Agda

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2025

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Agda

Agda is a programming language / proof assistant:

- one directly write programs of given type (no tactics),
- in order to help the user one gradually fills *holes*.

First proof

As a first proof, consider:

```
open import Prelude

-- The product is commutative
×-comm : {A B : Type} (A × B) → (B × A)
×-comm (a , b) = (b , a)
```

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We then type `C-c C-l` in order to have Agda check this for us.

Agda syntax

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Comments are of the form

```
-- one line
```

or

```
{-
multiple
lines
-}
```

Agda syntax

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We can use the notation `_` in order to have Agda guess part of the term.

Agda syntax

Agda allows for UTF-8 symbols which are mostly typed like in LaTeX:

×	\times	⊤	\top	→	\to	Π	\Pi	λ	\Gl	ℕ	\bN
⊎	\uplus	¬	\neg	∃	\ex	Σ	\Sigma	≡	\equiv	■	\qed

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The reason is that we can define new infix operators, e.g.

```
_+_ : ℕ → ℕ → ℕ
zero  + n = n
suc m + n = suc (m + n)
```

Agda syntax

In practice it is almost impossible to write a whole proof directly.

Agda offers holes which are typed as ?.

Agda shortcuts

We then have shortcuts to help us in proofs:

C-c C-l	typecheck and highlight the current file
C-c C-,	get information about the hole under the cursor
C-c C-.	same as above + the type of the proposed filler
C-c C-*space*	give a solution
C-c C-c	case analysis on a variable
C-c C-r	refine the hole
C-c C-a	automatic fill
middle click	go to the definition of the term

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Constructors are injective: Agda "knows" that

- `zero` is always different from `suc n`
- `suc m` is the same as `suc n` iff `m` is the same as `n`

Inductive types

The identity type can also be defined as an inductive type:

```
data _≡_ {A : Type} (x : A) : (y : A) → Type where
  refl : x ≡ x
```

Local definitions

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```
quadruple : ℕ → ℕ
quadruple n = double (double n)
  where
  double : ℕ → ℕ
  double zero = zero
  double (suc n) = suc (suc (double n))
```

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Agda can generate implicit arguments:

```
private variable
  ℓ ℓ' : Level

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arr A B = A → B
```

Capturing implicit arguments

We can capture implicit arguments

```
id : {ℓ : Level} {A : Type ℓ} → A → A
id {ℓ} {A} a = a
```

or

```
id : {ℓ : Level} {A : Type ℓ} → A → A
id {A = A} a = a
```

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We have a special syntax for pattern matching:

```
not : Bool → Bool
not = λ { true → false ; false → true }
```

(it is generally preferable to use `where`).

Modules

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```
module Int where
  int : Type
  int = ...

  add : int → int → int
  add = ...
```

Note the two spaces at the beginning of lines!

By default all files define modules.

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```
module _ {ℓ ℓ' : Level} (A : Type ℓ) (B : Type ℓ') where
  f : A → B
  f = ...

  g : B → A
  g = ...
```

equational reasoning