$$\frac{}{\langle\ \texttt{skip} \mid \sigma\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma}\ (\textsc{skip})$$

$$\frac{}{\langle\ x\ \texttt{:=}\ a \mid \sigma\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma[x \mapsto [\![a]\!]_\sigma^{\mathbf{Aexp}}]}\ (\textsc{set})$$

$$\frac{\langle\ c_1 \mid \sigma\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma' \qquad \langle\ c_2 \mid \sigma'\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma''}{\langle\ c_1\texttt{;}\ c_2 \mid \sigma\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma''}\ (\textsc{seq})$$

$$\frac{\langle\ c_1 \mid \sigma\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma'}{\langle\ \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2 \mid \sigma\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma'}\ (\textsc{if}_\top) \qquad \text{si } [\![b]\!]_\sigma^{\mathbf{Bexp}} = \top$$

$$\frac{\langle\ c_2 \mid \sigma\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma'}{\langle\ \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2 \mid \sigma\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma'}\ (\textsc{if}_\bot) \qquad \text{si } [\![b]\!]_\sigma^{\mathbf{Bexp}} = \bot$$

$$\frac{\langle\ c \mid \sigma\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma' \qquad \langle\ \texttt{while}\ b\ \texttt{do}\ c \mid \sigma'\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma''}{\langle\ \texttt{while}\ b\ \texttt{do}\ c \mid \sigma\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma''}\ (\textsc{while}_\top) \qquad \text{si } [\![b]\!]_\sigma^{\mathbf{Bexp}} = \top$$

$$\frac{}{\langle\ \texttt{while}\ b\ \texttt{do}\ c \mid \sigma\ \rangle \longrightarrow\!\!\!\!\!\rightarrow \sigma}\ (\textsc{while}_\bot) \qquad \text{si } [\![b]\!]_\sigma^{\mathbf{Bexp}} = \bot$$

Figure 1: Règles de l'évaluation des commandes.

$$\frac{}{\langle\ x \mid \sigma\ \rangle \longrightarrow \langle\ \underline{\sigma(x)} \mid \sigma\ \rangle}$$

$$\frac{\langle\ a_1 \mid \sigma\ \rangle \longrightarrow \langle\ a_1' \mid \sigma'\ \rangle}{\langle\ a_1\ \texttt{+}\ a_2 \mid \sigma\ \rangle \longrightarrow \langle\ a_1'\ \texttt{+}\ a_2 \mid \sigma'\ \rangle} \qquad \frac{\langle\ a_1 \mid \sigma\ \rangle \longrightarrow \langle\ a_1' \mid \sigma'\ \rangle}{\langle\ a_1\ \texttt{*}\ a_2 \mid \sigma\ \rangle \longrightarrow \langle\ a_1'\ \texttt{*}\ a_2 \mid \sigma'\ \rangle}$$

$$\frac{\langle\ a \mid \sigma\ \rangle \longrightarrow \langle\ a' \mid \sigma'\ \rangle}{\langle\ \underline{n}\ \texttt{+}\ a \mid \sigma\ \rangle \longrightarrow \langle\ \underline{n}\ \texttt{+}\ a' \mid \sigma'\ \rangle} \qquad \frac{\langle\ a \mid \sigma\ \rangle \longrightarrow \langle\ a' \mid \sigma'\ \rangle}{\langle\ \underline{n}\ \texttt{*}\ a \mid \sigma\ \rangle \longrightarrow \langle\ \underline{n}\ \texttt{*}\ a' \mid \sigma'\ \rangle}$$

$$\frac{}{\langle\ \underline{m}\ \texttt{+}\ \underline{n} \mid \sigma\ \rangle \longrightarrow \langle\ \underline{m+n} \mid \sigma\ \rangle} \qquad \frac{}{\langle\ \underline{m}\ \texttt{*}\ \underline{n} \mid \sigma\ \rangle \longrightarrow \langle\ \underline{m \times n} \mid \sigma\ \rangle}$$

Figure 2: Règles de réduction des expressions arithmétiques.

$$\frac{\langle\ a_1 \mid \sigma\ \rangle \longrightarrow \langle\ a_1' \mid \sigma'\ \rangle}{\langle\ a_1\ \texttt{<}\ a_2 \mid \sigma\ \rangle \longrightarrow \langle\ a_1'\ \texttt{<}\ a_2 \mid \sigma'\ \rangle}$$

$$\frac{\langle\ a \mid \sigma\ \rangle \longrightarrow \langle\ a' \mid \sigma'\ \rangle}{\langle\ \underline{n}\ \texttt{<}\ a \mid \sigma\ \rangle \longrightarrow \langle\ \underline{n}\ \texttt{<}\ a' \mid \sigma'\ \rangle} \qquad \frac{\langle\ b \mid \sigma\ \rangle \longrightarrow \langle\ b' \mid \sigma'\ \rangle}{\langle\ \texttt{not}\ b \mid \sigma\ \rangle \longrightarrow \langle\ \texttt{not}\ b' \mid \sigma'\ \rangle}$$

$$\frac{}{\langle\ \underline{m}\ \texttt{<}\ \underline{n} \mid \sigma\ \rangle \longrightarrow \langle\ \texttt{true} \mid \sigma\ \rangle} \quad \text{si } m < n \qquad \frac{}{\langle\ \texttt{not false} \mid \sigma\ \rangle \longrightarrow \langle\ \texttt{true} \mid \sigma\ \rangle}$$

$$\frac{}{\langle\ \underline{m}\ \texttt{<}\ \underline{n} \mid \sigma\ \rangle \longrightarrow \langle\ \texttt{false} \mid \sigma\ \rangle} \quad \text{si } m \geqslant n \qquad \frac{}{\langle\ \texttt{not true} \mid \sigma\ \rangle \longrightarrow \langle\ \texttt{false} \mid \sigma\ \rangle}$$

Figure 3: Règles de réduction des expressions booléennes.

$$\frac{\langle\ a \mid \sigma\ \rangle \longrightarrow \langle\ a' \mid \sigma'\ \rangle}{\langle\ x\ \texttt{:=}\ a \mid \sigma\ \rangle \longrightarrow \langle\ x\ \texttt{:=}\ a' \mid \sigma'\ \rangle}\ (\textsc{set})$$

$$\frac{}{\langle\ x\ \texttt{:=}\ \underline{n} \mid \sigma\ \rangle \longrightarrow \langle\ \texttt{skip} \mid \sigma[x \mapsto n]\ \rangle}\ (\textsc{set}_n)$$

$$\frac{\langle\ c_1 \mid \sigma\ \rangle \longrightarrow \langle\ c_1' \mid \sigma'\ \rangle}{\langle\ c_1\texttt{;}\ c_2 \mid \sigma\ \rangle \longrightarrow \langle\ c_1'\texttt{;}\ c_2 \mid \sigma'\ \rangle}\ (\textsc{seq}_\textsc{g}) \qquad \frac{}{\langle\ \texttt{skip;}\ c \mid \sigma\ \rangle \longrightarrow \langle\ c \mid \sigma\ \rangle}\ (\textsc{seq}_\textsc{d})$$

$$\frac{\langle\ b \mid \sigma\ \rangle \longrightarrow \langle\ b' \mid \sigma'\ \rangle}{\langle\ \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2 \mid \sigma\ \rangle \longrightarrow \langle\ \texttt{if}\ b'\ \texttt{then}\ c_1\ \texttt{else}\ c_2 \mid \sigma'\ \rangle}\ (\textsc{if})$$

$$\frac{}{\langle\ \texttt{if true then}\ c_1\ \texttt{else}\ c_2 \mid \sigma\ \rangle \longrightarrow \langle\ c_1 \mid \sigma\ \rangle}\ (\textsc{if}_\top)$$

$$\frac{}{\langle\ \texttt{if false then}\ c_1\ \texttt{else}\ c_2 \mid \sigma\ \rangle \longrightarrow \langle\ c_2 \mid \sigma\ \rangle}\ (\textsc{if}_\bot)$$

$$\frac{}{\langle\ \texttt{while}\ b\ \texttt{do}\ c \mid \sigma\ \rangle \longrightarrow \langle\ \texttt{if}\ b\ \texttt{then}\ (c\texttt{; while}\ b\ \texttt{do}\ c)\ \texttt{else skip} \mid \sigma\ \rangle}\ (\textsc{while})$$

Figure 4: Règles de réduction des commandes.

$$\frac{}{\Gamma \vdash \underline{n} : \texttt{int}}\ (\textsc{int}) \qquad \frac{}{\Gamma \vdash \underline{b} : \texttt{bool}}\ (\textsc{bool})$$

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A}\ (\textsc{var}) \qquad \frac{}{\Gamma \vdash \texttt{add} : \texttt{int} \times \texttt{int} \Rightarrow \texttt{int}}\ (\textsc{add})$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \texttt{fun}\ \ x\ \ \rightarrow\ \ t : A \Rightarrow B}\ (\textsc{fun}) \qquad \frac{\Gamma \vdash t : A \Rightarrow B \qquad \Gamma \vdash u : A}{\Gamma \vdash t\ u : B}\ (\textsc{app})$$

$$\frac{}{\Gamma \vdash \texttt{fst} : A \times B \Rightarrow A}\ (\textsc{fst})$$

$$\frac{\Gamma \vdash t : A \qquad \Gamma \vdash u : B}{\Gamma \vdash (t, u) : A \times B}\ (\textsc{pair}) \qquad \frac{}{\Gamma \vdash \texttt{snd} : A \times B \Rightarrow B}\ (\textsc{snd})$$

Figure 5: Règles de typage.

$$\frac{}{\underline{n} \longrightarrow\!\!\!\!\!\rightarrow \underline{n}}\ (\textsc{int}) \qquad \frac{}{\underline{b} \longrightarrow\!\!\!\!\!\rightarrow \underline{b}}\ (\textsc{bool})$$

$$\frac{}{\texttt{add} \longrightarrow\!\!\!\!\!\rightarrow \texttt{add}}\ (\textsc{add}) \qquad \frac{t \longrightarrow\!\!\!\!\!\rightarrow \texttt{add} \qquad u \longrightarrow\!\!\!\!\!\rightarrow (\underline{m}, \underline{n})}{t\ u \longrightarrow\!\!\!\!\!\rightarrow \underline{m+n}}\ (\textsc{sum})$$

$$\frac{}{(\texttt{fun}\ \ x\ \ \rightarrow\ \ t) \longrightarrow\!\!\!\!\!\rightarrow (\texttt{fun}\ \ x\ \ \rightarrow\ \ t)}\ (\textsc{fun}) \qquad \frac{t \longrightarrow\!\!\!\!\!\rightarrow (\texttt{fun}\ \ x\ \ \rightarrow\ \ t') \qquad u \longrightarrow\!\!\!\!\!\rightarrow u' \qquad t'[x \mapsto u'] \longrightarrow\!\!\!\!\!\rightarrow v}{t\ u \longrightarrow\!\!\!\!\!\rightarrow v}\ (\textsc{app})$$

$$\frac{t \longrightarrow\!\!\!\!\!\rightarrow t' \qquad u \longrightarrow\!\!\!\!\!\rightarrow u'}{(t, u) \longrightarrow\!\!\!\!\!\rightarrow (t', u')}\ (\textsc{pair})$$

$$\frac{}{\texttt{fst} \longrightarrow\!\!\!\!\!\rightarrow \texttt{fst}}\ (\textsc{fst}) \qquad \frac{t \longrightarrow\!\!\!\!\!\rightarrow \texttt{fst} \qquad u \longrightarrow\!\!\!\!\!\rightarrow (v_1, v_2)}{t\ u \longrightarrow\!\!\!\!\!\rightarrow v_1}\ (\textsc{fstp})$$

$$\frac{}{\texttt{snd} \longrightarrow\!\!\!\!\!\rightarrow \texttt{snd}}\ (\textsc{snd}) \qquad \frac{t \longrightarrow\!\!\!\!\!\rightarrow \texttt{snd} \qquad u \longrightarrow\!\!\!\!\!\rightarrow (v_1, v_2)}{t\ u \longrightarrow\!\!\!\!\!\rightarrow v_2}\ (\textsc{sndp})$$

Figure 6: Évaluation en mini-ML.

Pour v, v_1 et v_2 des valeurs :

$$\frac{t \longrightarrow t'}{t\ v \longrightarrow t'\ v}\ (\textsc{appl}) \qquad \frac{u \longrightarrow u'}{t\ u \longrightarrow t\ u'}\ (\textsc{appr})$$

$$\frac{}{(\texttt{fun}\ \ x\ \ \rightarrow\ \ t)\ v \longrightarrow t[x \mapsto v]}\ (\textsc{app}) \qquad \frac{}{\texttt{add}\ (\underline{m}\texttt{,}\underline{n}\texttt{)} \longrightarrow \underline{m+n}}\ (\textsc{add})$$

$$\frac{t \longrightarrow t'}{\texttt{(}t\texttt{,}v\texttt{)} \longrightarrow \texttt{(}t'\texttt{,}v\texttt{)}}\ (\textsc{pairl}) \qquad \frac{u \longrightarrow u'}{\texttt{(}t\texttt{,}u\texttt{)} \longrightarrow \texttt{(}t\texttt{,}u'\texttt{)}}\ (\textsc{pairr})$$

$$\frac{}{\texttt{fst (}v_1\texttt{,}v_2\texttt{)} \longrightarrow v_1}\ (\textsc{fst}) \qquad \frac{}{\texttt{snd (}v_1\texttt{,}v_2\texttt{)} \longrightarrow v_2}\ (\textsc{snd})$$

Figure 7: Réduction en mini-ML.

Typage:

$$\frac{}{\Gamma \vdash \texttt{fix} : (A \Rightarrow A) \Rightarrow A}\ (\textsc{fix})$$

Évaluation:

$$\frac{t \longrightarrow\!\!\!\!\rightarrow \texttt{fun}\ \ x\ \ \rightarrow\ \ t' \qquad t'[x \mapsto \texttt{fix}\ (\texttt{fun}\ \ x\ \ \rightarrow\ \ t')] \longrightarrow\!\!\!\!\rightarrow v}{\texttt{fix}\ t \longrightarrow\!\!\!\!\rightarrow v}$$

Réduction:

$$\frac{t \longrightarrow t'}{\texttt{fix}\ t \longrightarrow \texttt{fix}\ t'} \qquad \frac{}{\texttt{fix}\ (\texttt{fun}\ \ x\ \ \rightarrow\ \ t) \longrightarrow t[x \mapsto \texttt{fix}\ (\texttt{fun}\ \ x\ \ \rightarrow\ \ t)]}$$

Figure 8: Point fixe en mini-ML.