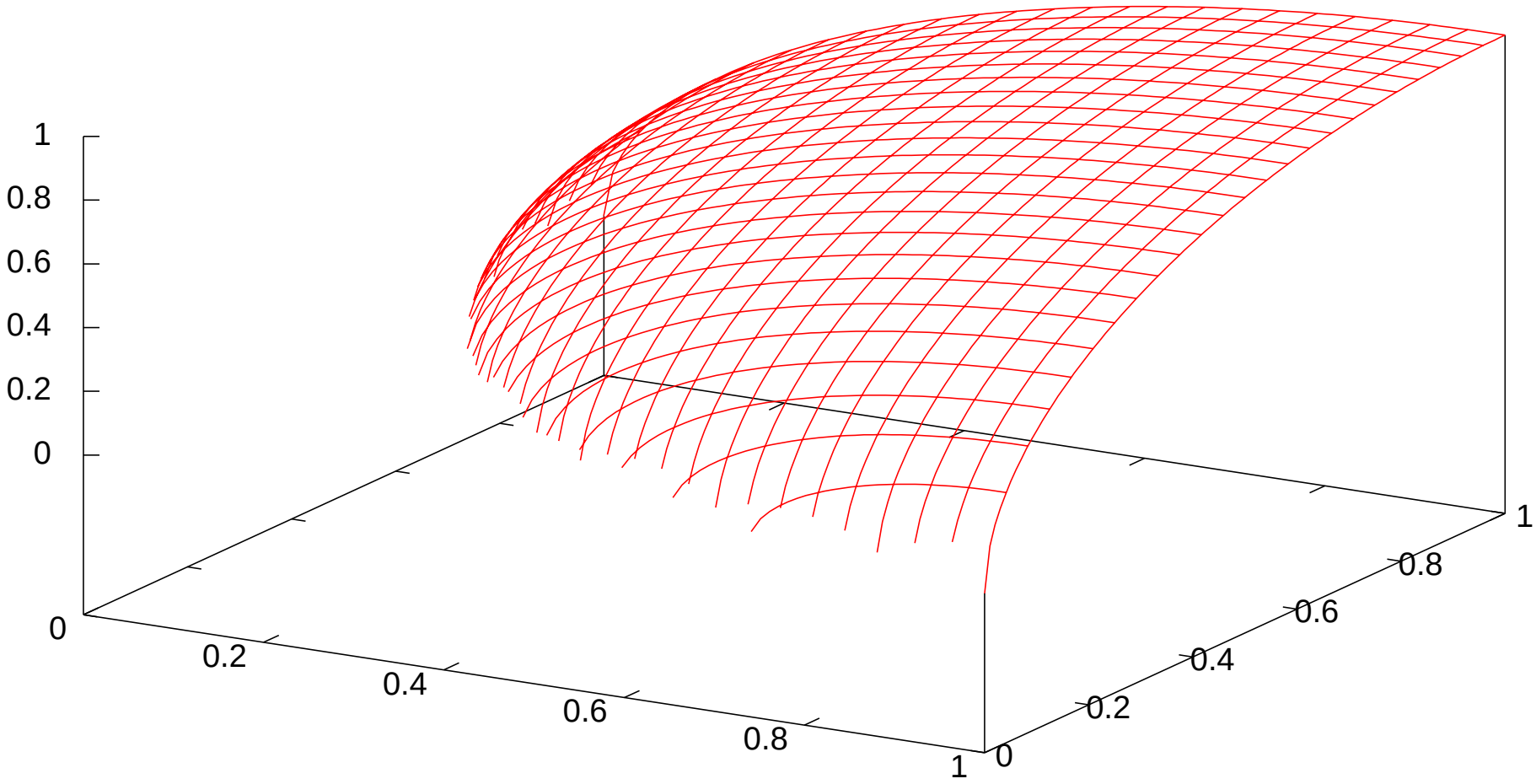

f(x)
X


f(x)
x


f(x)
x


f(x)
x

A **Monotone** and **Concave** Function $f : \overline{\mathbb{R}}^2 \to \overline{\mathbb{R}}$



Summary: Our Problem reduces to

Compute **Minimal Solution** of

$$\begin{aligned} \mathbf{x}_1 \geq\ & f_{1,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \quad \ldots, \quad f_{1,k_1}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \\ \vdots\ & \\ \mathbf{x}_n \geq\ & f_{n,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \quad \ldots, \quad f_{n,k_n}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \end{aligned}$$

where

- $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \overline{\mathbb{R}} := \mathbb{R} \cup \{-\infty, \infty\}$ are the variables
- The $f_{i,j}$'s are **monotone** and **concave**

Summary: Our Problem reduces to

Compute **Minimal Solution** of

$$\begin{aligned} \mathbf{x}_1 &\geq \mathbf{max}\ \{ f_{1,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \quad \ldots, \quad f_{1,k_1}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \\ &\vdots \\ \mathbf{x}_n &\geq \mathbf{max}\ \{ f_{n,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \quad \ldots, \quad f_{n,k_n}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \end{aligned}$$

where

- $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \overline{\mathbb{R}} := \mathbb{R} \cup \{-\infty, \infty\}$ are the variables
- The $f_{i,j}$'s are **monotone** and **concave**

Summary: Our Problem reduces to

Compute **Minimal Solution** of

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{max}\ \{ f_{1,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \quad \ldots, \quad f_{1,k_1}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \\ &\vdots \\ \mathbf{x}_n &= \mathbf{max}\ \{ f_{n,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \quad \ldots, \quad f_{n,k_n}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \end{aligned}$$

where

- $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \overline{\mathbb{R}} := \mathbb{R} \cup \{-\infty, \infty\}$ are the variables
- The $f_{i,j}$'s are **monotone** and **concave**

1-dim Example:

1-dim Example:

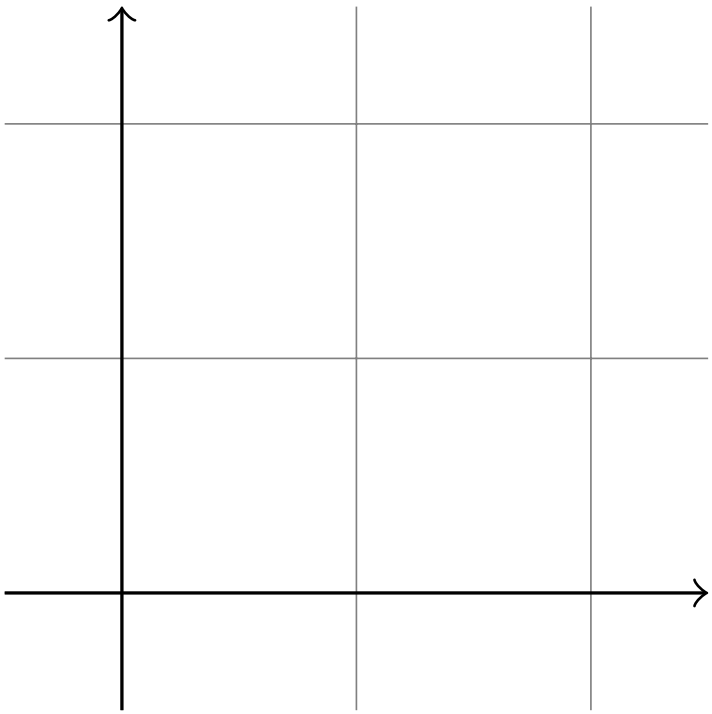
Compute **Minimal Solution** of

$$\mathbf{x} = \mathbf{max}\ \{\, 0.4,\ \sqrt{\mathbf{x}},\ 1 + \sqrt{\mathbf{x} - 1}\,\}$$

1-dim Example:

Compute **Minimal Solution** of

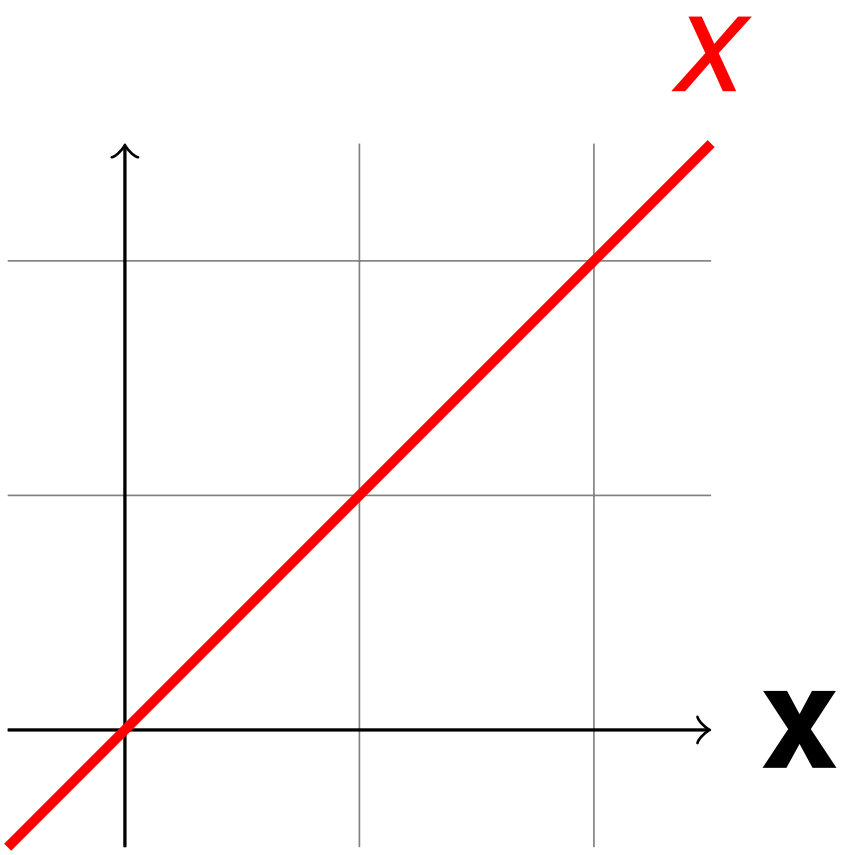
$= \max \{ \quad , \quad , \quad \}$



1-dim Example:

Compute **Minimal Solution** of

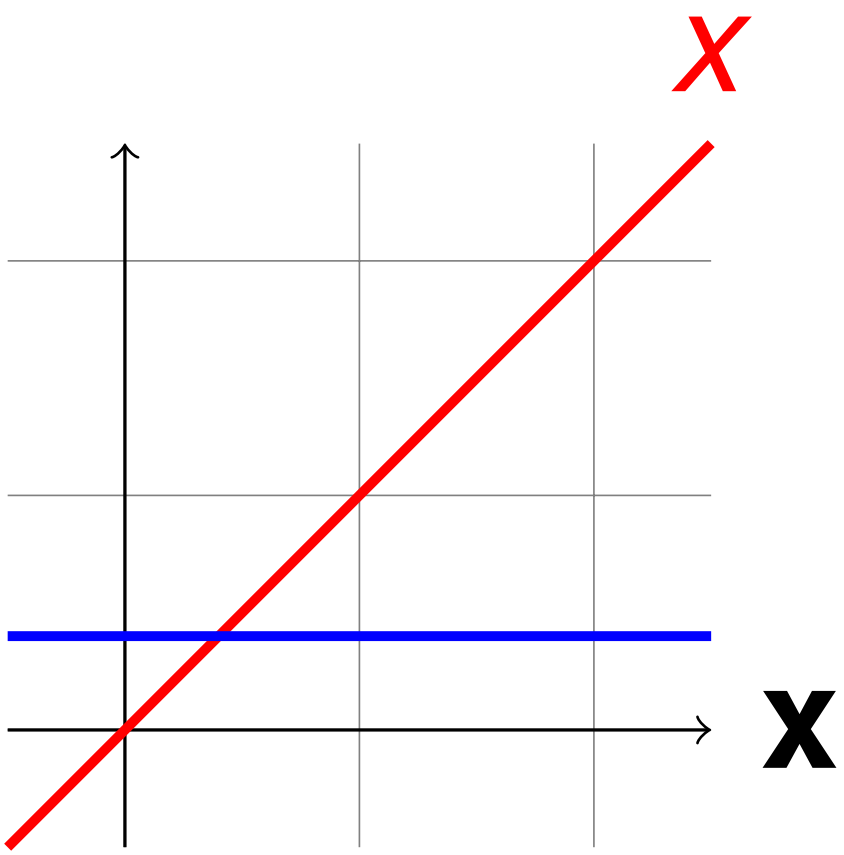
$$x = \max\{\quad,\quad,\quad\}$$



1-dim Example:

Compute **Minimal Solution** of

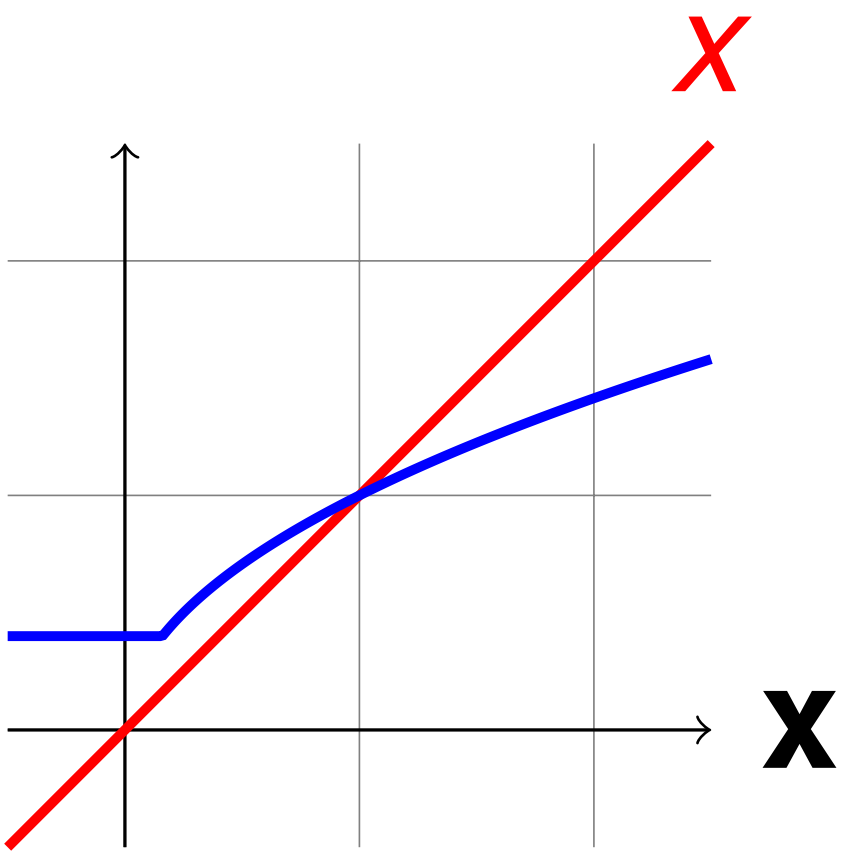
$$\mathbf{x} = \mathbf{max}\ \{\, 0.4, \quad , \quad \}$$



1-dim Example:

Compute **Minimal Solution** of

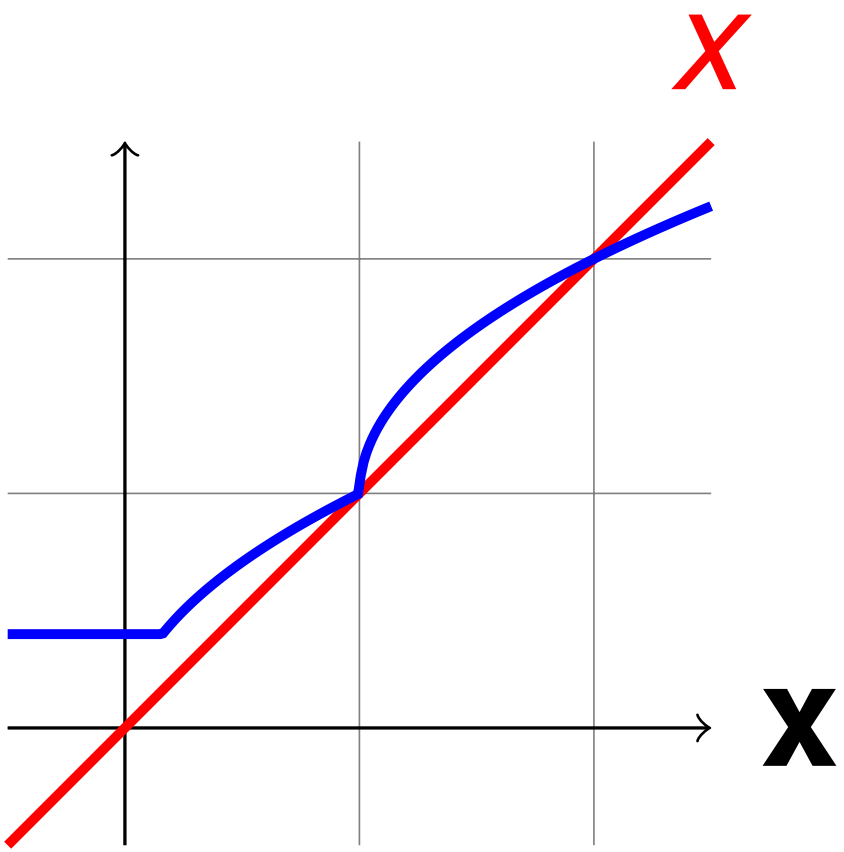
$$\mathbf{x} = \mathbf{max}\ \{\, 0.4,\ \sqrt{\mathbf{x}},\ \ldots \,\}$$



1-dim Example:

Compute **Minimal Solution** of

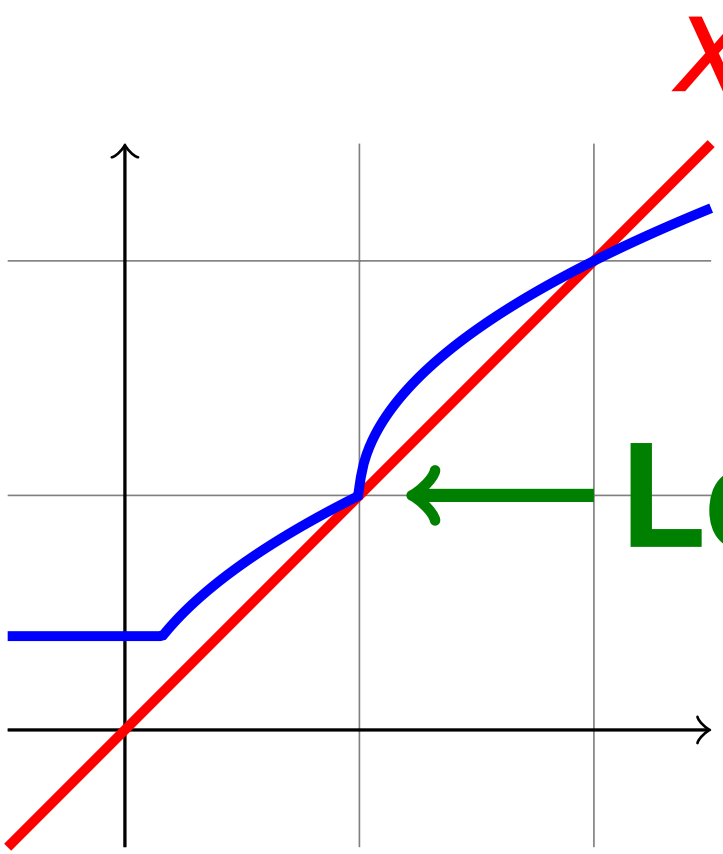
$$\mathbf{x} = \mathbf{max}\ \{\, 0.4,\ \sqrt{\mathbf{x}},\ 1 + \sqrt{\mathbf{x} - 1}\,\}$$



1-dim Example:

Compute **Minimal Solution** of

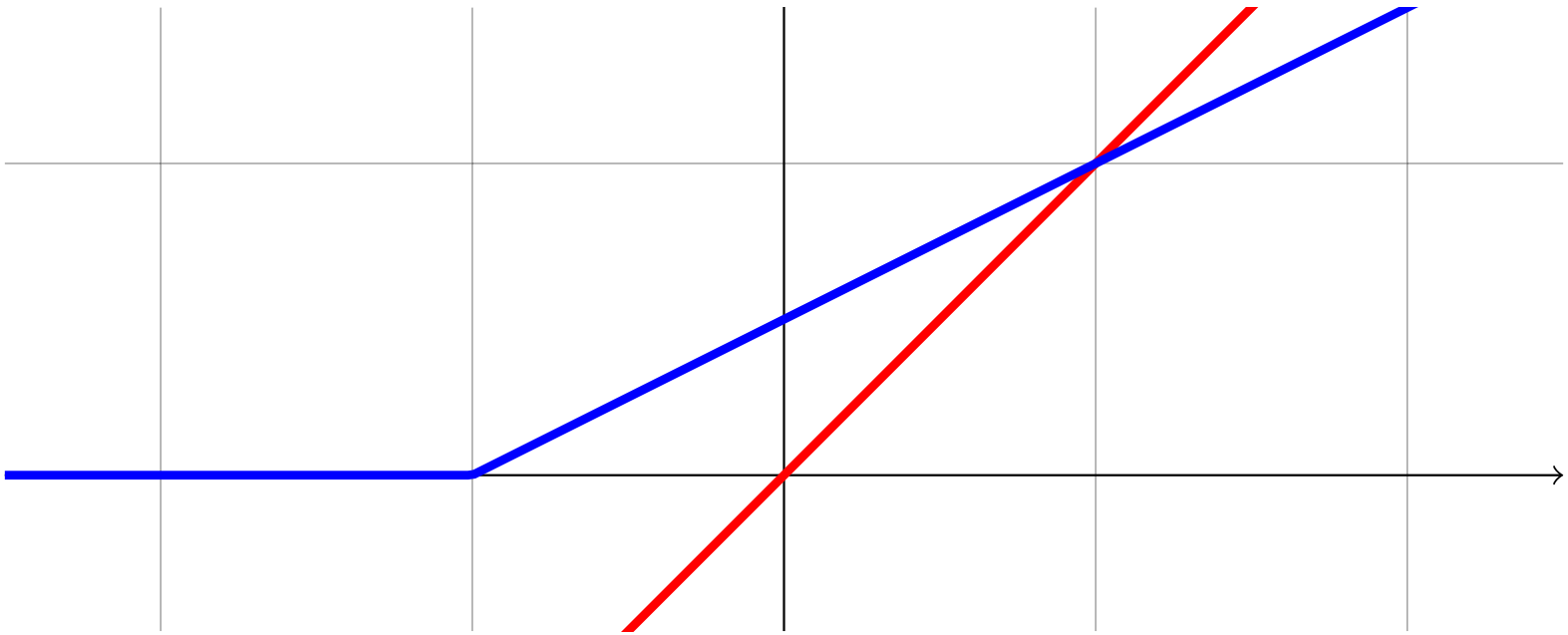
$$\mathbf{x} = \mathbf{max}\ \{\, 0.4,\ \sqrt{\mathbf{x}},\ 1 + \sqrt{\mathbf{x} - 1}\,\}$$



Least Solution

Kleene + Widening/Narrowing?

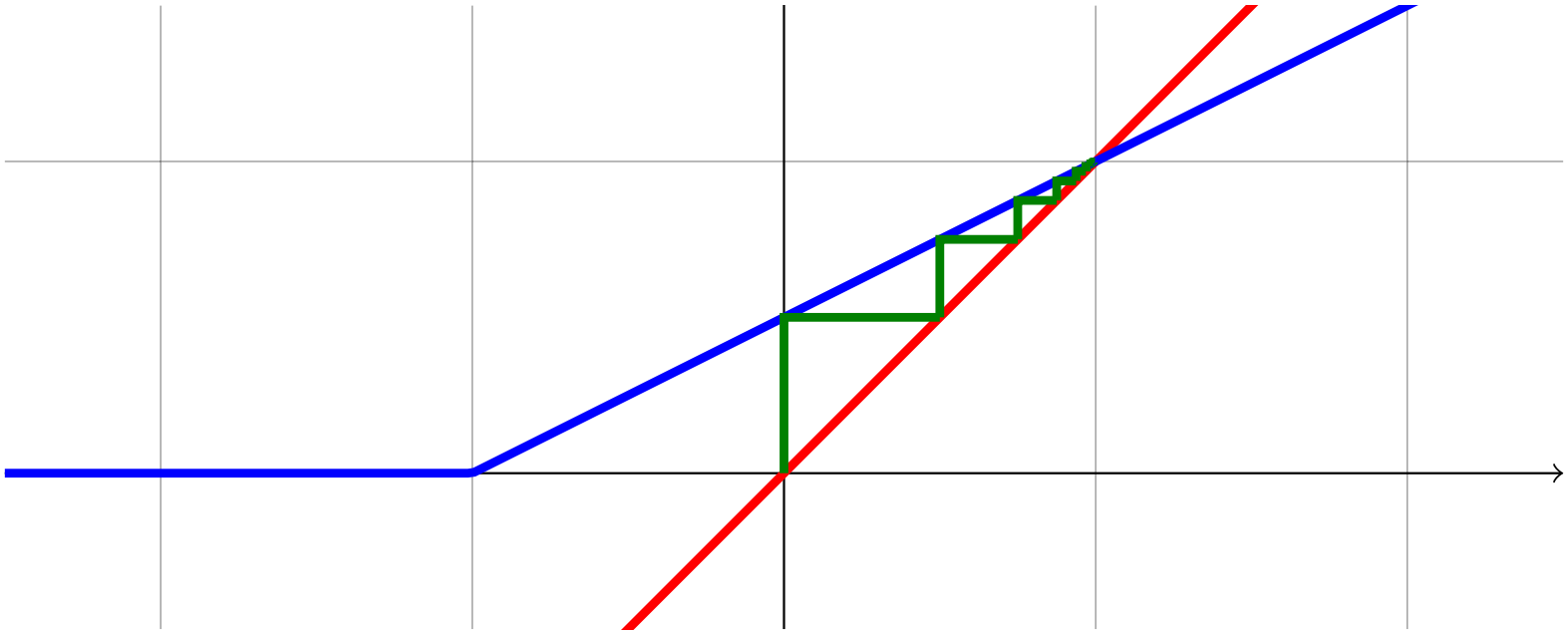
$$\mathbf{x} = \mathbf{max}\ \left\{ 0\,,\ \ \frac{1}{2}\mathbf{x} + \frac{1}{2} \right\}$$



Kleene + Widening/Narrowing?

$$\mathbf{x} = \mathbf{max}\ \left\{ 0\,,\ \ \frac{1}{2}\mathbf{x} + \frac{1}{2} \right\}$$

Kleene (without widening) gives us:



Outline

Baguette

Minette

Assalé Adjé
Alexandru Costan
Stéphane Gaubert
Éric Goubault
Matthieu Martel
Sylvie Putot
Ankur Taly
Sarah Zennou

Recall:

$$\begin{aligned}
\mathbf{x}_1 &= \mathbf{max}\ \{ f_{1,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \ldots, f_{1,k_1}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \\
&\vdots \\
\mathbf{x}_n &= \mathbf{max}\ \{ f_{n,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \ldots, f_{n,k_n}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \}
\end{aligned}$$

Recall:

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{max}\ \{ f_{1,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \ldots, f_{1,k_1}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \\ &\vdots \\ \mathbf{x}_n &= \mathbf{max}\ \{ f_{n,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \ldots, f_{n,k_n}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \end{aligned}$$

Minette's View

Recall:

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{max}\ \{ f_{1,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \ldots, f_{1,k_1}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \\ &\vdots \\ \mathbf{x}_n &= \mathbf{max}\ \{ f_{n,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \ldots, f_{n,k_n}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \end{aligned}$$

Minette's View

- **Minimal Solution** of $\mathbf{x} = F(\mathbf{x})$ $\quad \forall \mathbf{x} \in \overline{\mathbb{R}}^n$

Recall:

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{max}\ \{ f_{1,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \ldots, f_{1,k_1}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \\ &\vdots \\ \mathbf{x}_n &= \mathbf{max}\ \{ f_{n,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \ldots, f_{n,k_n}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \end{aligned}$$

Minette's View

- **Minimal Solution** of $\mathbf{x} = F(\mathbf{x})$ $\quad \forall \mathbf{x} \in \overline{\mathbb{R}}^n$
- $F(\mathbf{x}) = \mathbf{min}\ \{\pi(x) \mid \pi \in \Pi\}$ $\quad \forall \mathbf{x} \in \overline{\mathbb{R}}^n$

Set of **simpler** monotonic operators

Recall:

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{max}\ \{ f_{1,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \ldots, f_{1,k_1}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \\ &\vdots \\ \mathbf{x}_n &= \mathbf{max}\ \{ f_{n,1}(\mathbf{x}_1, \ldots, \mathbf{x}_n), \ldots, f_{n,k_n}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \} \end{aligned}$$

Minette's View

- **Minimal Solution** of $\mathbf{x} = F(\mathbf{x})$ $\quad \forall \mathbf{x} \in \overline{\mathbb{R}}^n$
- $F(\mathbf{x}) = \mathbf{min}\ \{\pi(x) \mid \pi \in \Pi\}$ $\quad \forall \mathbf{x} \in \overline{\mathbb{R}}^n$

  ↑

  Set of **simpler** monotonic operators
- **Consequence:** $\mu F = \mathbf{min}\ \{\mu\pi \mid \pi \in \Pi\}$

π's simpler than F?

π's simpler than F?

F

π's simpler than F?

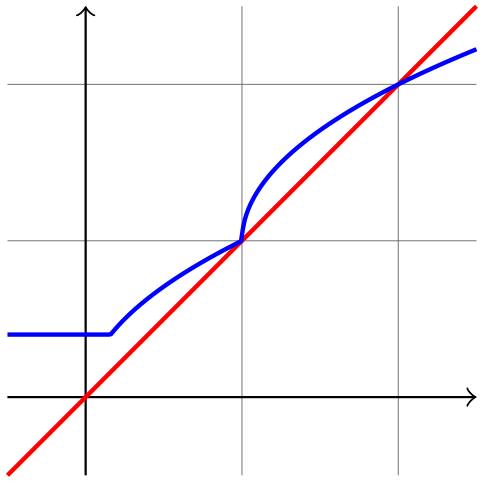
F

point-wise maximum
of
monotone and **concave**
operators

π's simpler than F?

F

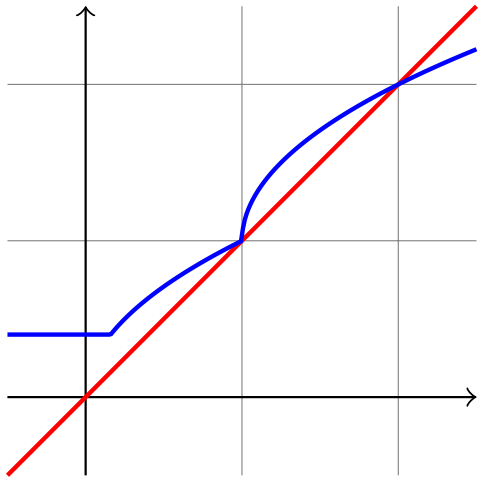
point-wise maximum
of
monotone and **concave**
operators



π's simpler than F?

F

point-wise maximum
of
monotone and **concave**
operators

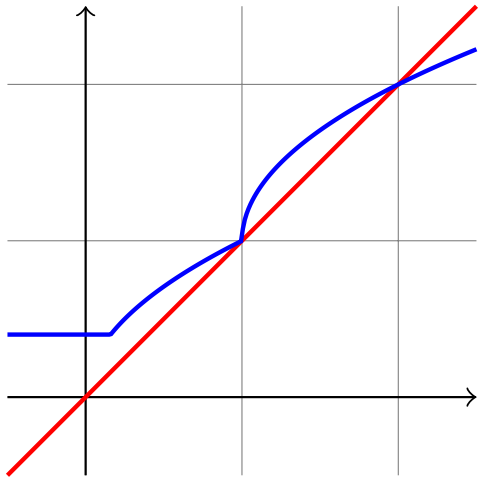


π

π's simpler than F?

F

point-wise maximum
of
monotone and **concave**
operators



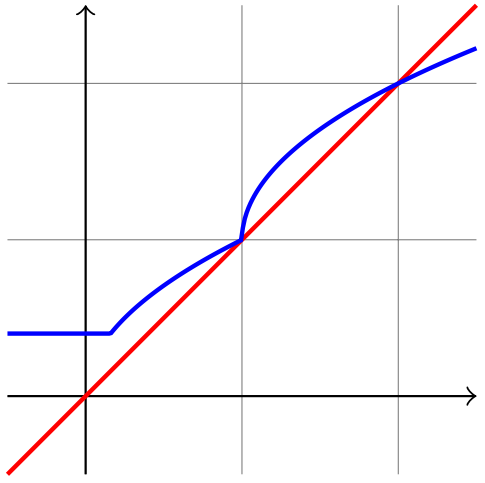
π

point-wise maximum
of
monotone and **concave**
operators

π's simpler than F?

F

point-wise maximum
of
monotone and **concave**
operators



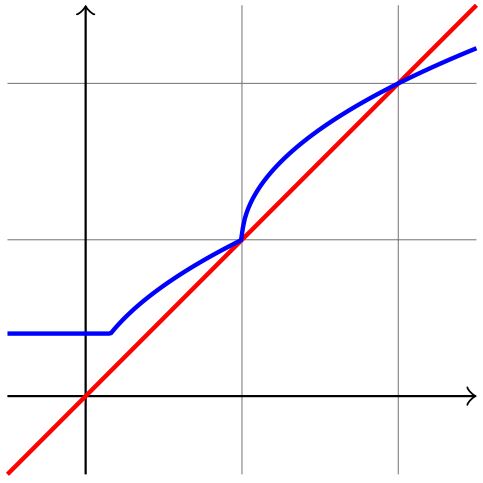
π

point-wise maximum
of
monotone and ~~**concave**~~
operators

π's simpler than F?

F

point-wise maximum
of
monotone and **concave**
operators



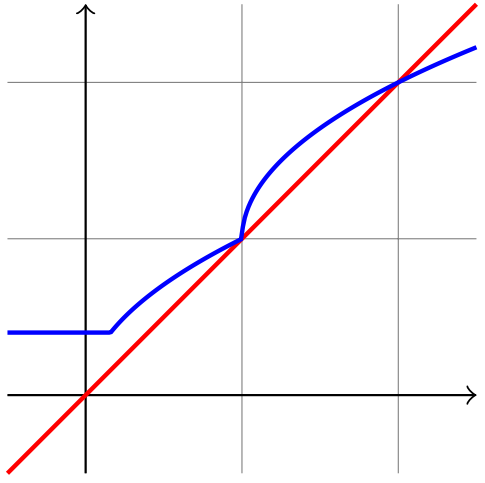
π

point-wise maximum
of
monotone and ~~concave~~ **affine**
operators

π's simpler than F?

F

point-wise maximum
of
monotone and **concave**
operators



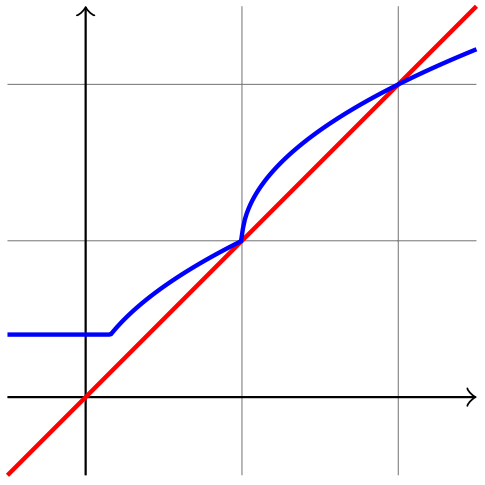
π

point-wise maximum
of
monotone and ~~concave~~ **affine**
operators

π's simpler than F?

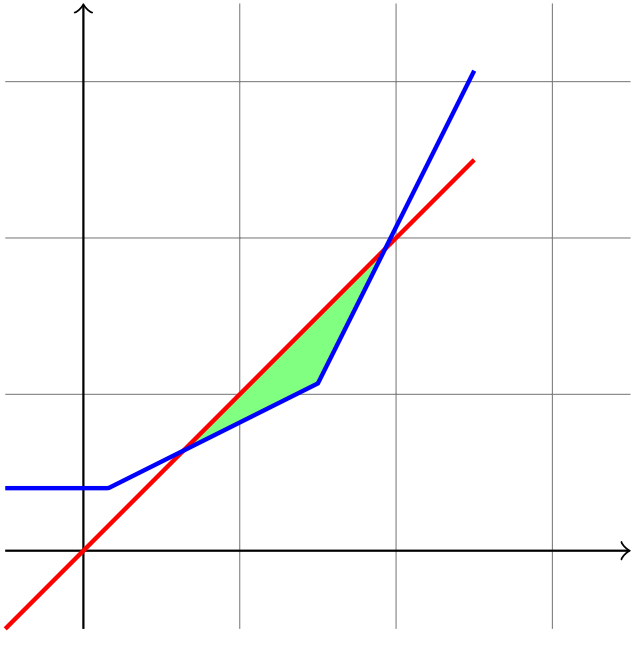
F

point-wise maximum
of
monotone and **concave**
operators



π

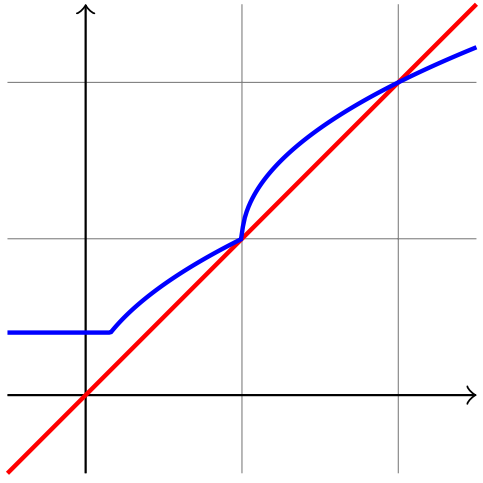
point-wise maximum
of
monotone and ~~concave~~ **affine**
operators



π's simpler than F?

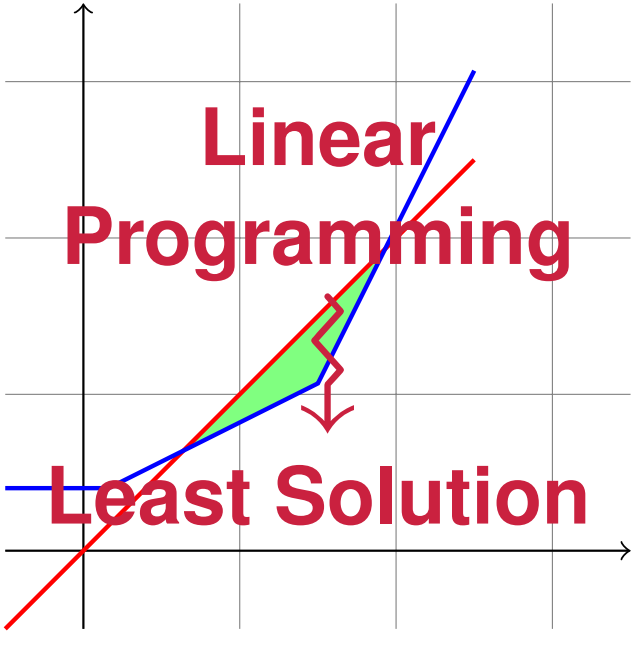
F

point-wise maximum
of
monotone and **concave**
operators

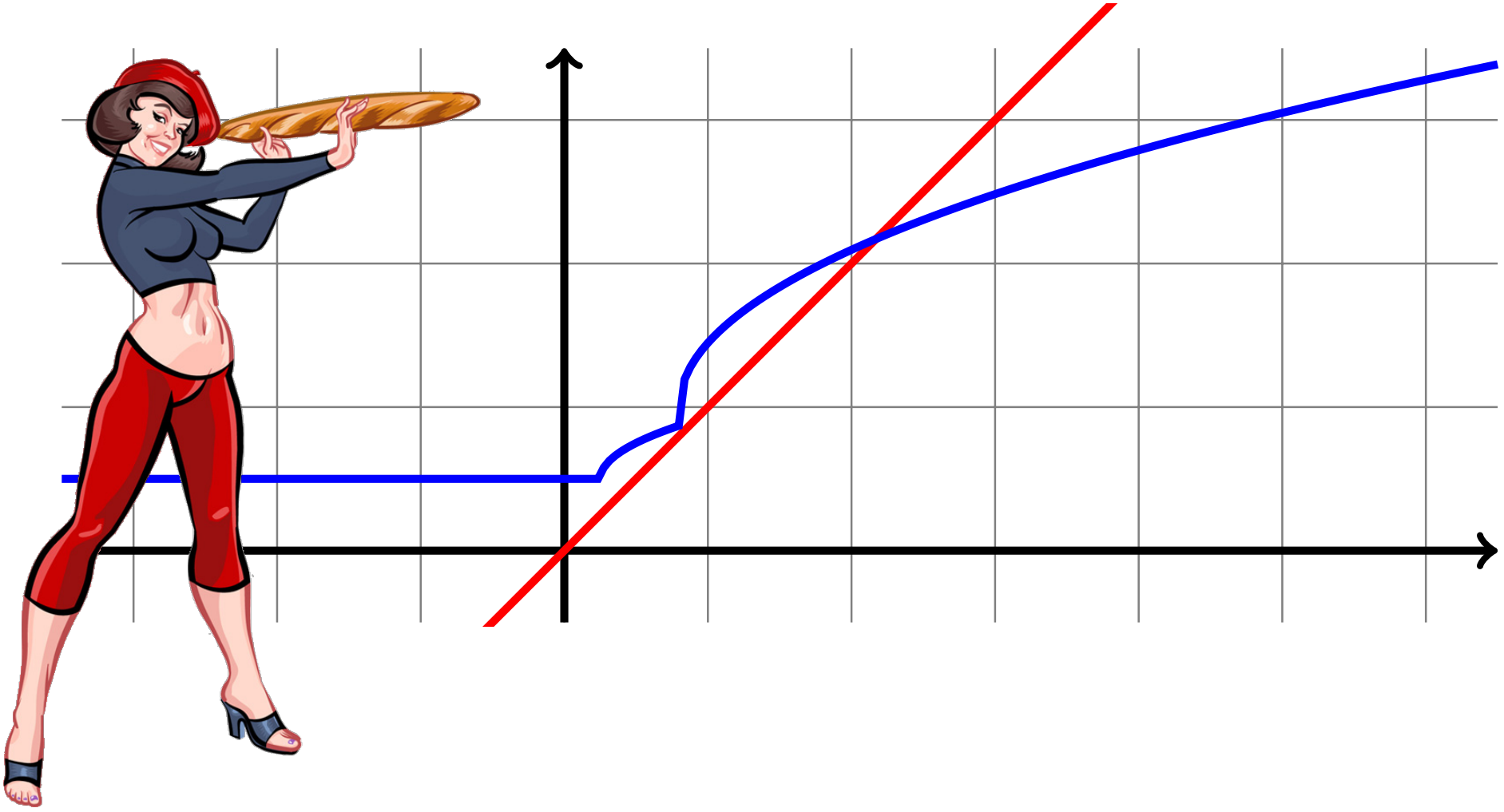


π

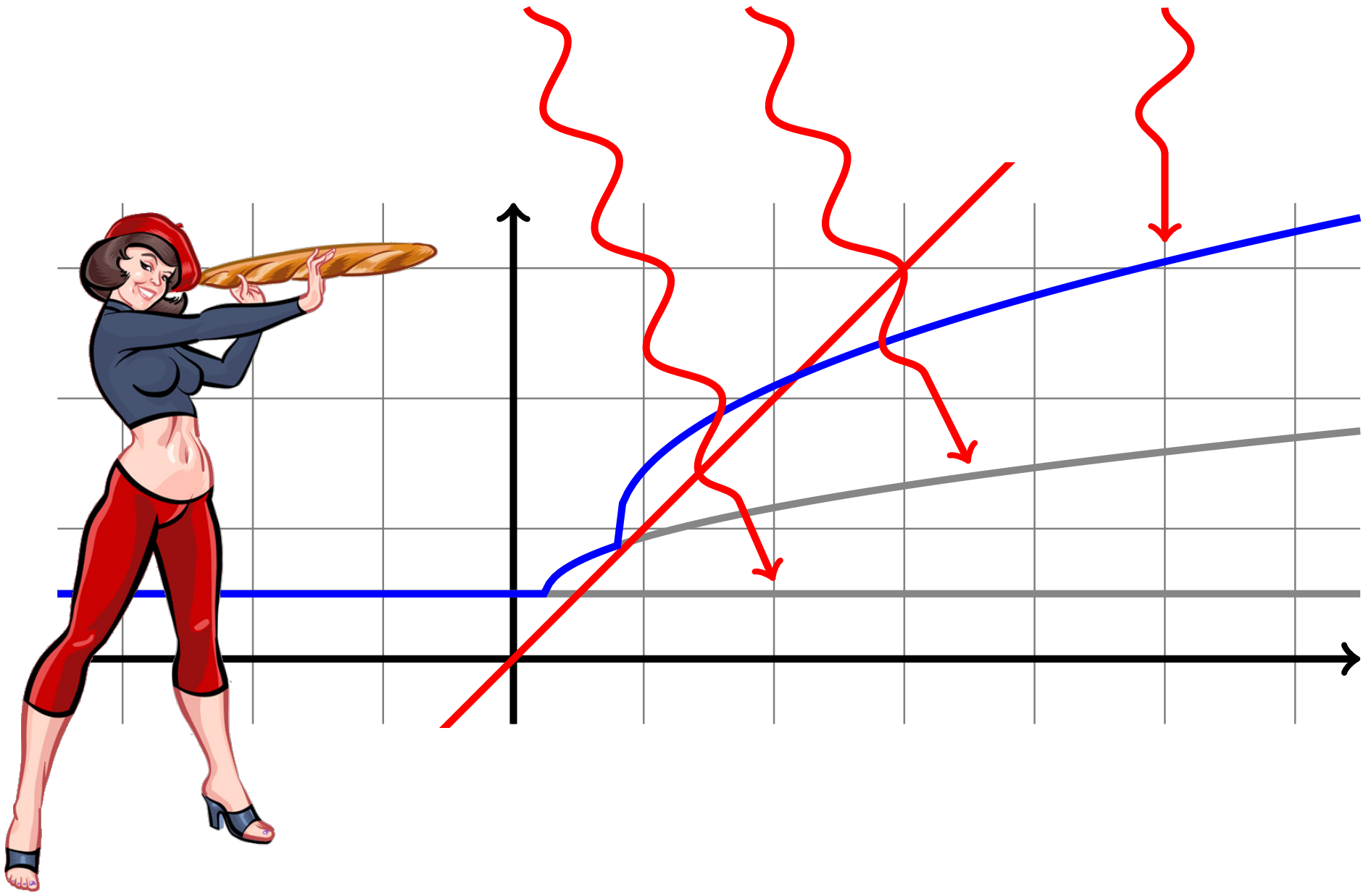
point-wise maximum
of
monotone and ~~concave~~ **affine**
operators



$$\mathbf{x} = \mathbf{max}\ \{\ \tfrac{1}{2},\ \tfrac{1}{2} \cdot \sqrt{\mathbf{x} - \tfrac{1}{4}} + \tfrac{1}{2},\ 1 + \sqrt{\mathbf{x} - 0.8}\ \}$$

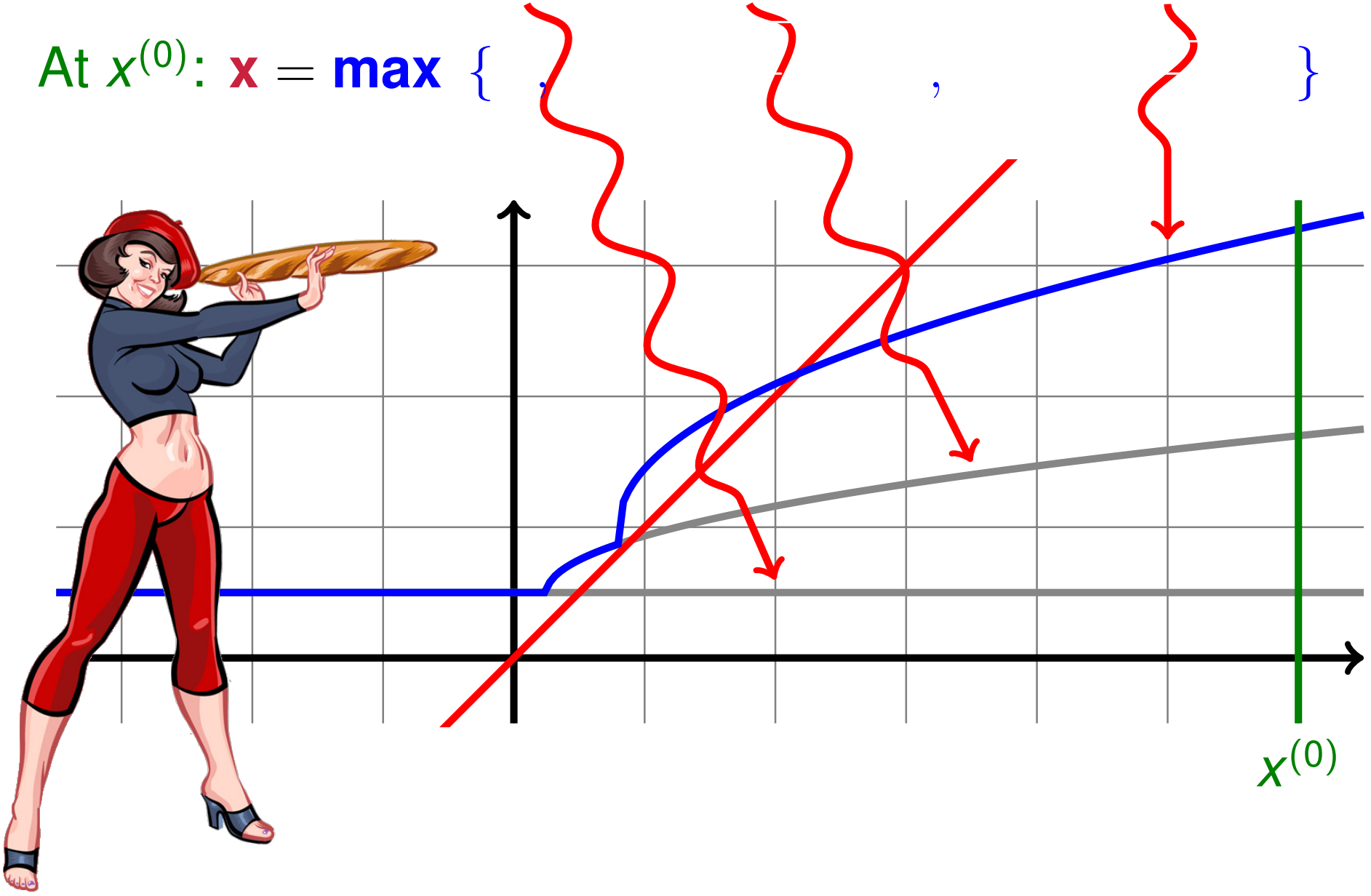


$$\mathbf{x} = \mathbf{max}\ \{\ \tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\ \}$$



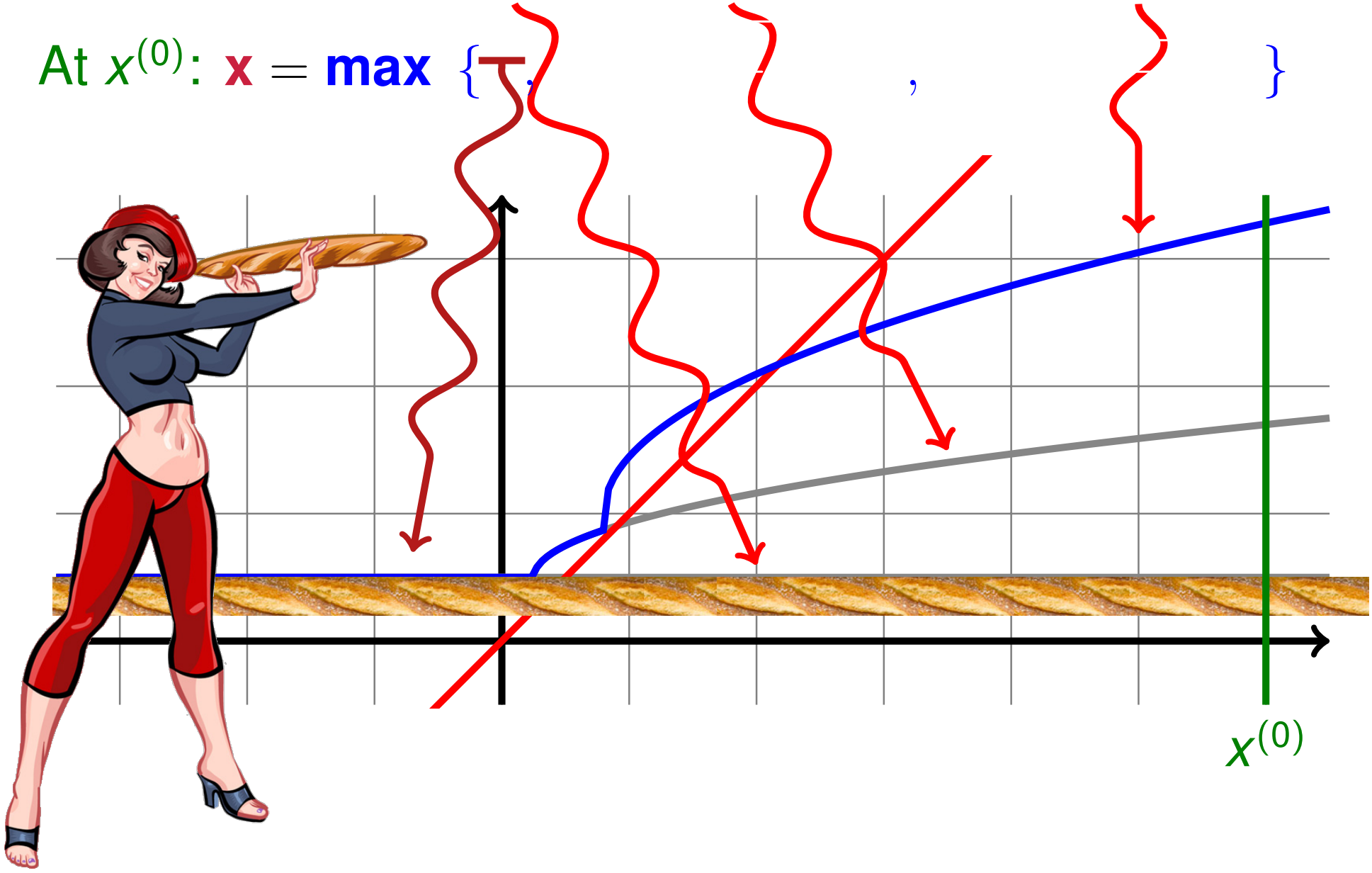
$$\mathbf{x} = \mathbf{max}\ \{\ \tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\ \}$$

At $x^{(0)}$: $\mathbf{x} = \mathbf{max}\ \{\ ,\ ,\ \}$



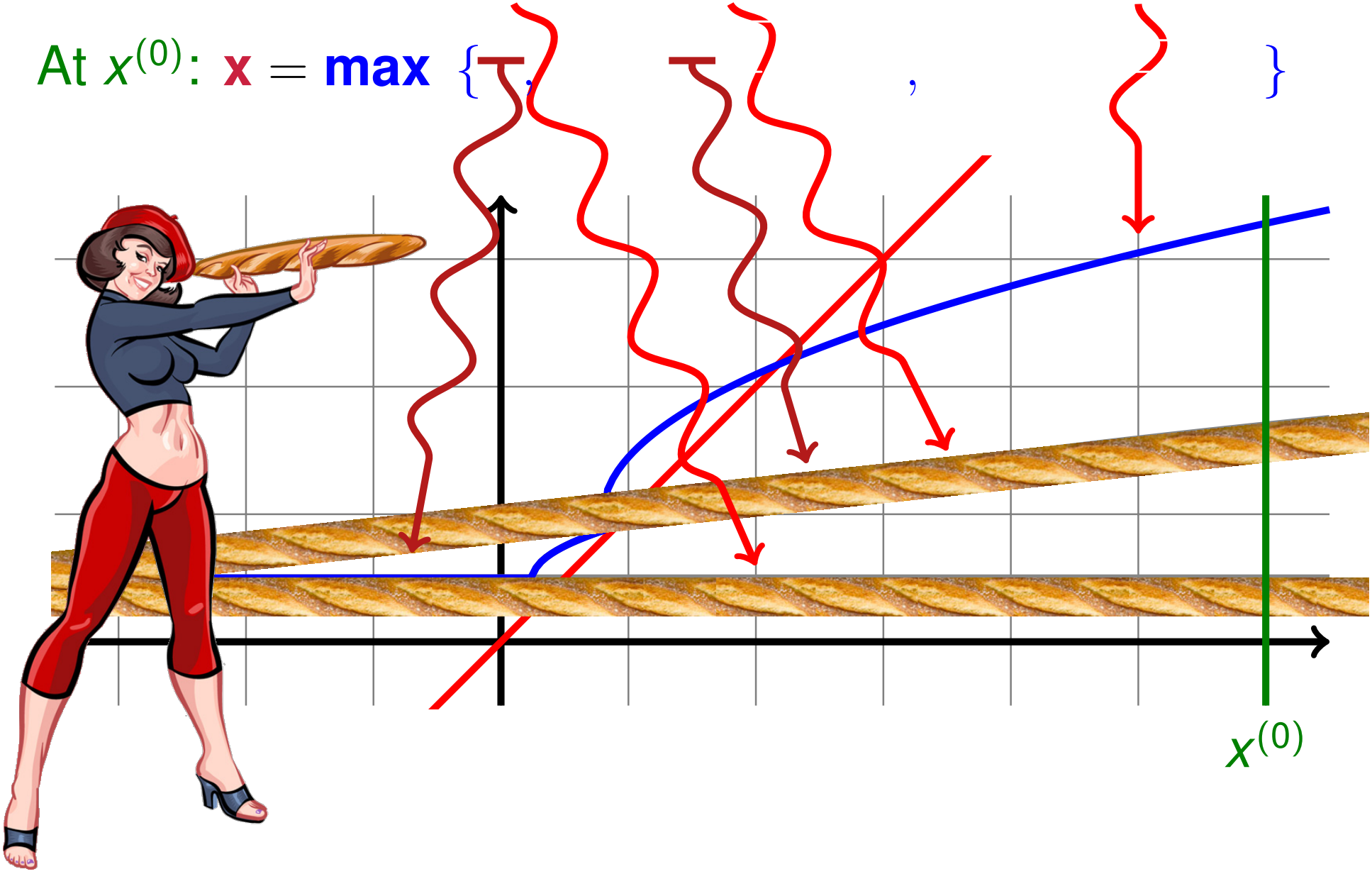
$$\mathbf{x} = \mathbf{max}\ \{\ \tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\ \}$$

At $x^{(0)}$: $\mathbf{x} = \mathbf{max}\ \{\ \ ,\ \ ,\ \ \}$



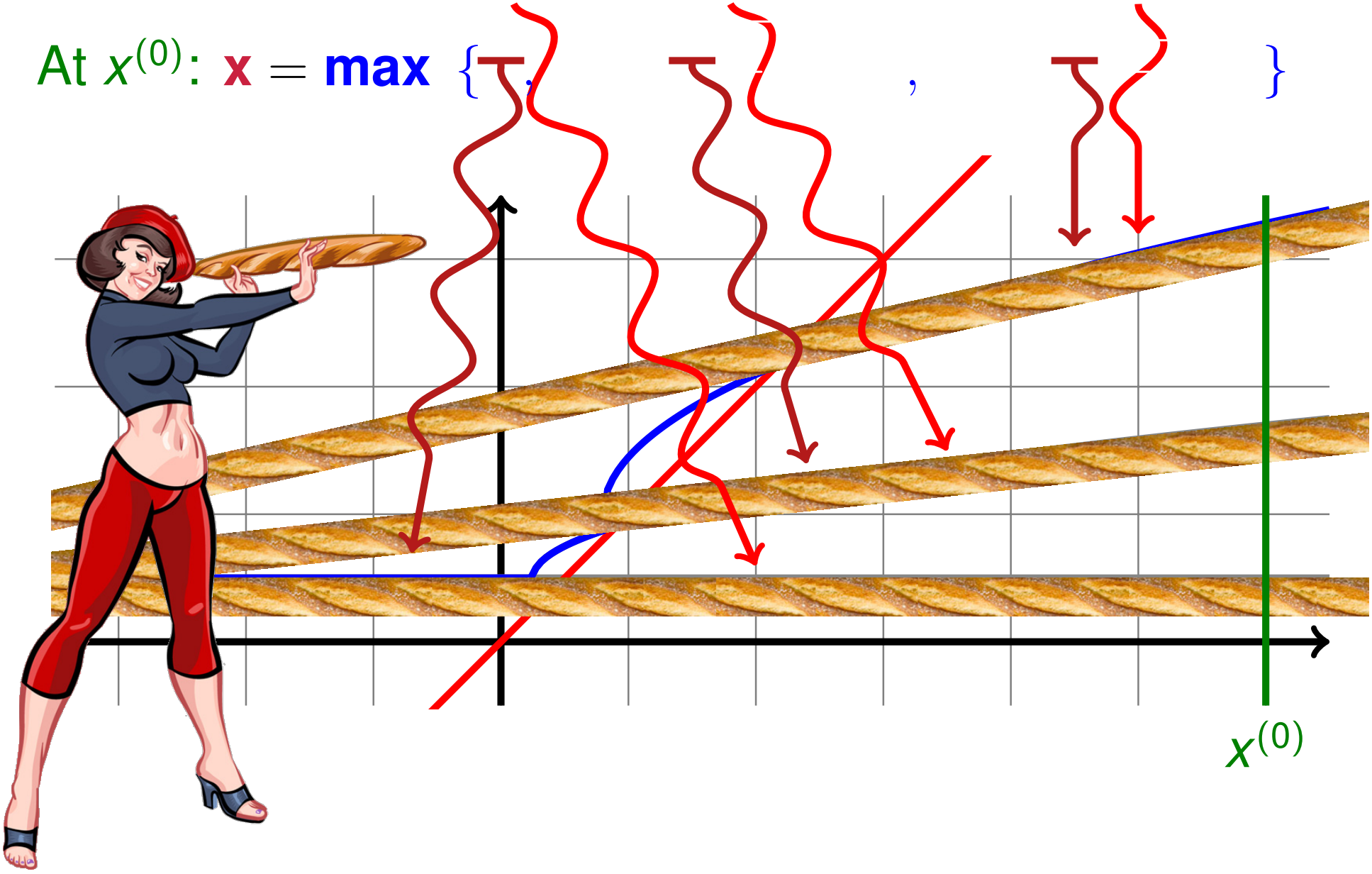
$$\mathbf{x} = \mathbf{max}\ \{\ \tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\ \}$$

At $x^{(0)}$: $\mathbf{x} = \mathbf{max}\ \{\ \ ,\ \ ,\ \ \}$



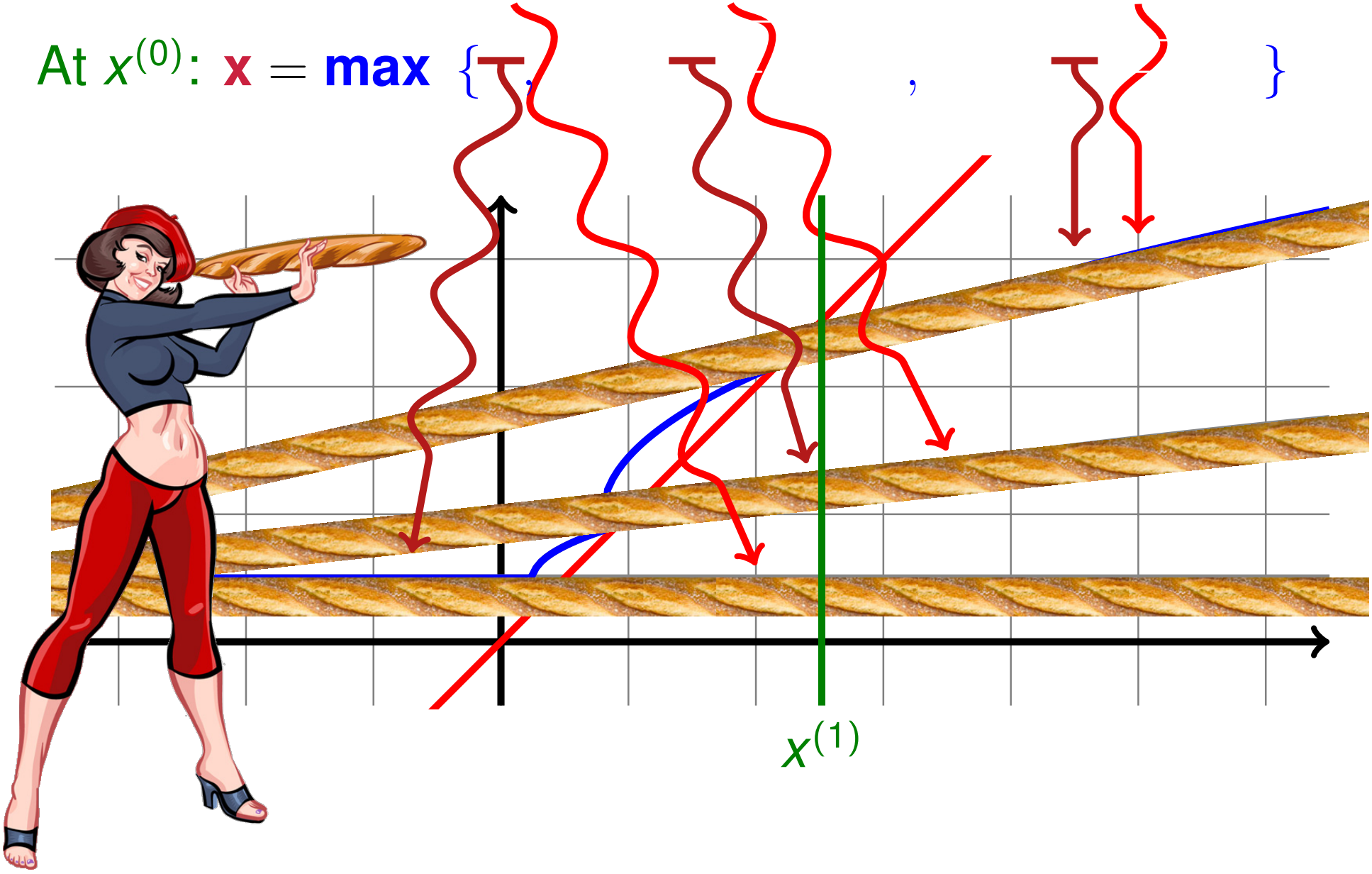
$$\mathbf{x} = \mathbf{max}\ \{\ \tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\ \}$$

At $x^{(0)}$: $\mathbf{x} = \mathbf{max}\ \{\ \top,\ \top,\ \top\ \}$



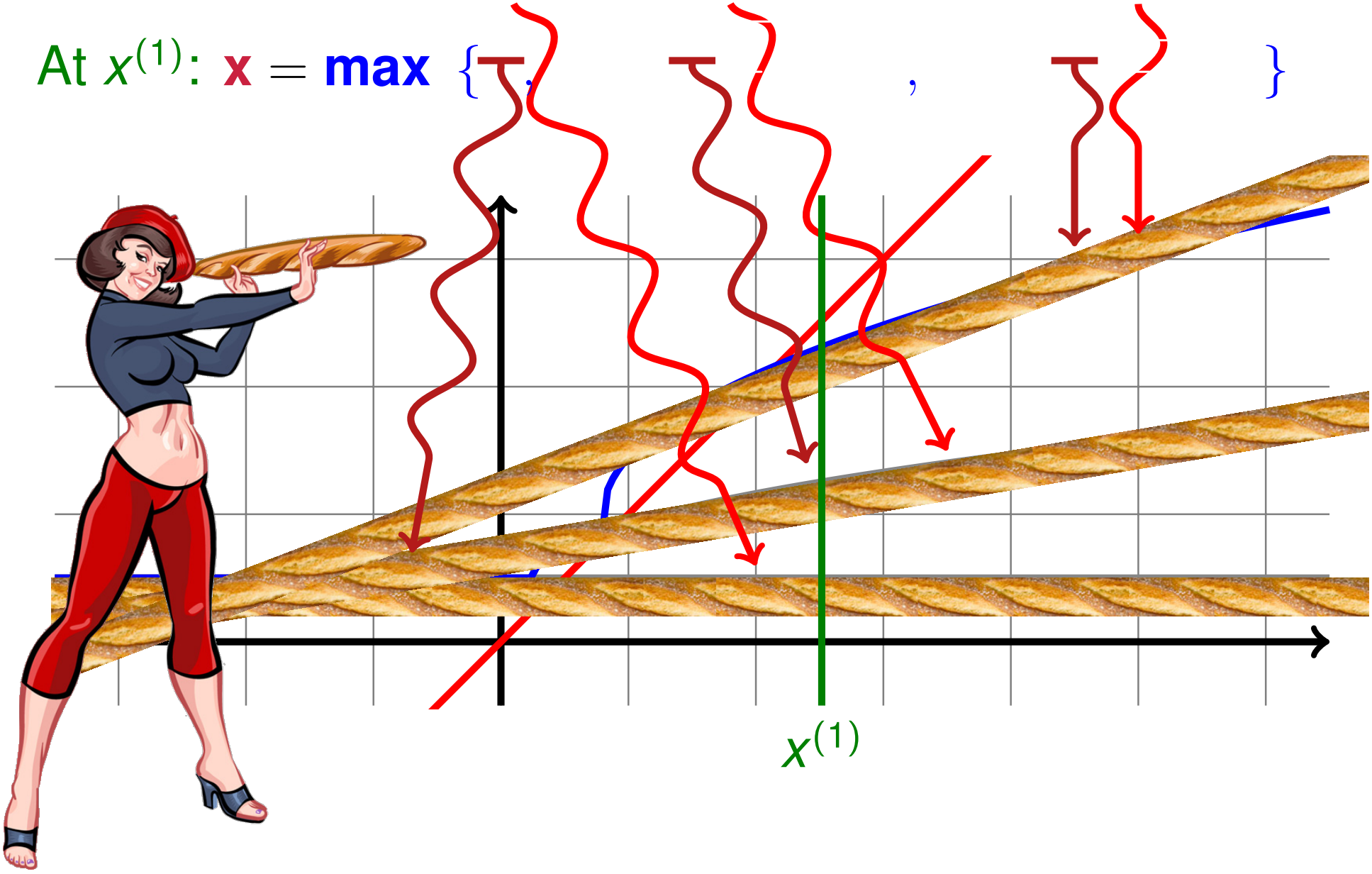
$$\mathbf{x} = \mathbf{max}\ \{\ \tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\ \}$$

At $x^{(0)}$: $\mathbf{x} = \mathbf{max}\ \{\ \top,\ \top,\ \top\ \}$



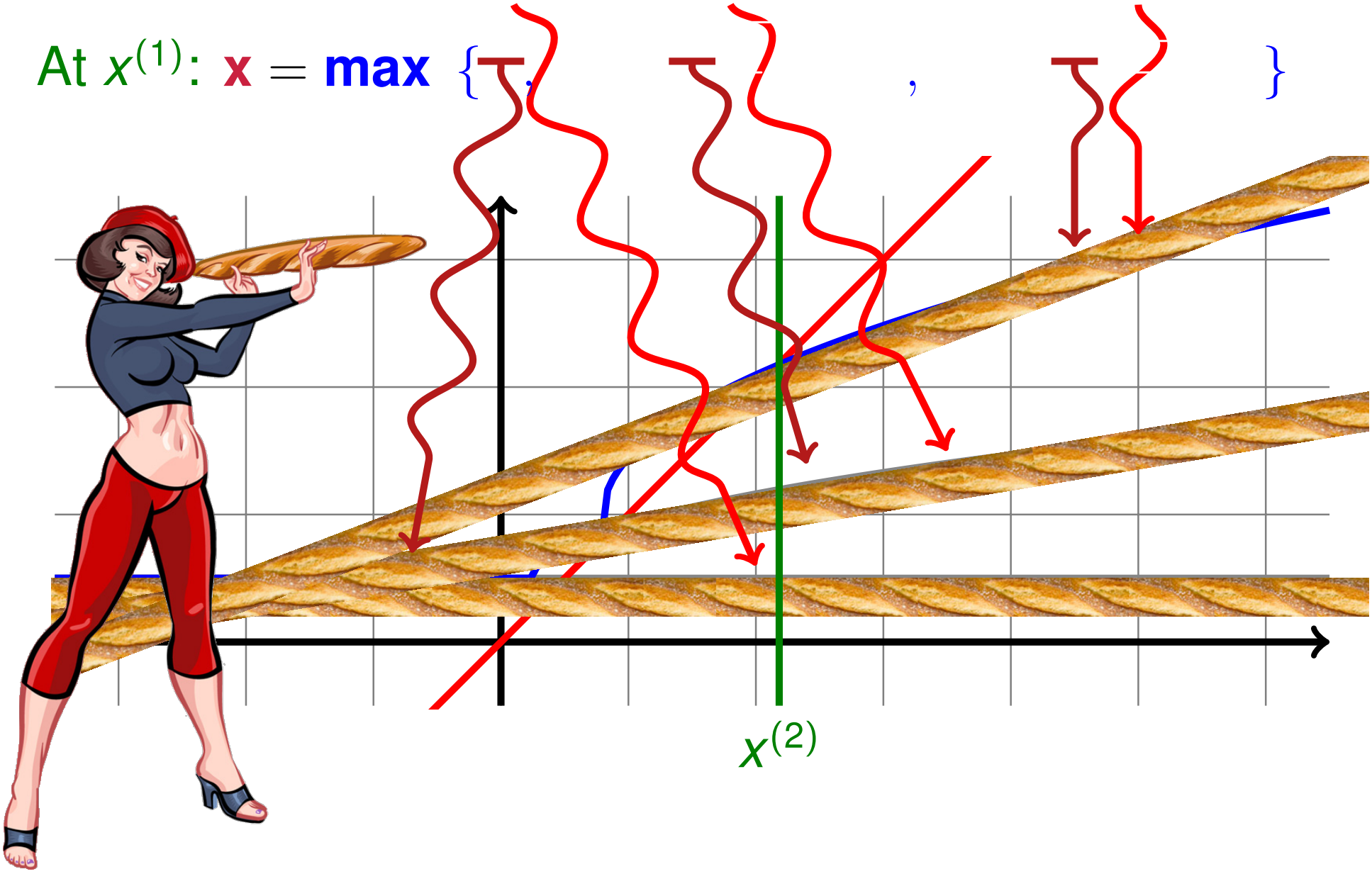
$$\mathbf{x} = \mathbf{max}\ \{\ \tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\ \}$$

At $x^{(1)}$: $\mathbf{x} = \mathbf{max}\ \{\ \ ,\ \ ,\ \ \}$



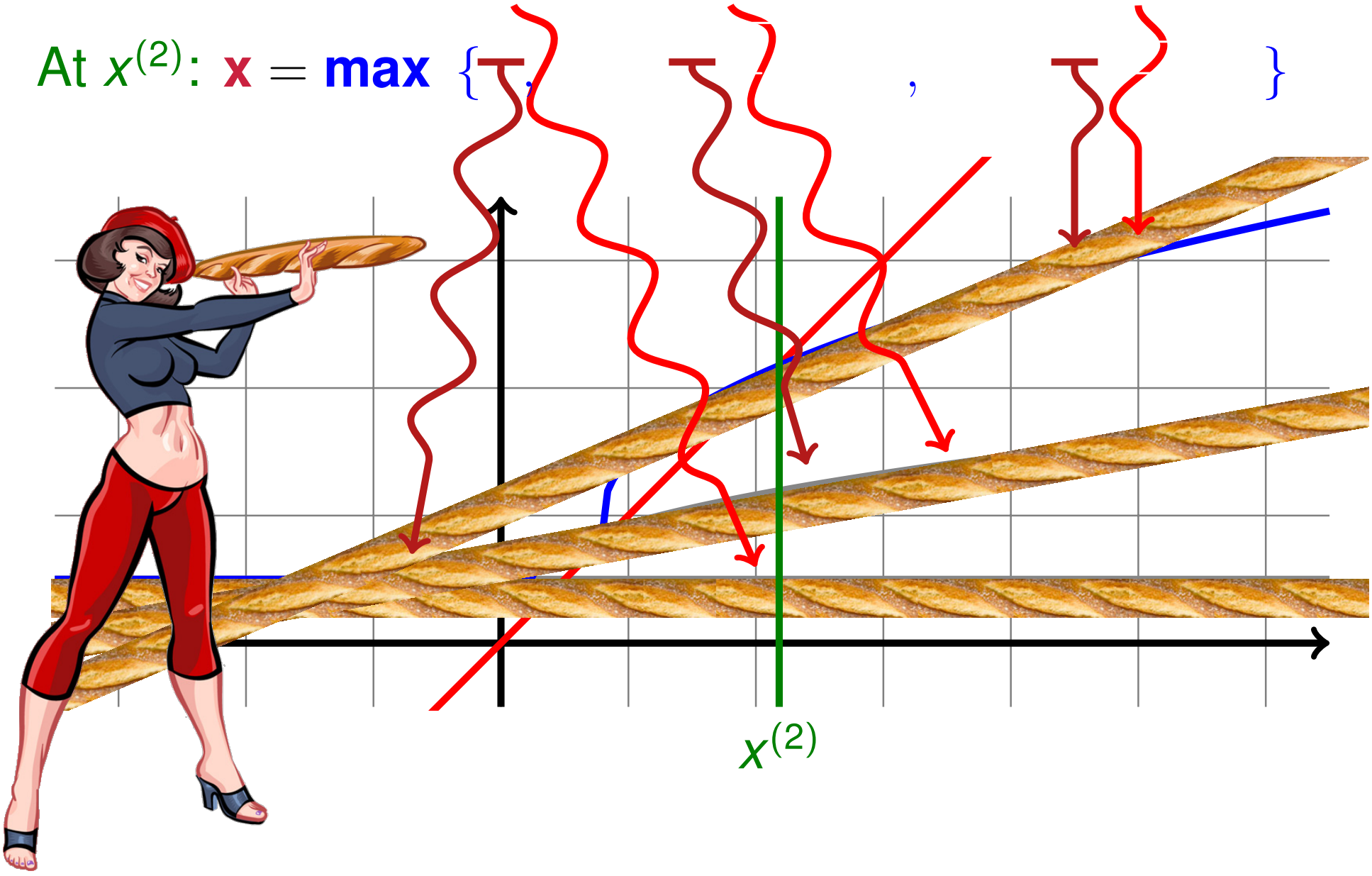
$$\mathbf{x} = \mathbf{max}\ \{\ \tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\ \}$$

At $x^{(1)}$: $\mathbf{x} = \mathbf{max}\ \{\ \top,\ \top,\ \top\ \}$



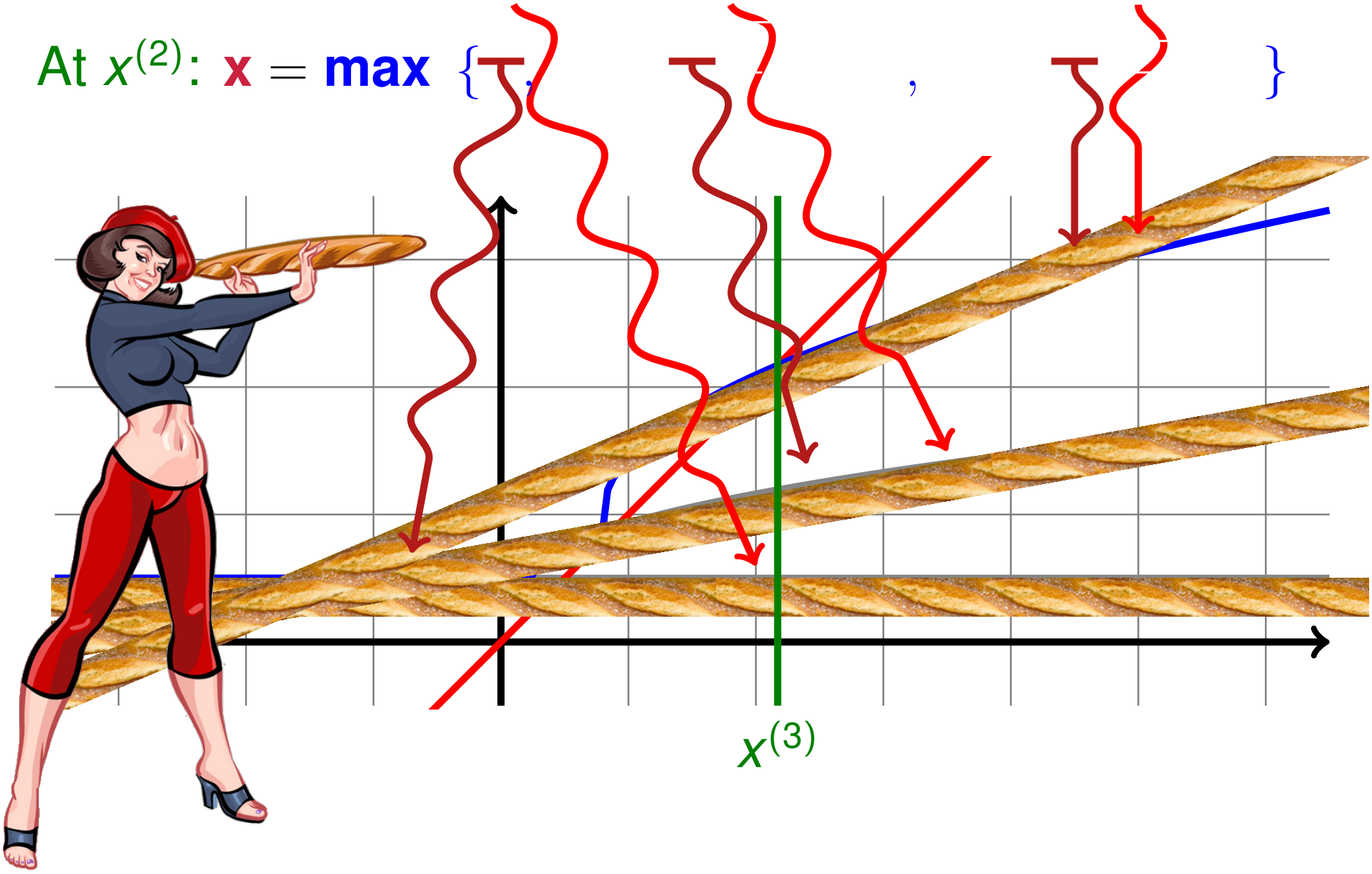
$$\mathbf{x} = \mathbf{max}\ \{\ \tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\ \}$$

At $x^{(2)}$: $\mathbf{x} = \mathbf{max}\ \{\ \ ,\ \ ,\ \ \}$



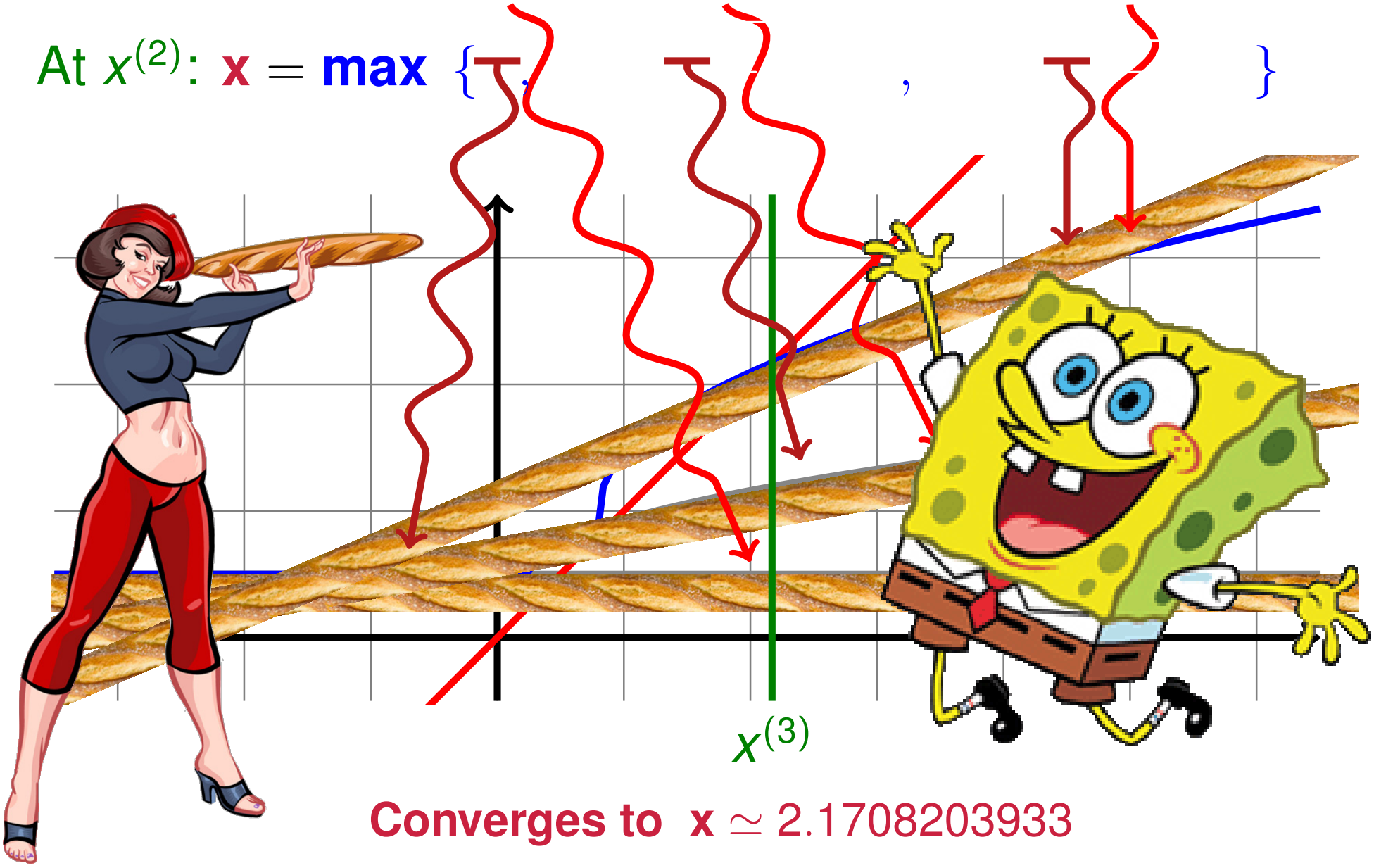
$$\mathbf{x} = \mathbf{max}\ \{\ \tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\ \}$$

At $x^{(2)}$: $\mathbf{x} = \mathbf{max}\ \{\ \ ,\ \ ,\ \ \}$



$$\mathbf{x} = \mathbf{max}\left\{\tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\right\}$$

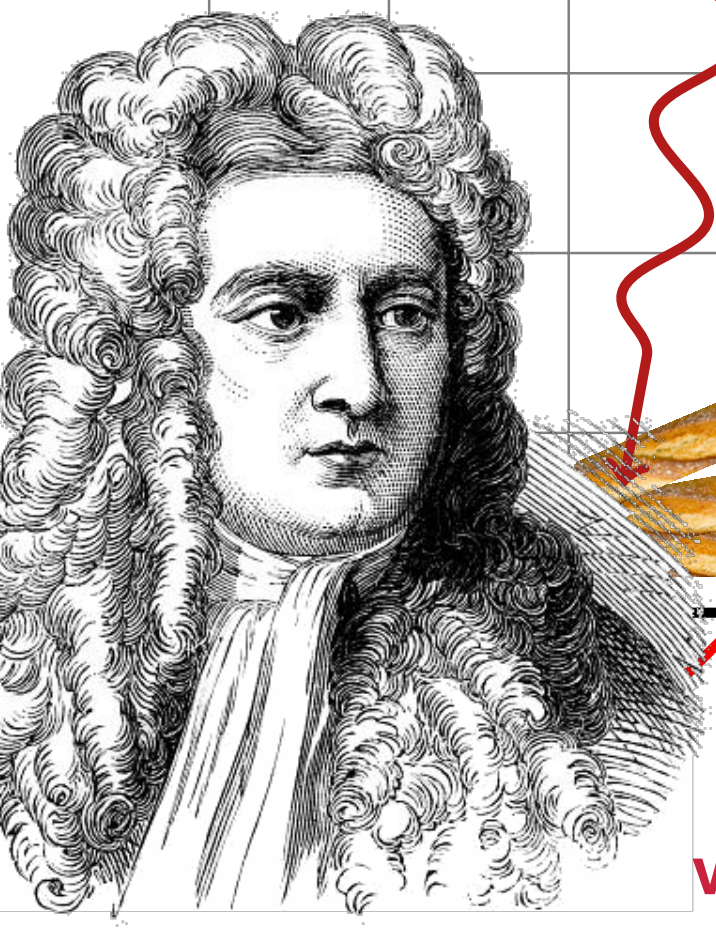
At $x^{(2)}$: $\mathbf{x} = \mathbf{max}\ \{\ ,\ ,\ \}$

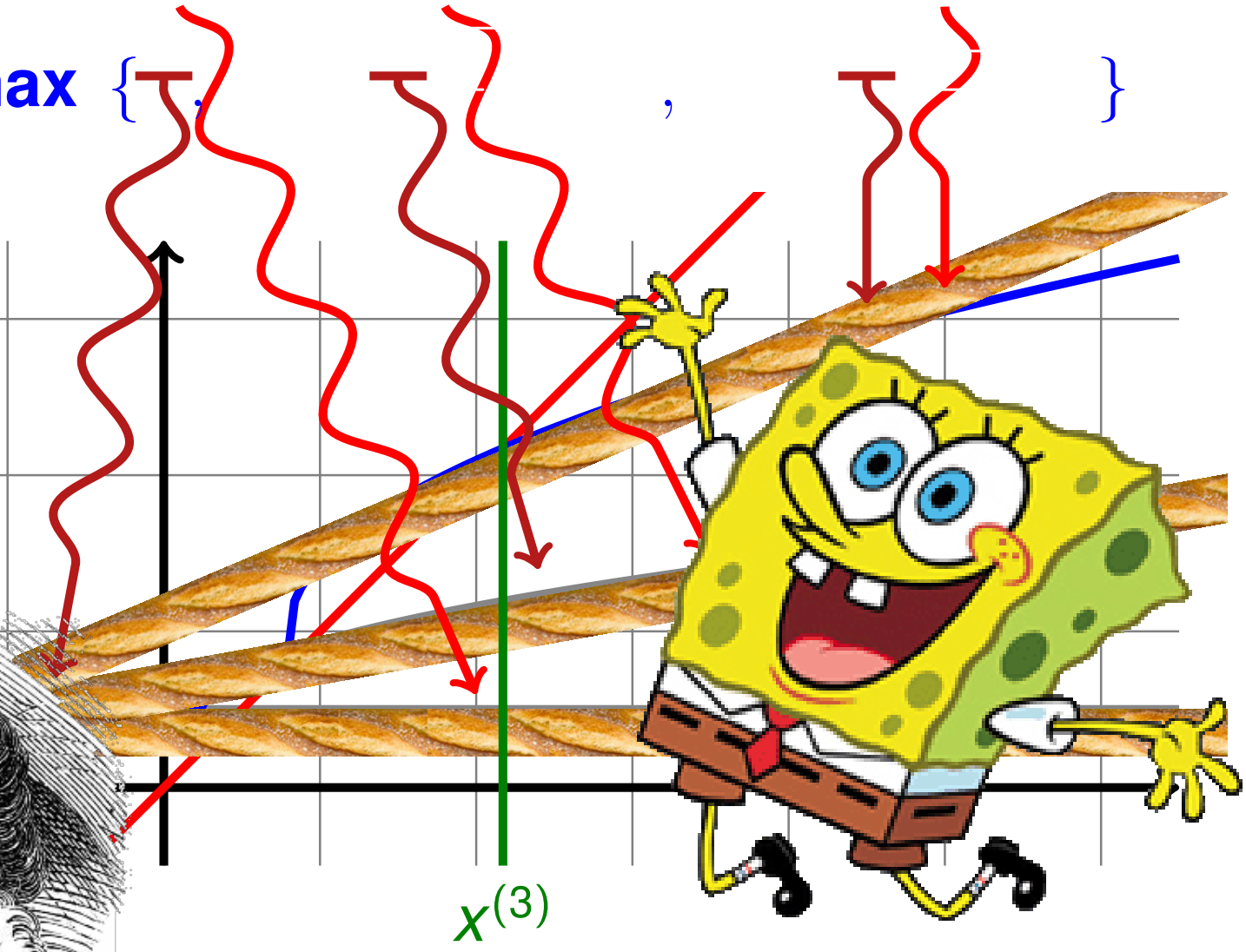


Converges to $\mathbf{x} \simeq 2.1708203933$

$$\mathbf{x} = \mathbf{max}\left\{\tfrac{1}{2},\ \tfrac{1}{2}\cdot\sqrt{\mathbf{x}-\tfrac{1}{4}}+\tfrac{1}{2},\ 1+\sqrt{\mathbf{x}-0.8}\right\}$$

At $x^{(2)}$: $\mathbf{x} = \mathbf{max}\ \{\ ,\ ,\ \}$

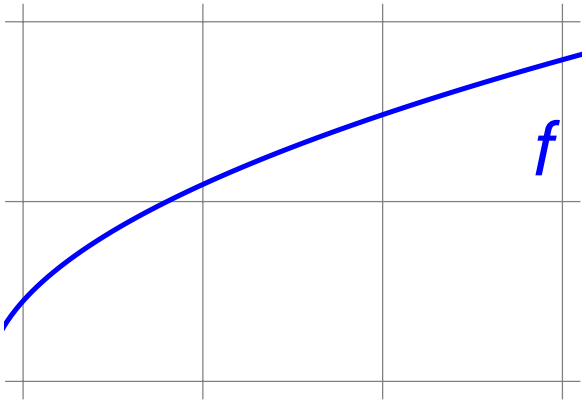




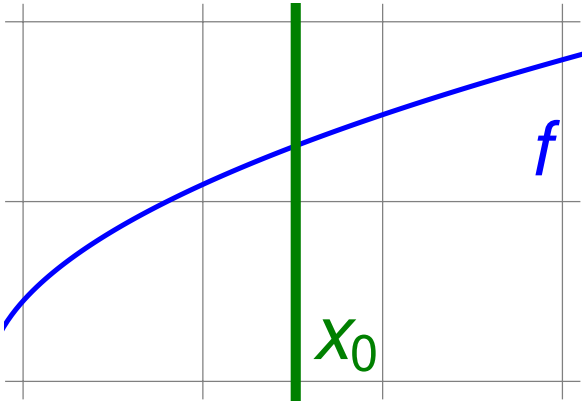
verges to $\mathbf{x} \simeq 2.1708203933$

How can  compute the  s?

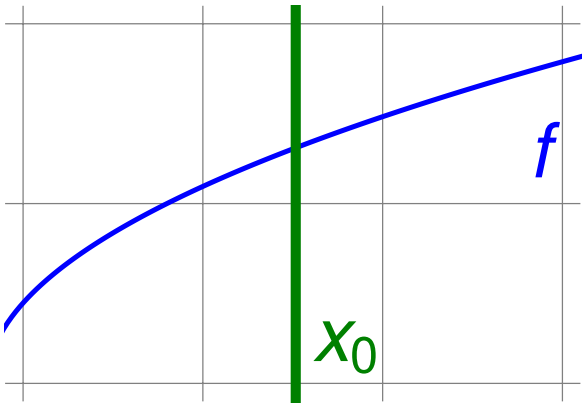
How can  compute the  s?



How can  compute the  s?



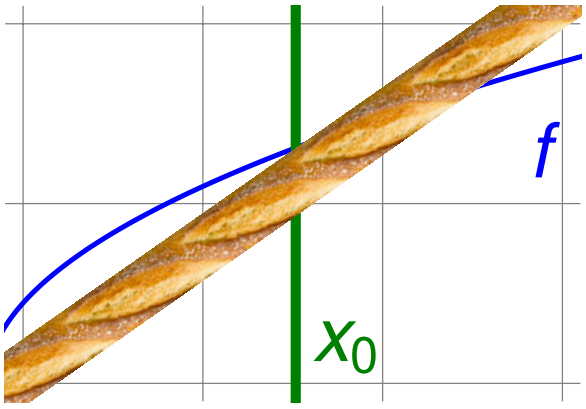
How can  compute the  s?



Requirement 1:

 $(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$

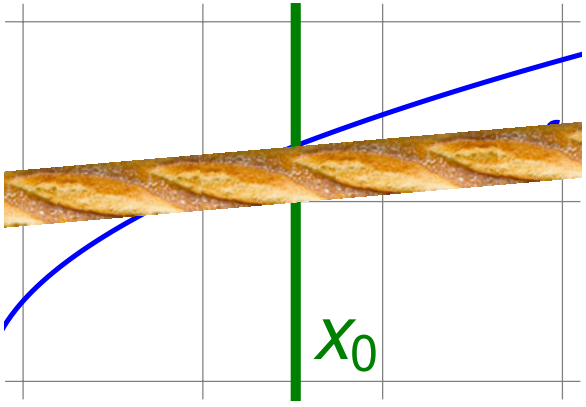
How can  compute the  s?



Requirement 1:

 $(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$

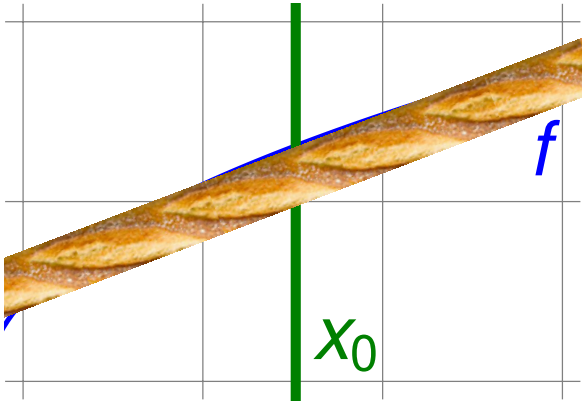
How can  compute the  s?



Requirement 1:

 $(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$

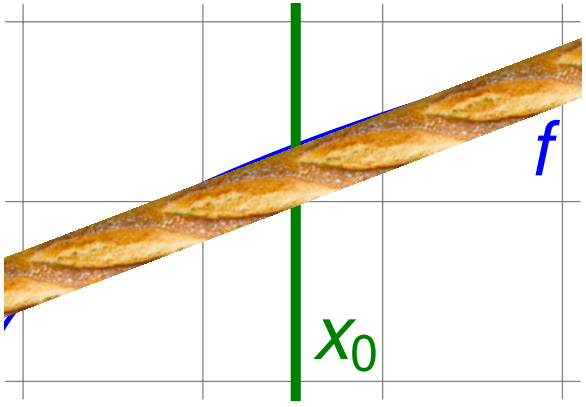
How can  compute the  s?



Requirement 1:

 $(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$

How can  compute the  s?



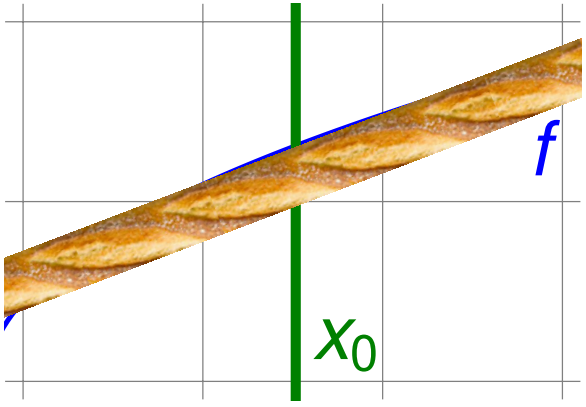
Requirement 1:

$$\boxed{}(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$$

Requirement 2:

$$f(x) \leq \boxed{}(x) \qquad \forall x$$

How can  compute the  s?



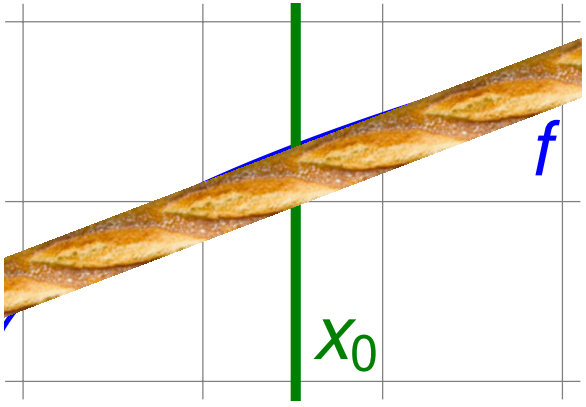
Requirement 1:

 $(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$

Requirement 2:

$f(x) \leq$  $(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$

How can  compute the  s?



Requirement 1:

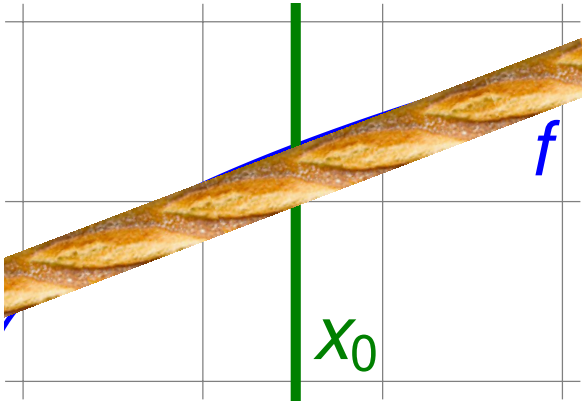
 $(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$

Requirement 2:

$$f(x) \leq \text{  }(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$$

$$\iff \quad f(x) - \mathbf{d}^\top (x - x_0) \leq f(x_0) \quad \forall x$$

How can  compute the  s?



Requirement 1:

 $(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$

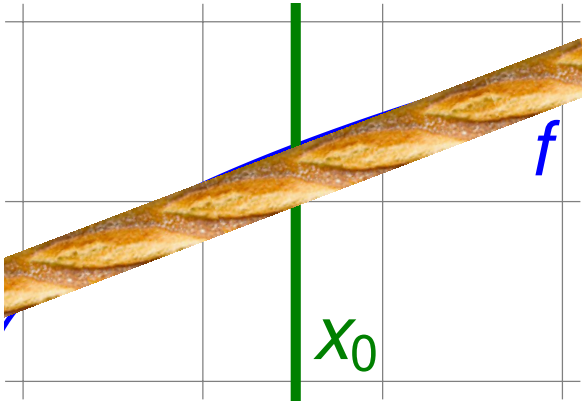
Requirement 2:

$$f(x) \leq \text{  }(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$$

$$\iff \quad f(x) - \mathbf{d}^\top (x - x_0) \leq f(x_0) \quad \forall x$$

$$\iff \quad \sup_x \; f(x) - \mathbf{d}^\top (x - x_0) \leq f(x_0)$$

How can  compute the  s?



Requirement 1:

 $(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$

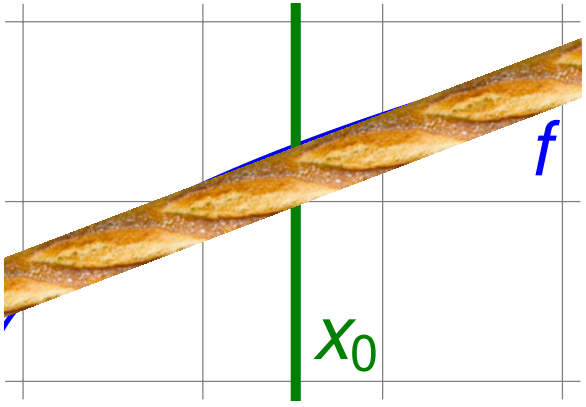
Requirement 2:

$$f(x) \leq \text{  }(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$$

$$\iff \quad f(x) - \mathbf{d}^\top (x - x_0) \leq f(x_0) \quad \forall x$$

$$\iff \quad \mathbf{g}(\mathbf{d}) := \sup_x \; f(x) - \mathbf{d}^\top (x - x_0) \leq f(x_0)$$

How can  compute the  s?



Requirement 1:

 $(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$

Requirement 2:

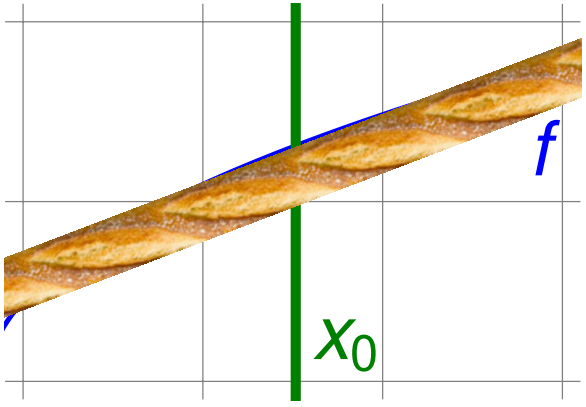
$$f(x) \leq \text{  }(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$$

$$\iff \quad f(x) - \mathbf{d}^\top (x - x_0) \leq f(x_0) \quad \forall x$$

$$\iff \quad \mathbf{g}(\mathbf{d}) := \sup_x \; f(x) - \mathbf{d}^\top (x - x_0) \leq f(x_0)$$

$\Rightarrow$ **g convex**

How can  compute the  s?



Requirement 1:

 $(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$

Requirement 2:

$$f(x) \leq \text{  }(x) = f(x_0) + \mathbf{d}^\top (x - x_0) \quad \forall x$$

$$\iff \quad f(x) - \mathbf{d}^\top (x - x_0) \leq f(x_0) \quad \forall x$$

$$\iff \quad \mathbf{g}(\mathbf{d}) := \sup_x \; f(x) - \mathbf{d}^\top (x - x_0) \leq f(x_0)$$

$\Rightarrow$ **g convex** $\Rightarrow$ $\inf_{\mathbf{d}} \mathbf{g}(\mathbf{d})$ is a **convex optimization problem**

Are their Special Cases?

Are their Special Cases?

Linear Programming (As in our Application):

$$f(d) = \sup \left\{ c^\top x \mid Ax \leq d \right\}$$

Then:

Are their Special Cases?

Linear Programming (As in our Application):

$$f(d) = \sup \left\{ c^\top x \mid Ax \leq d \right\}$$

Then:

Use **Strong Duality**

to compute

the  s

Are their Special Cases?

Linear Programming (As in our Application):

$$f(d) = \sup \left\{ c^\top x \mid Ax \leq d \right\}$$

Then:

Use **Strong Duality**

to compute

the  s

Semidefinite Programming: . . .

Advantages:

Advantages:

- **Min**ette can be **stopped** at **any time**

  ⇝

  Safe Over-approximation

Advantages:

- **Min**ette can be **stopped** at **any time**

  ⇝

  Safe Over-approximation

-  s can be computed efficiently

  ⇑

  Small Convex Optimization Problems

Advantages:

- **Min**ette can be **stopped** at **any time**

  ⇝

  Safe Over-approximation

-  s can be computed efficiently

  ⇑

  Small Convex Optimization Problems

- Each Min-Strategy π (point-wise max of  s) can be evaluated efficiently

  ⇑

  Linear Programming

Advantages:

- **Min**ette can be **stopped** at **any time**

  ⇝

  Safe Over-approximation

-  s can be computed efficiently

  ⇑

  Small Convex Optimization Problems

- Each Min-Strategy π (point-wise max of  s) can be evaluated efficiently

  ⇑

  Linear Programming

- **Min**ette's iteration "usually" convergences fast

  ⇑

  Newton's iteration "usually" convergences fast

Advantages:

- **Min**ette can be **stopped** at **any time**

  ⇝

  Safe Over-approximation

-  s can be computed efficiently

  ⇑

  Small Convex Optimization Problems

- Each Min-Strategy π (point-wise max of  s) can be evaluated efficiently

  ⇑

  Linear Programming

- **Min**ette's iteration "usually" convergences fast

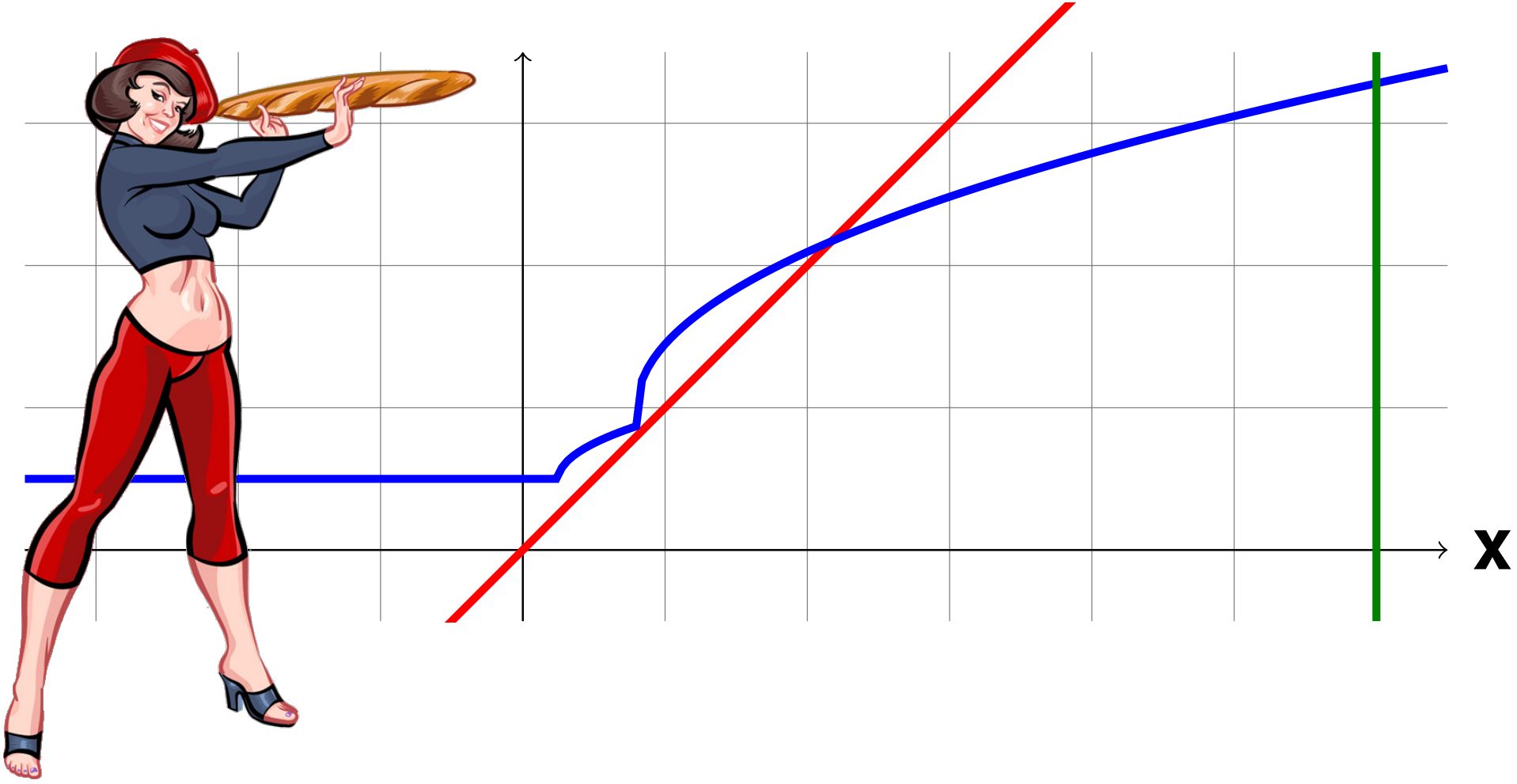
  ⇑

  Newton's iteration "usually" convergences fast

- **The Hybrid Systems Case: Min**ette performs at most **exponentially** many min-strategy iteration steps.

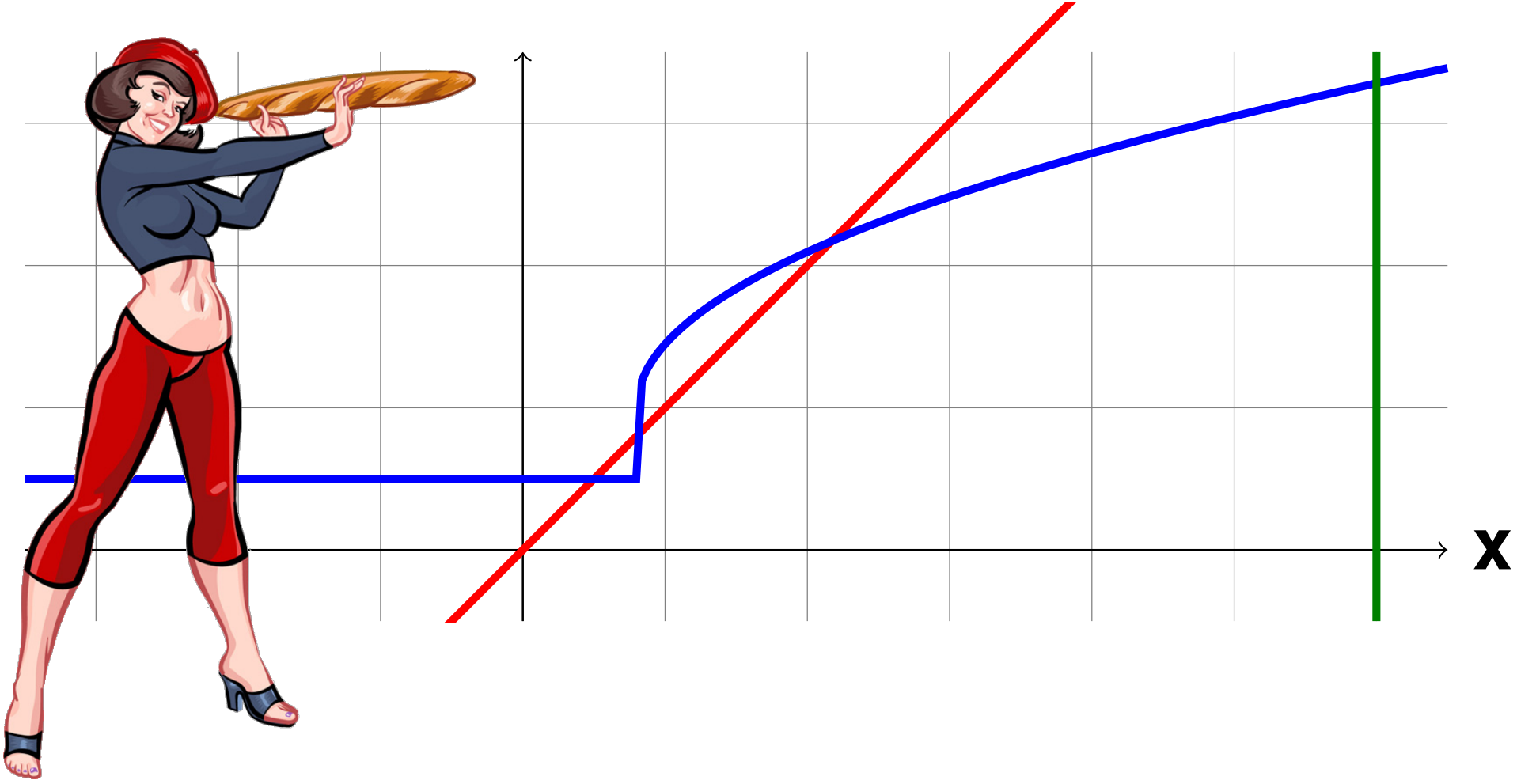
Disadvantages:

Minette is a bit **short-sighted**  !



Disadvantages:

Minette is a bit **short-sighted**  !

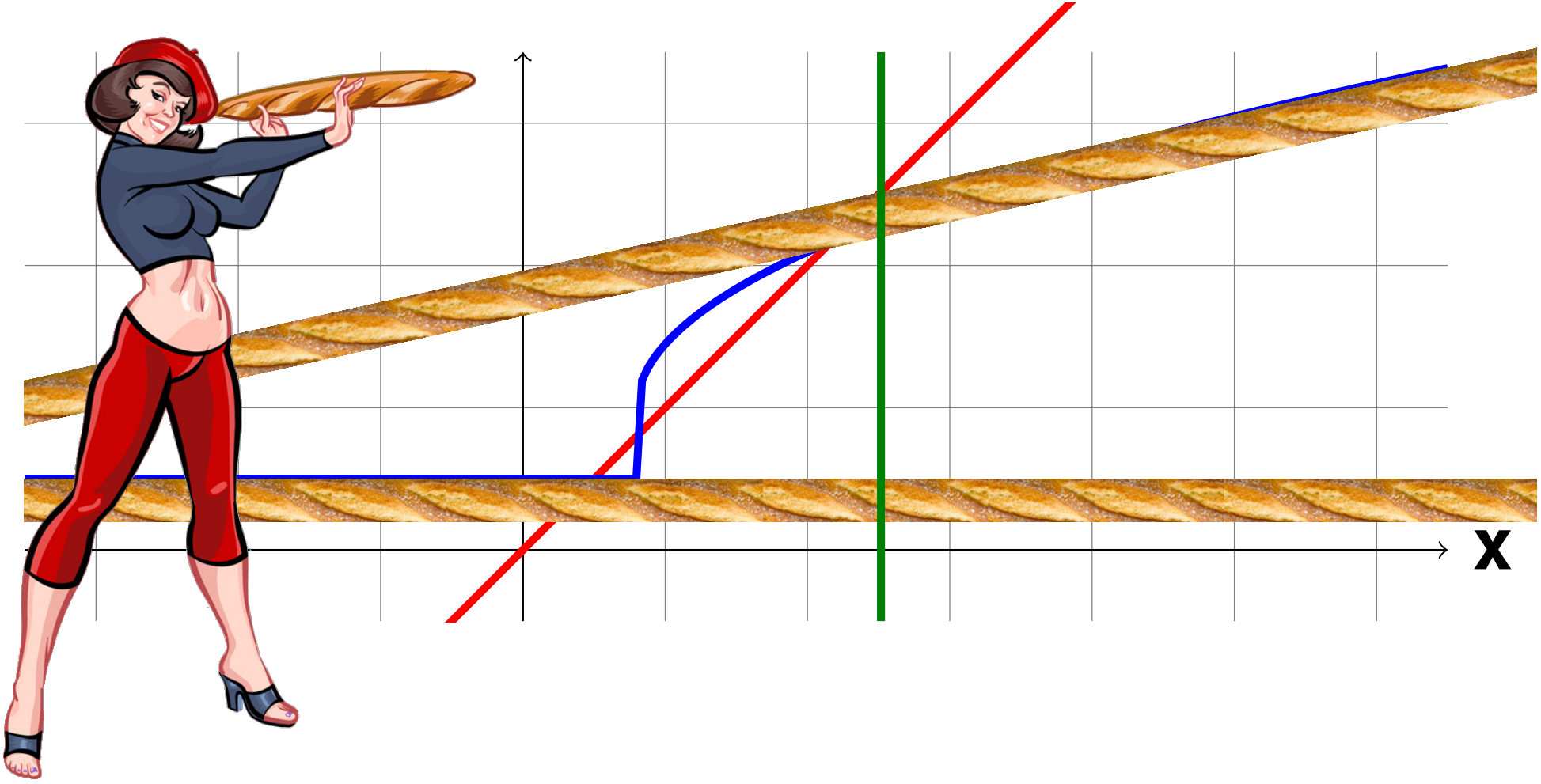


Disadvantages:

Minette is a bit **short-sighted** ()!

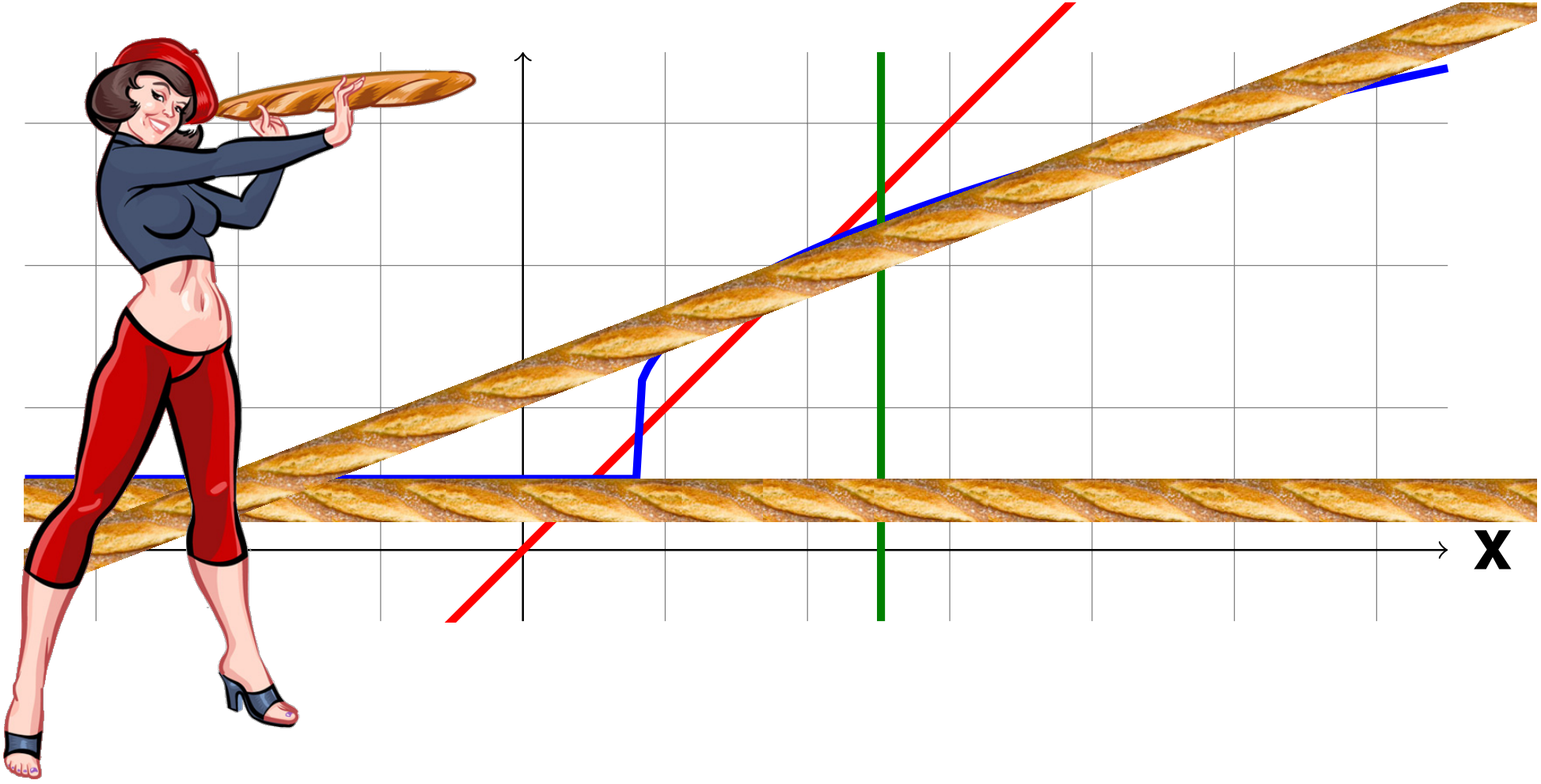
Disadvantages:

Minette is a bit **short-sighted** ()!



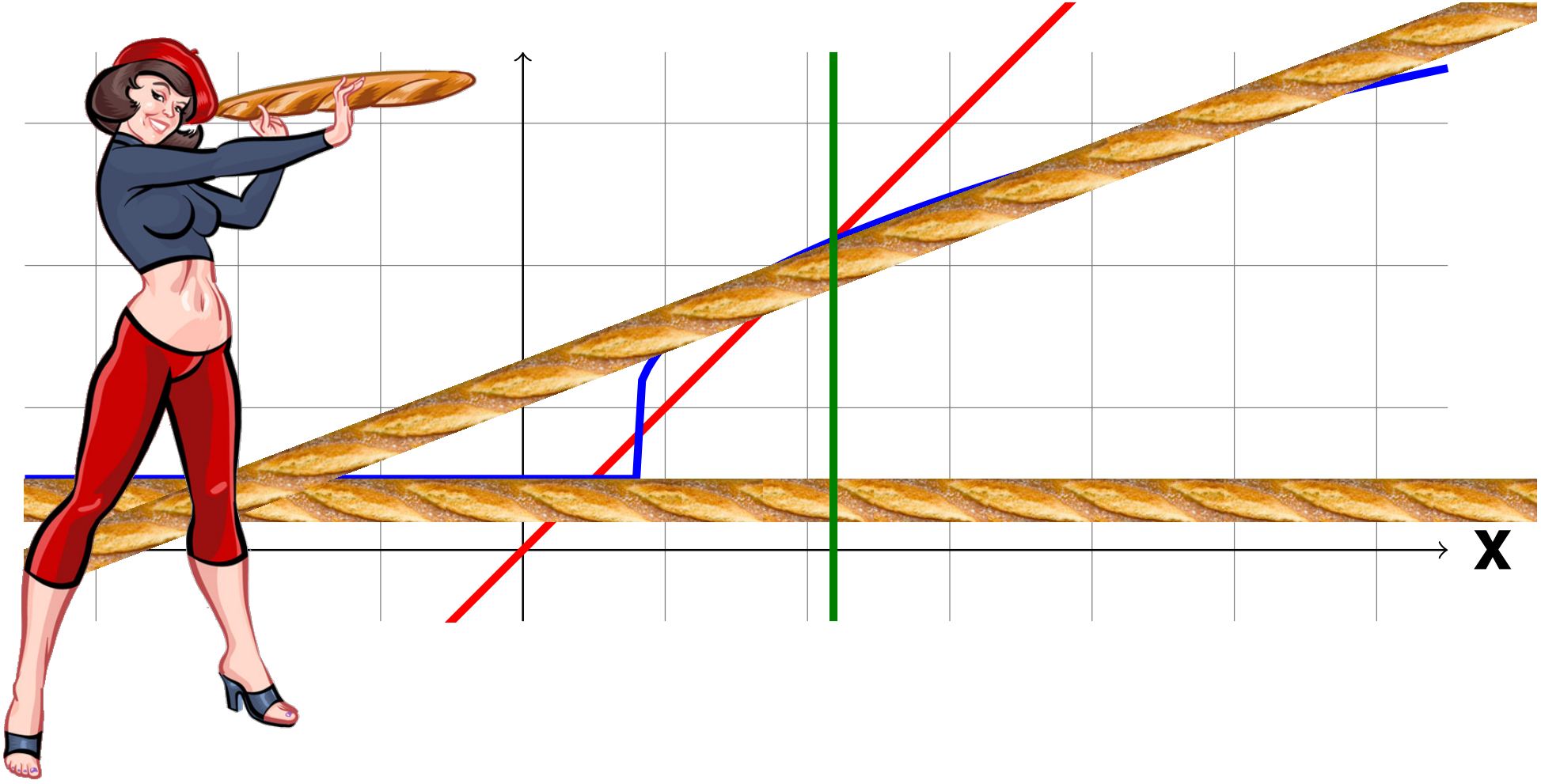
Disadvantages:

Minette is a bit **short-sighted** ()!



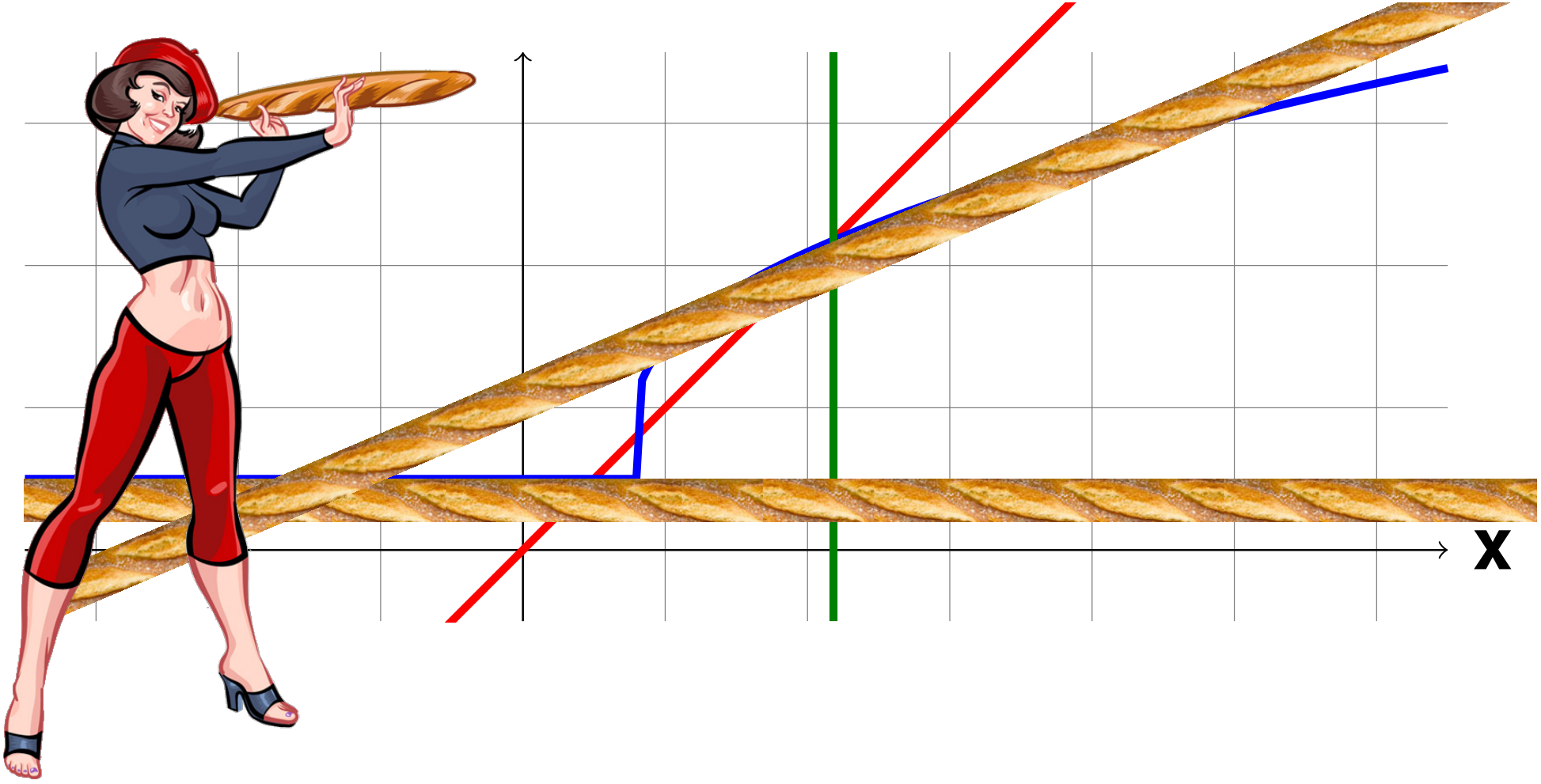
Disadvantages:

Minette is a bit **short-sighted** ()!



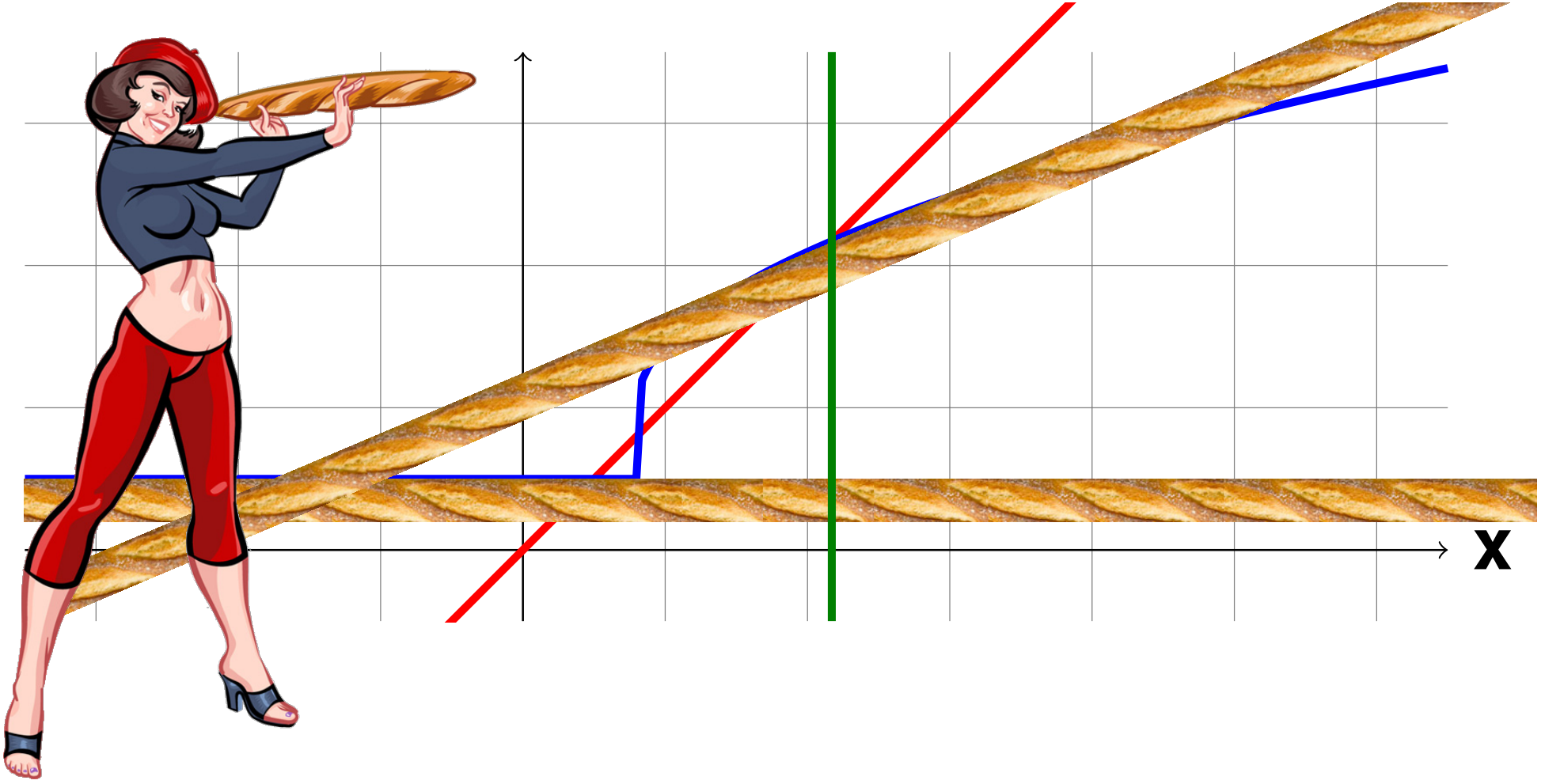
Disadvantages:

Minette is a bit **short-sighted** ()!



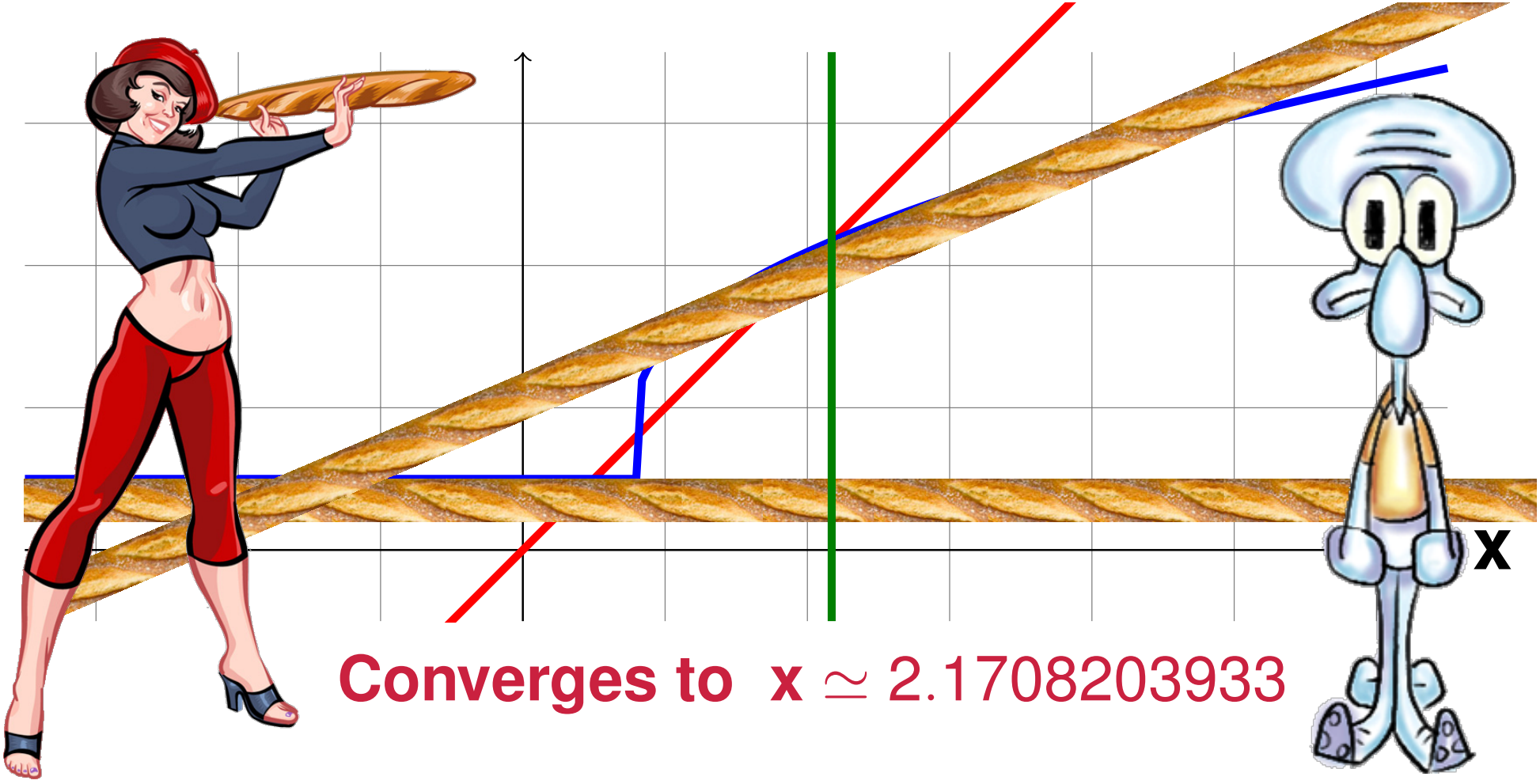
Disadvantages:

Minette is a bit **short-sighted** ()!



Disadvantages:

Minette is a bit **short-sighted** ()!

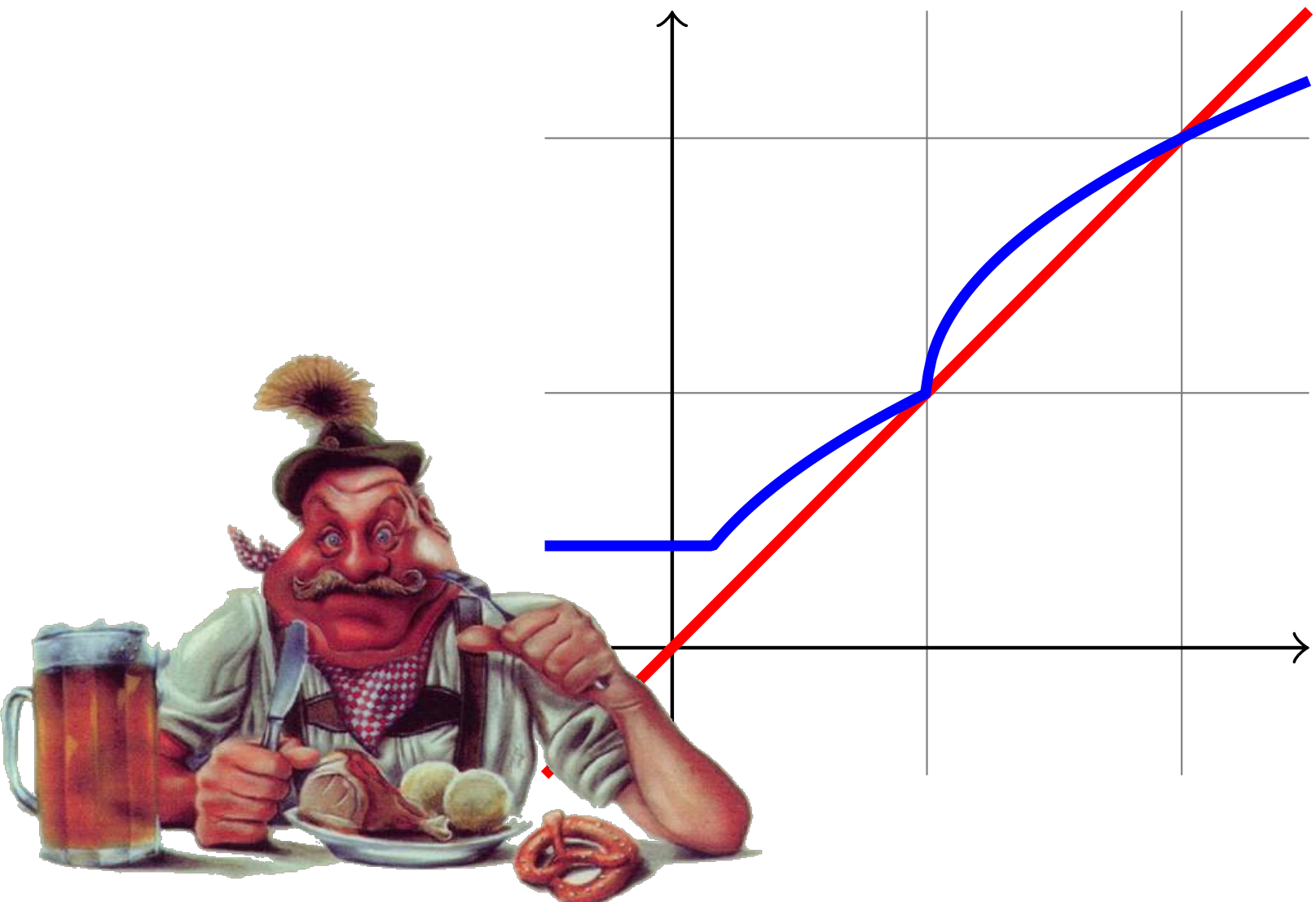


Converges to $x \simeq 2.1708203933$

Outline

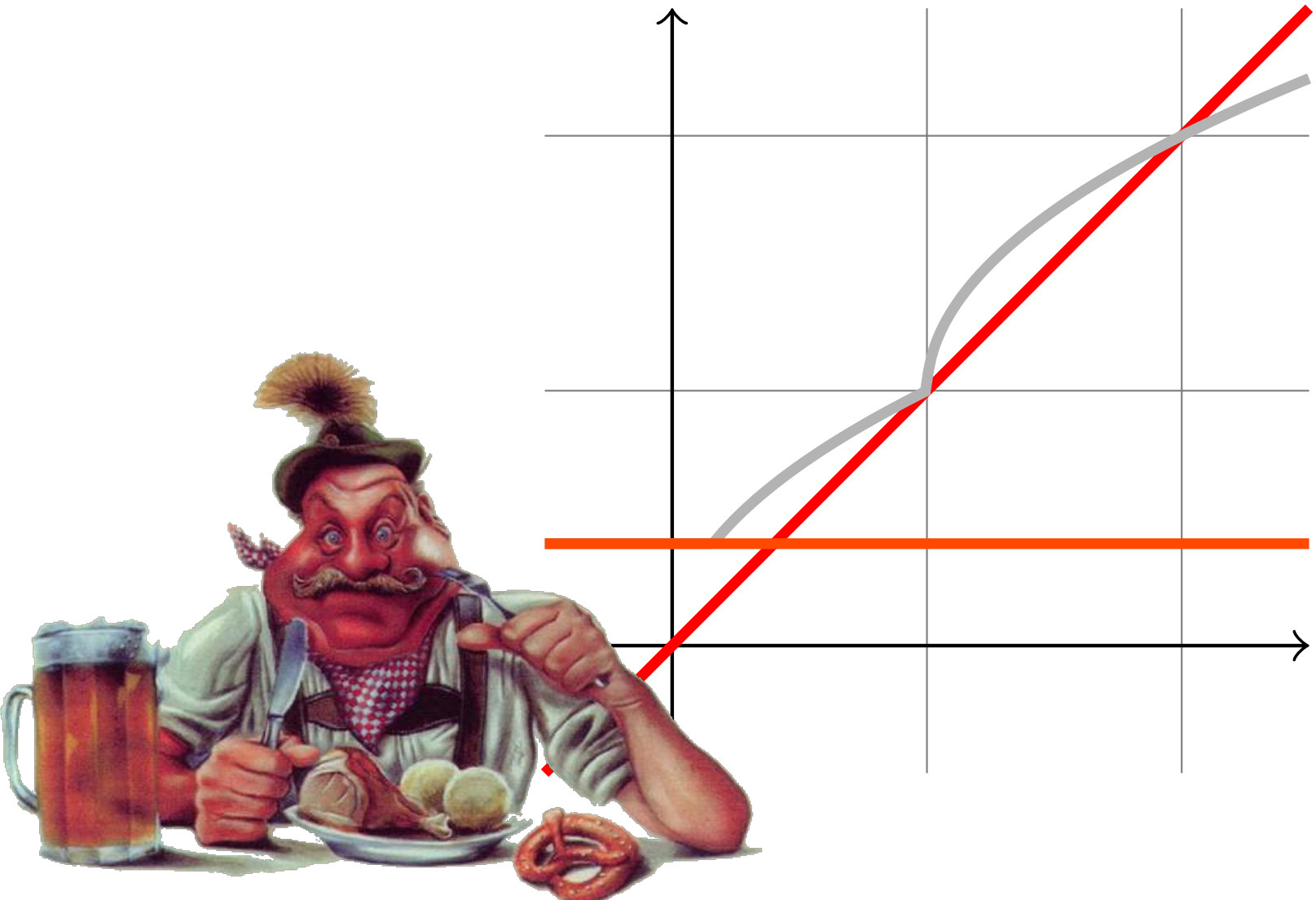
T.M.G.
Helmut Seidl

$$\mathbf{x} = \mathbf{max}\ \{-\infty\ ,\ 0.4\ ,\ \sqrt{\mathbf{x}}\ ,\ 1+\sqrt{\mathbf{x}-1}\}$$

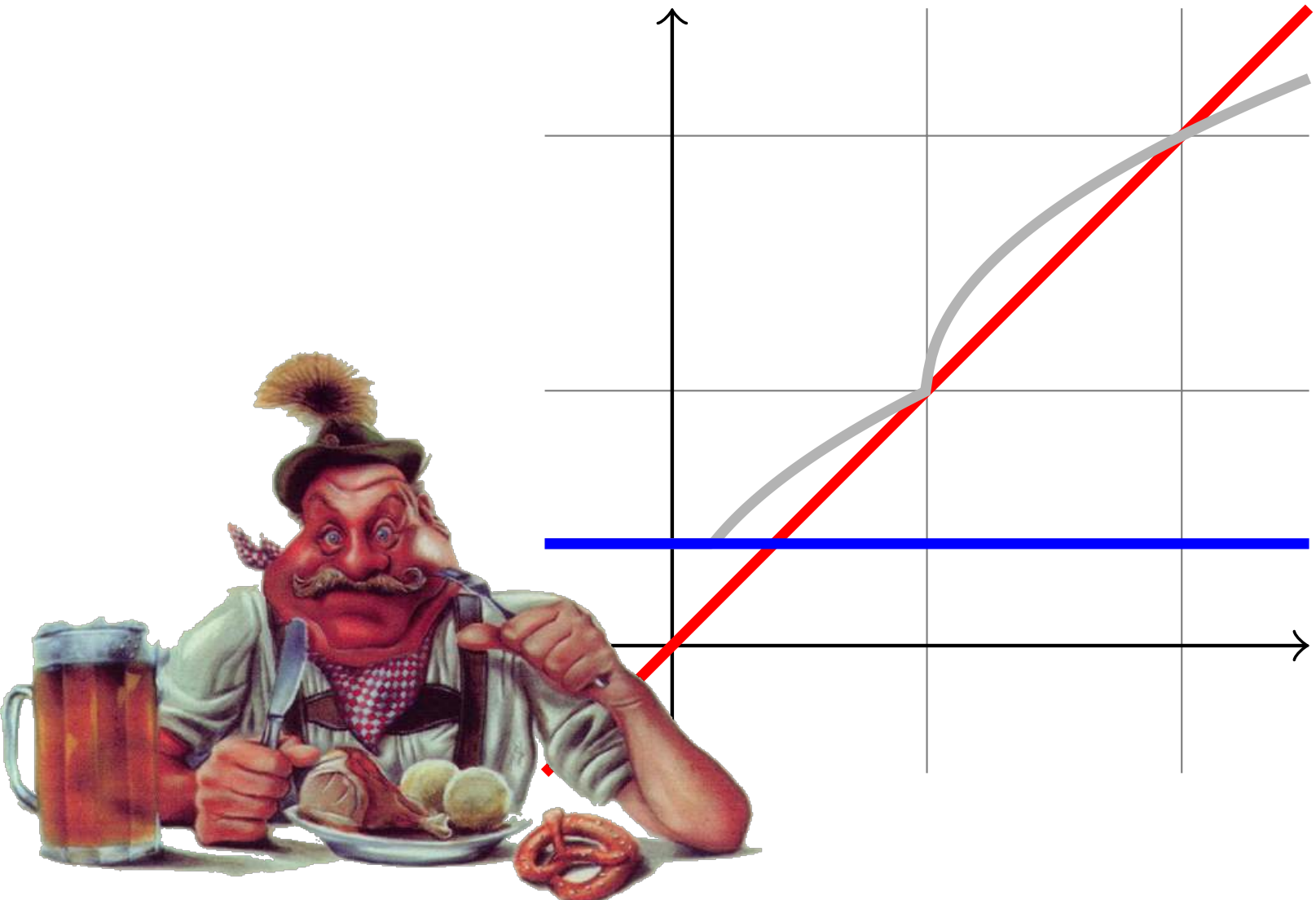


$$x = \max\{-\infty\ ,\ 0.4\ ,\ \sqrt{x}\ ,\ 1+\sqrt{x-1}\}$$

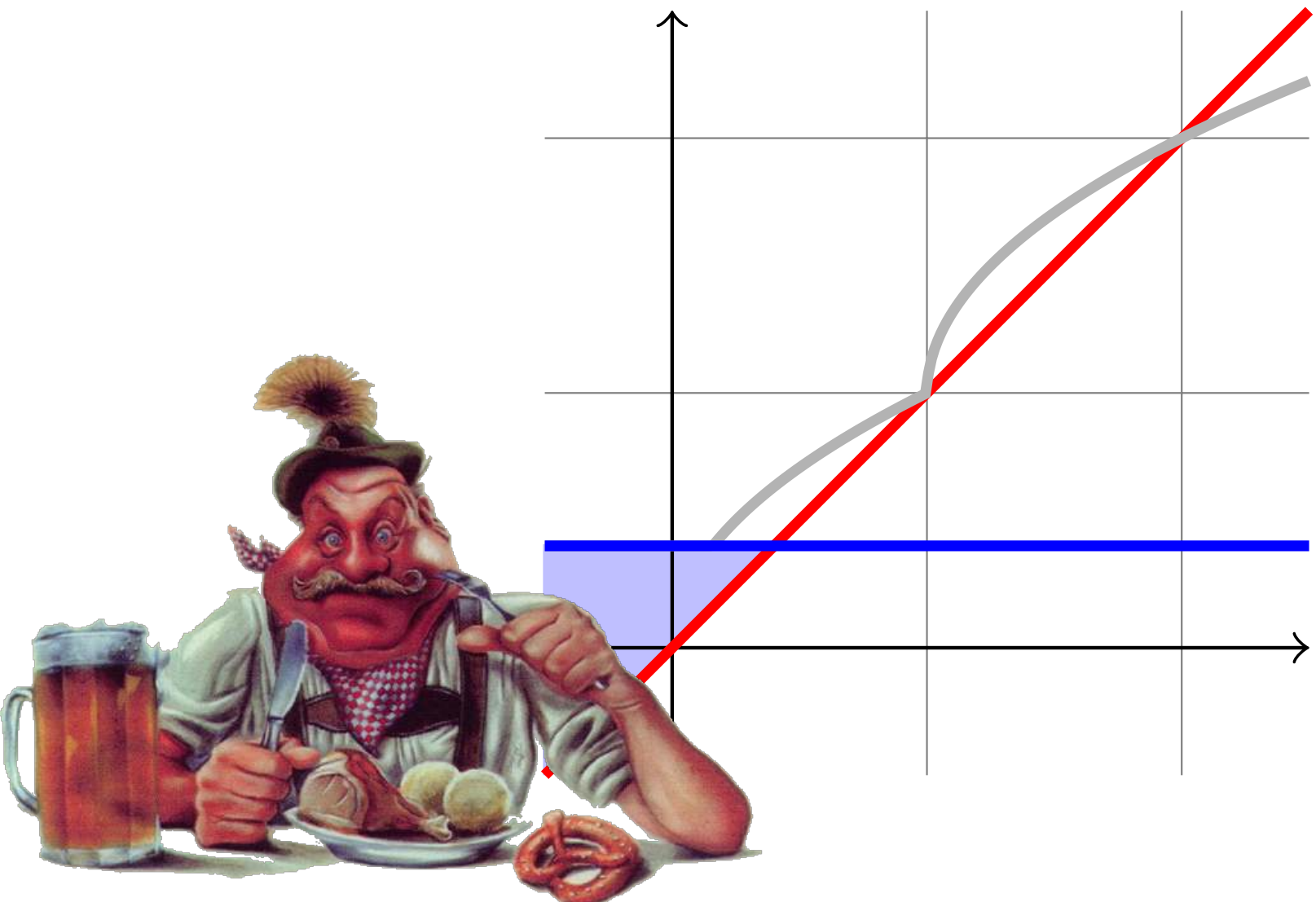
$$x = \max\{-\infty\ ,\ 0.4\ ,\ \sqrt{x}\ ,\ 1+\sqrt{x-1}\}$$



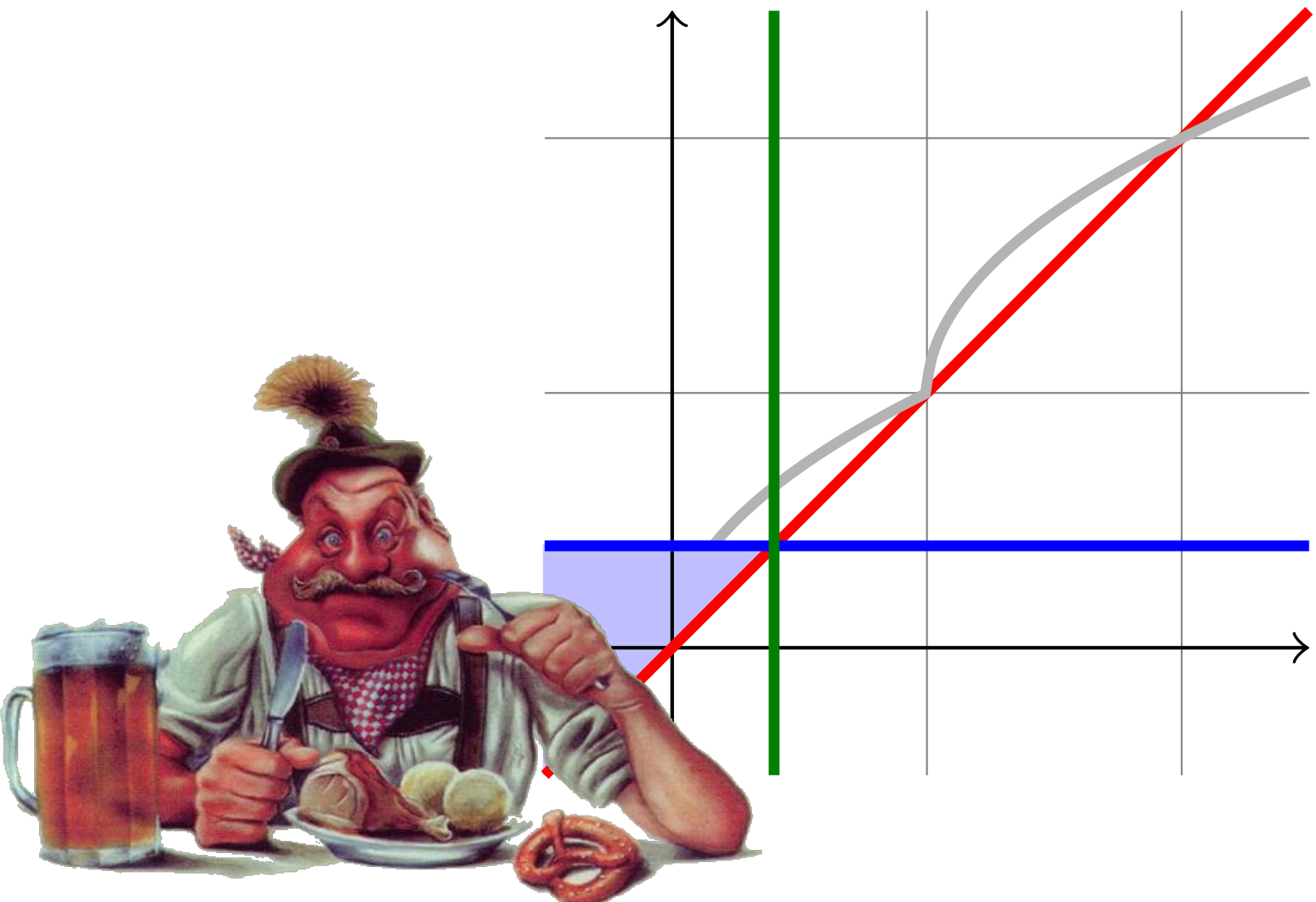
$$x = \max\{-\infty\ ,\ 0.4\ ,\ \sqrt{x}\ ,\ 1+\sqrt{x-1}\}$$



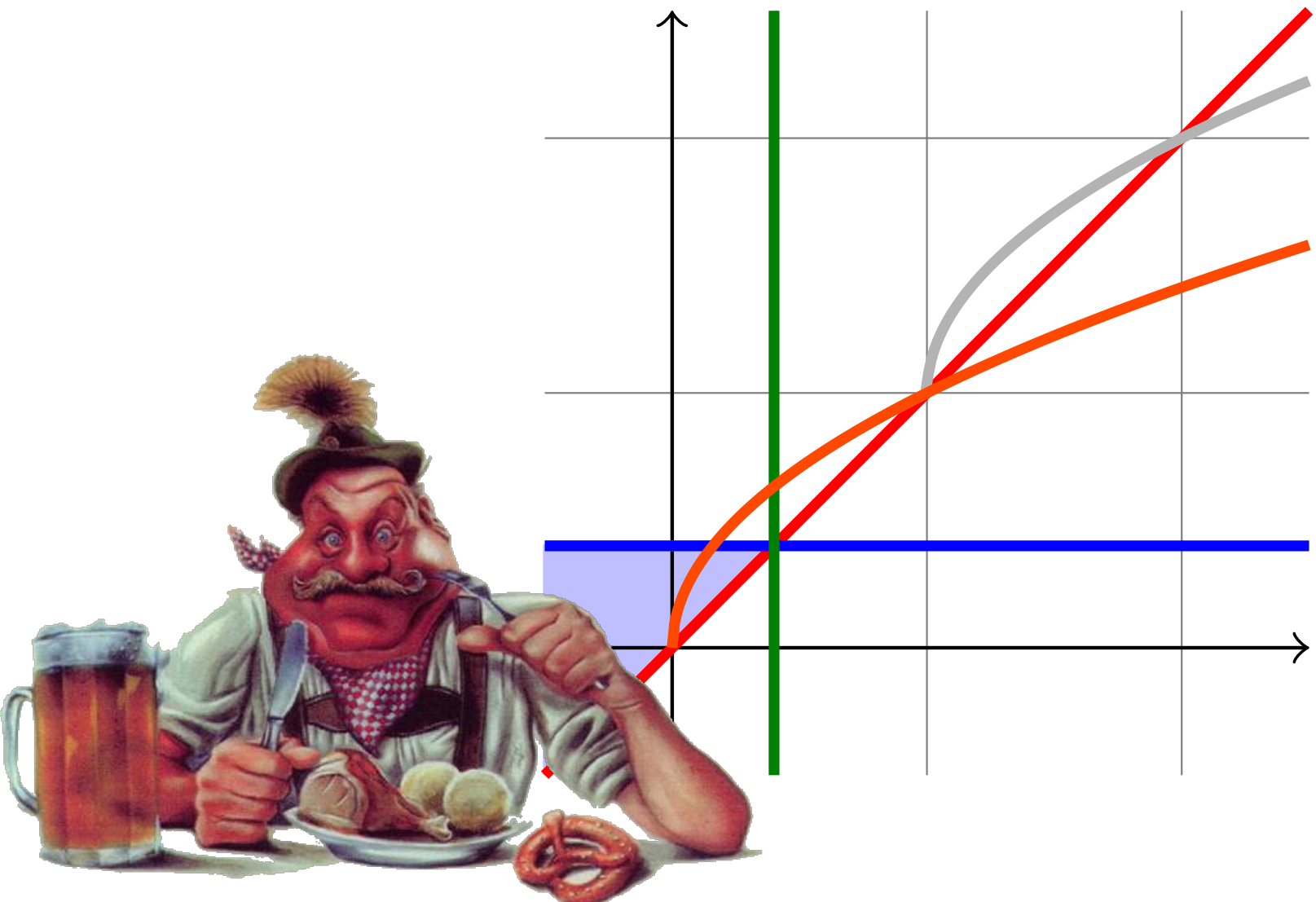
$$x = \max\{-\infty\ ,\ 0.4\ ,\ \sqrt{x}\ ,\ 1+\sqrt{x-1}\}$$



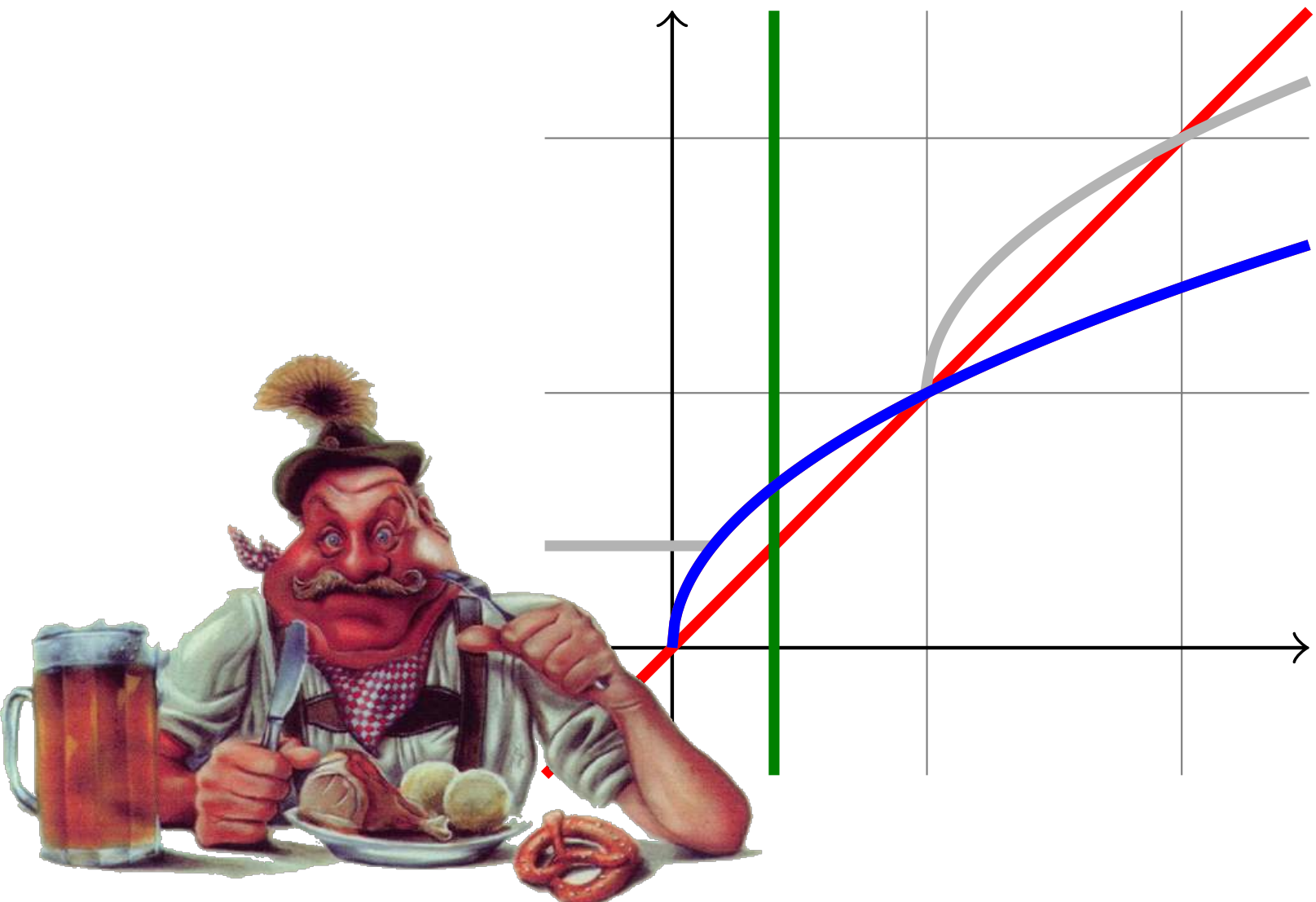
$$x = \max\{-\infty\ ,\ 0.4\ ,\ \sqrt{x}\ ,\ 1+\sqrt{x-1}\}$$



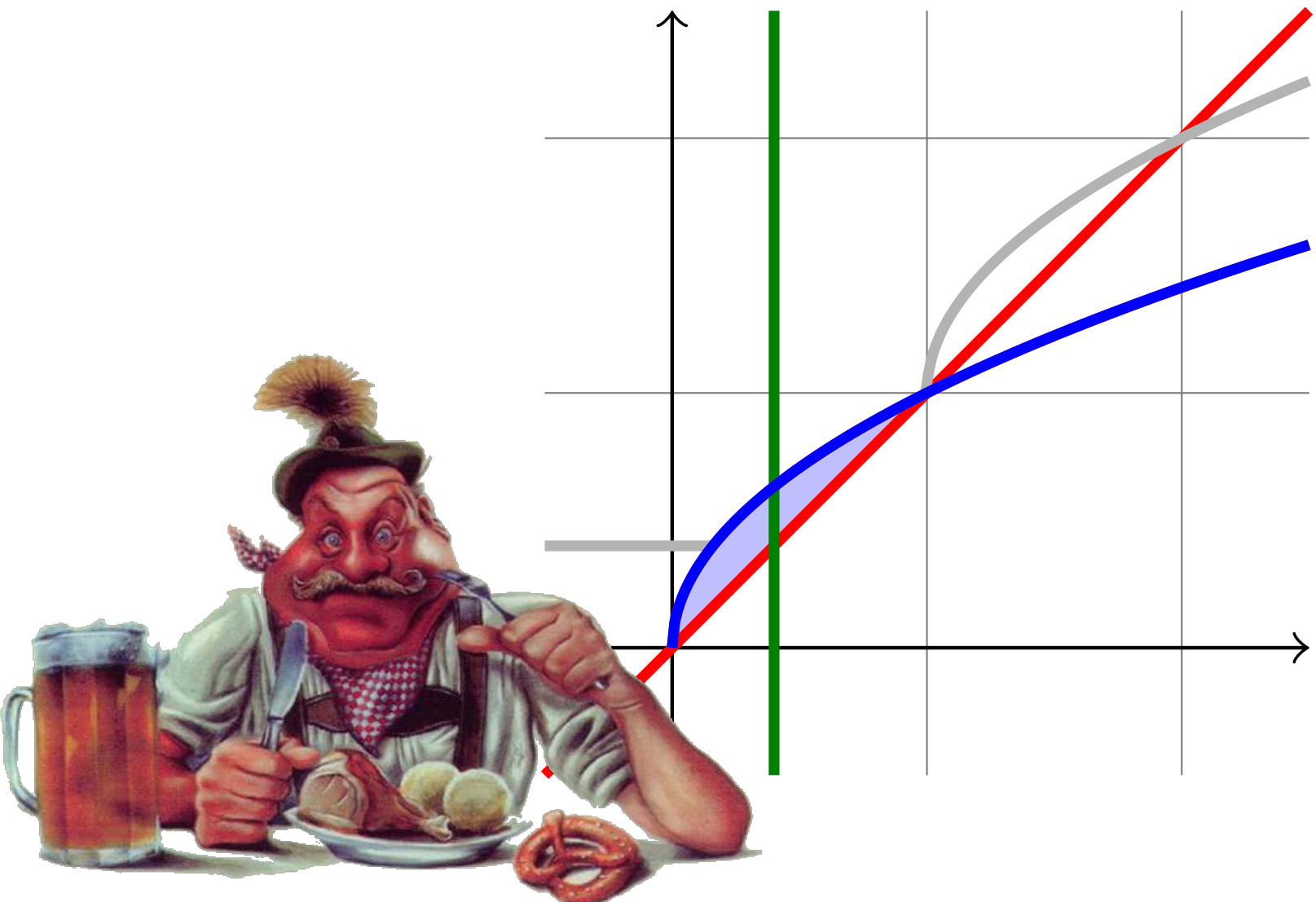
$$x = \max\{-\infty\ ,\ 0.4\ ,\ \sqrt{x}\ ,\ 1+\sqrt{x-1}\}$$



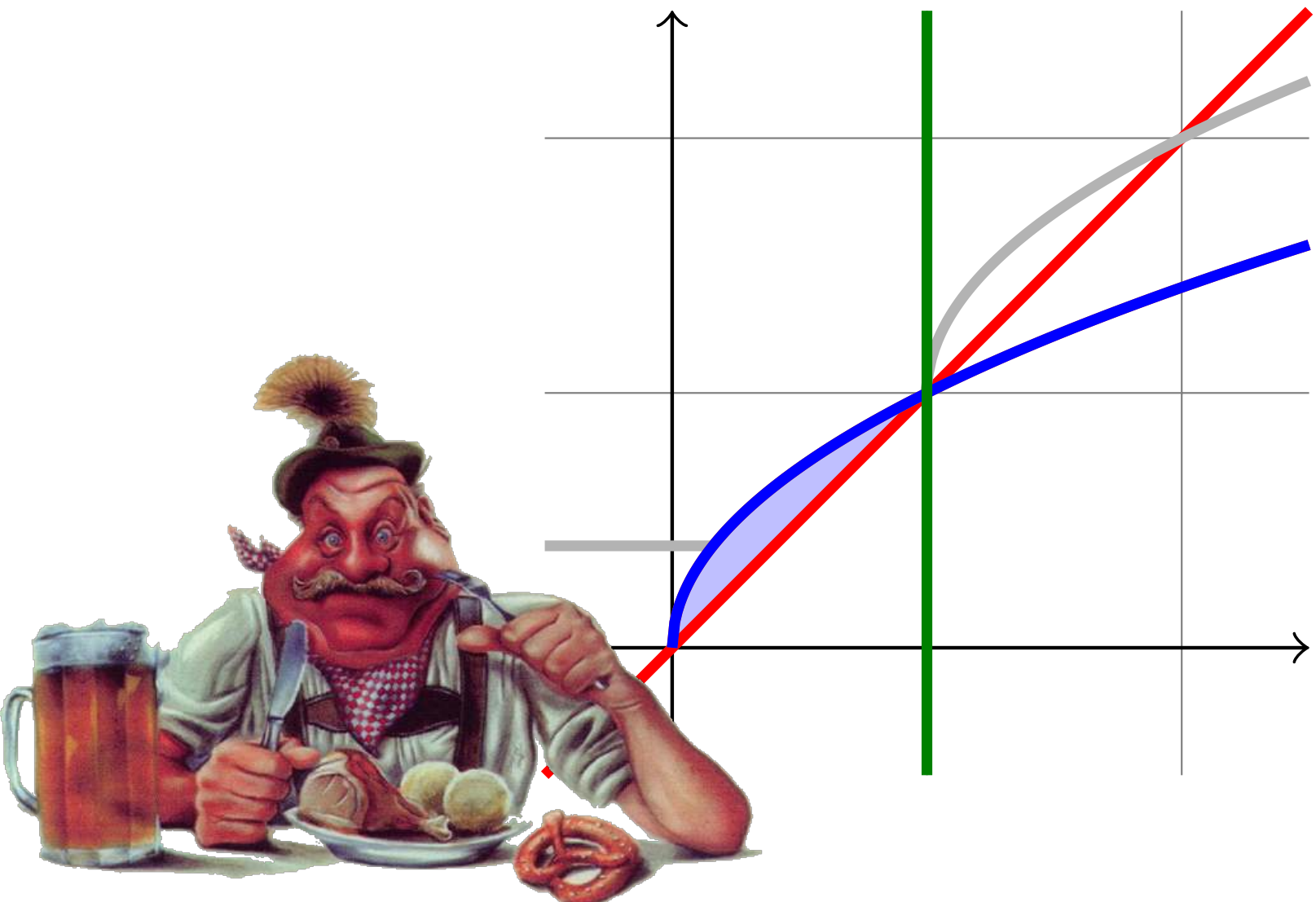
$$\mathbf{x} = \mathbf{max}\ \{-\infty\ ,\ 0.4\ ,\ \sqrt{\mathbf{x}}\ ,\ 1+\sqrt{\mathbf{x}-1}\}$$



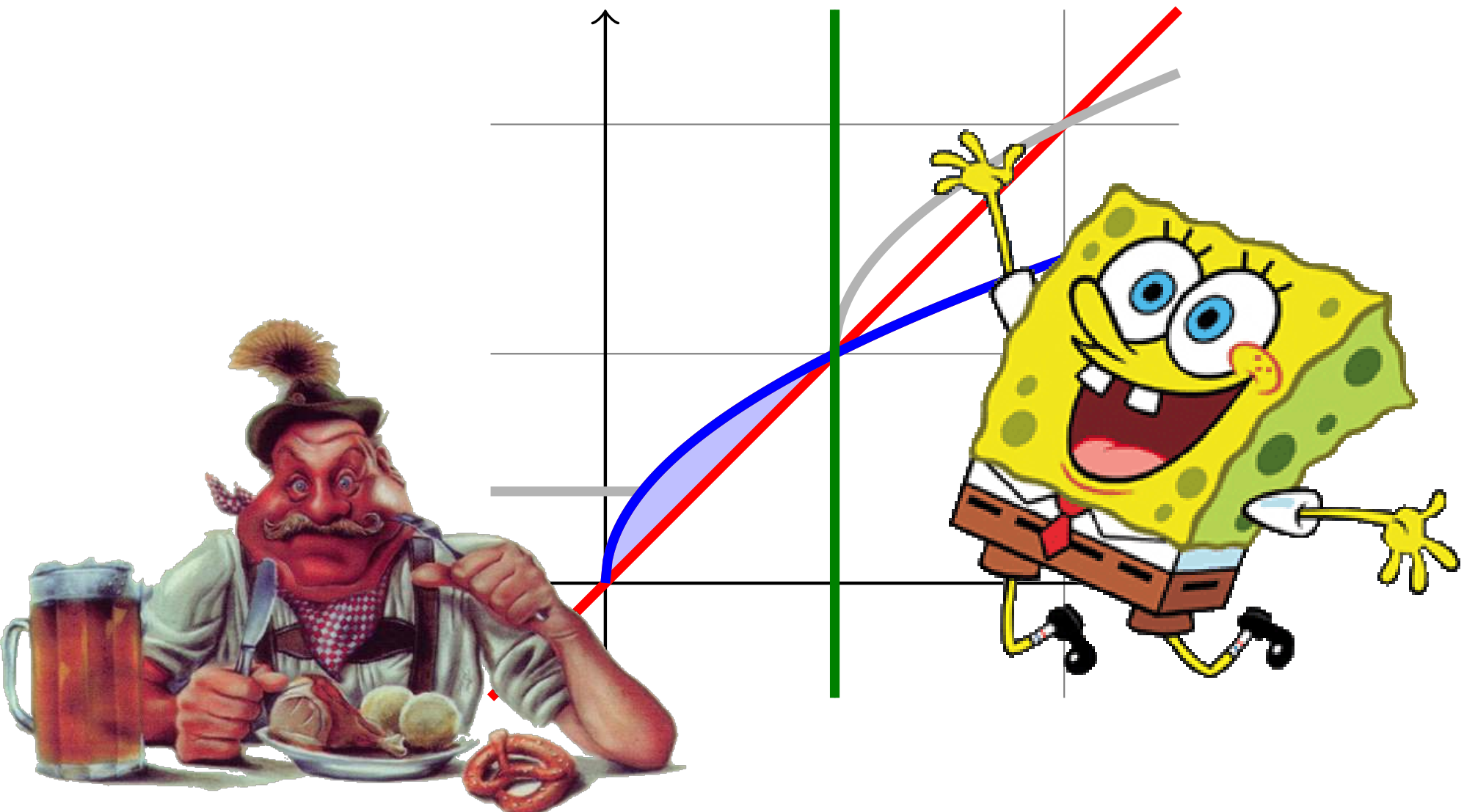
$$\mathbf{x} = \mathbf{max}\ \{-\infty\ ,\ 0.4\ ,\ \sqrt{\mathbf{x}}\ ,\ 1+\sqrt{\mathbf{x}-1}\}$$



$$\mathbf{x} = \mathbf{max}\ \{-\infty\ ,\ 0.4\ ,\ \sqrt{\mathbf{x}}\ ,\ 1+\sqrt{\mathbf{x}-1}\}$$



$$x = \max\{-\infty\ ,\ 0.4\ ,\ \sqrt{x}\ ,\ 1+\sqrt{x-1}\}$$



Observations

Observations

- **Max**imilian's iteration **always terminates**

  (after at most exponentially many improvement steps).

Observations

- **Max**imilian's iteration **always terminates**

  (after at most exponentially many improvement steps).

- **Max**imilian **always** finds the **Minimal Solution**.

Observations

- **Max**imilian's iteration **always terminates**

  (after at most exponentially many improvement steps).

- **Max**imilian **always** finds the **Minimal Solution**.

- Each **Max-Strategy** $\rightsquigarrow$ **convex optimization**.

Observations

- **Max**imilian's iteration **always terminates**

  (after at most exponentially many improvement steps).

- **Max**imilian **always** finds the **Minimal Solution**.

- Each **Max-Strategy** $\rightsquigarrow$ **convex optimization**.

- **For our Hybrid-Systems-Application:**
  Each **Max-Strategy** $\rightsquigarrow$ **linear programming**.

Observations

- **Max**imilian's iteration **always terminates**

  (after at most exponentially many improvement steps).

- **Max**imilian **always** finds the **Minimal Solution**.

- Each **Max-Strategy** $\rightsquigarrow$ **convex optimization**.

- **For our Hybrid-Systems-Application:**
  Each **Max-Strategy** $\rightsquigarrow$ **linear programming**.

- **Abstract Reachability/Verification** $\in$ **coNP**.

Outline

Conclusion

Conclusion

Mean Payoff Games $\leq_{\mathbf{P}}$ Abstract Reachability $\in$ coNP

Conclusion

Mean Payoff Games $\leq_P$ Abstract Reachability $\in$ coNP

Conclusion

Mean Payoff Games $\leq_P$ Abstract Reachability $\in$ coNP

Linear

Terminates with **Safe Over-approximation**

Can be stopped at any time

Terminates with Least Solution

—

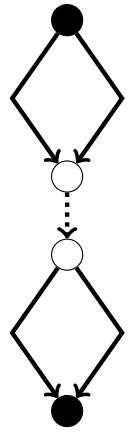
Conclusion

Mean Payoff Games $\leq_P$ Abstract Reachability $\in$ coNP

Linear	**Terminates** with **Safe Over-approximation**	**Terminates** with Least Solution
	Can be stopped at any time	—
Non-Linear	**Converges** to a **Safe Over-approximation**	**Terminates** with Least Solution
	Can be stopped at any time	—
	Small Convex Optimization Problems	**Big** Convex Optimization Problems

What else?

- More sophisticated **Discrete Transitions** [Gawlitza & Monniaux, ESOP'11]:



Linear Inequalities
$(x, y, \ldots, x', y', \ldots)$
$+$
$\wedge$

Linear Inequalities
$(x, y, \ldots, x', y', \ldots)$
$+$
$\wedge + \vee$

coNP

Π_2^p-complete

What else?

- More sophisticated **Abstract Domains**
  [Adjé et al, ESOP'10; Gawlitza & Seidl, SAS'10]:

  linear templates $\rightsquigarrow$ **non-linear** templates

  Idea: Semi-definite Relaxation of the Abstract Semantics

- **Least Fixpoints** $\rightsquigarrow$ **Nested Fixpoints**
  [Gawlitza & Seidl, CAV'09]

- **Recursive Stochastic Games**
  [Esparza & Gawlitza & Kiefer & Seidl, ICALP'08]

Perspectives?

- **Numerical** Issues (Exact Arithmetic necessary?)
- **Practical** Evaluations?
- **Combinations** with Other techniques?
- **Flow-Pipe** Constructions?

Thanks for Your Attention!

Thanks for Your Attention!

Questions