A Local View on Innocence

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Workshop in Roma

April 24th 2009

Congratulations Paolo!

Part I

Game Semantics

Denotational semantics

What are those programs/proofs doing?

Denotational semantics

What are those **concurrent** programs/proofs doing?

Game semantics

An *interactive trace semantics*:

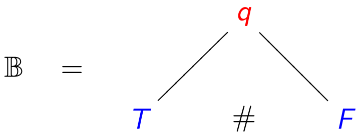
- types are interpreted by **games**
- programs are interpreted by **strategies**

A **game**

$$(M, \leq, \#, \lambda)$$

consists of

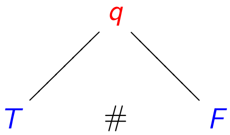
- a set of **moves** M
- a partial order $\leq$ expressing **causal dependencies**
- a symmetric relation $\#$ expressing **incompatibilities**
- a **polarization** of the moves $\lambda : M \to \{O, P\}$



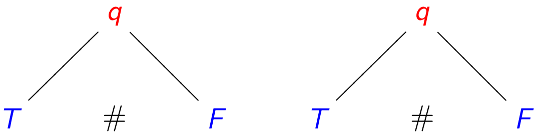
Strategies

- A **play** is a sequence of moves respecting the dependencies and the incompatibility.
- A **strategy** is a set of plays.

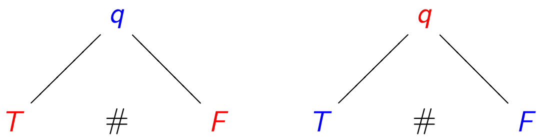
The game $\mathbb{B}$:



The game $\mathbb{B} \otimes \mathbb{B}$:



The game $\mathbb{B} \multimap \mathbb{B}$:



The strategy `not`

$$\mathbb{B} \quad \multimap \quad \mathbb{B}$$

The strategy `not`

$$\mathbb{B} \quad \multimap \quad \mathbb{B}$$

$\color{blue}{q}$

$\color{red}{q}$

The strategy `not`

$$\mathbb{B} \quad \multimap \quad \mathbb{B}$$

		q
q		
T		
		F

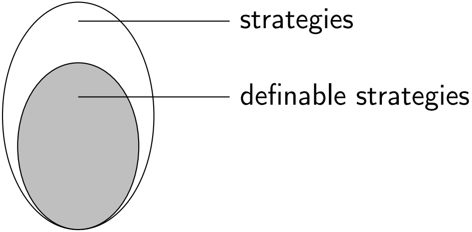
The strategy `not`

$\mathbb{B}$	$\multimap$	$\mathbb{B}$
		q
q		
F		
		T

We get a model (of PCF / MLL / . . .),
but it's not very informative (yet).

Definable strategies

We have to characterize **definable** strategies
(= strategies which are the interpretation of a proof)



Definable strategies

We have to characterize **definable** strategies
(= strategies which are the interpretation of a proof)

Two series of work laid the foundations of game semantics:

- fully abstract models of PCF [HON,AJM]
- fully complete models of MLL [AJ,HO]

extended later on (references, control, non-determinism, ...)

Definable strategies

We have to characterize **definable** strategies
(= strategies which are the interpretation of a proof)

In these models, definable strategies are characterized using conditions such as **innocence**, bracketing, ...

innocent strategy $\approx$ strategy behaving like a λ-term

We want to:

- build a model of MLL adapted to concurrency
- reformulate innocence in this model using local principles
- extend innocence to get a fully complete model of MLL
- define a unifying framework: recover preexisting models

Part II

Innocence

In the framework of HO games:

- plays are alternating (and start with an Opponent move)
- strategies contain only even-length plays
- plays are pointed
- strategies σ are deterministic:

$$s \cdot m \in \sigma \quad \text{and} \quad s \cdot n \in \sigma \quad \text{implies} \quad m = n$$

Innocence

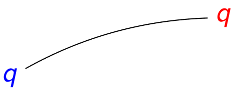
A strategy σ is **innocent** when it reacts only according to its view of the play:

$$s \cdot m \in \sigma \quad \text{and} \quad \ulcorner s \urcorner = \ulcorner t \urcorner \quad \text{implies} \quad t \cdot m \in \sigma$$

Here, $\ulcorner s \urcorner \sqsubseteq s$ is the **view** of the play s.

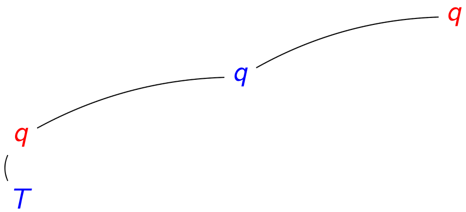
$\texttt{fun}\ f \rightarrow \cdots f \quad \cdots$

$(\mathbb{B} \quad \Rightarrow \quad \mathbb{B}) \quad \Rightarrow \quad \mathbb{B}$



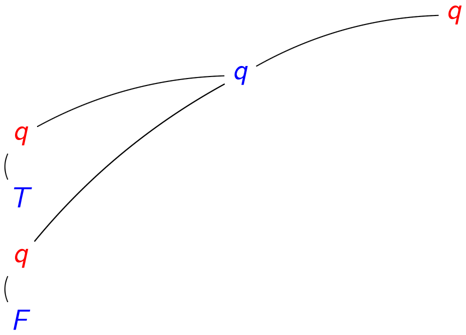
$$\texttt{fun } f \rightarrow \cdots f \texttt{ true} \cdots$$

$$(\mathbb{B} \quad \Rightarrow \quad \mathbb{B}) \quad \Rightarrow \quad \mathbb{B}$$



$$\texttt{fun}\ f \rightarrow \cdots f\ \texttt{?} \cdots$$

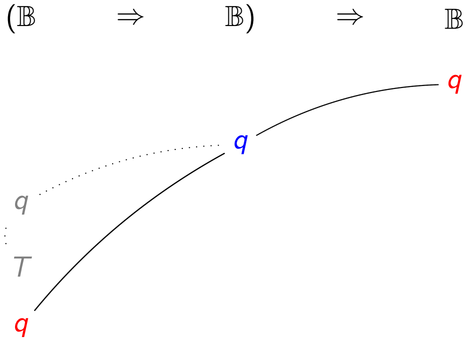
$$(\mathbb{B} \Rightarrow \mathbb{B}) \Rightarrow \mathbb{B}$$



Innocence

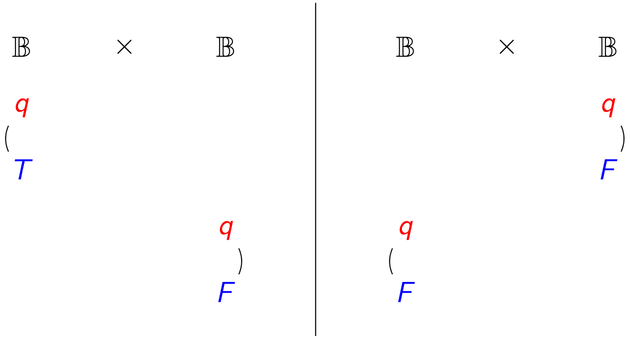
1st counter-example: uniformity

$$\texttt{fun}\ f \rightarrow \cdots f\ \texttt{?} \cdots$$



Innocence

2^{nd} counter example: history-freeness



Decomposing innocence

$$A \Rightarrow B \quad = \quad !A \multimap B$$

A linear decomposition of innocence:

- $!A$ is many copies of A which are handled uniformly

$$!A = \bigotimes_{\omega} A / \approx$$

- **innocent strategies on $A \multimap B$ are history-free**

Part III

Asynchronous games

Implementations of conjunction

	$\mathbb{B}$	$\otimes$	$\mathbb{B}$	$\multimap$	$\mathbb{B}$
					q
	q_L				
left conjunction	T_L				
			q_R		
			F_R		
					F

Implementations of conjunction

	$\mathbb{B}$	$\otimes$	$\mathbb{B}$	$\multimap$	$\mathbb{B}$
					q
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right conjunction			F_R		
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Implementations of conjunction

$$\mathbb{B} \otimes \mathbb{B} \multimap \mathbb{B}$$

q

q_R

parallel conjunction q_L

F_R

T_L

F

Implementations of conjunction

$$\mathbb{B} \quad \otimes \quad \mathbb{B} \quad \multimap \quad \mathbb{B}$$

q

q_L

parallel conjunction $\quad q_R$

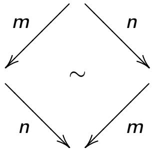
T_L

F_R

F

partial order vs asynchronous graphs

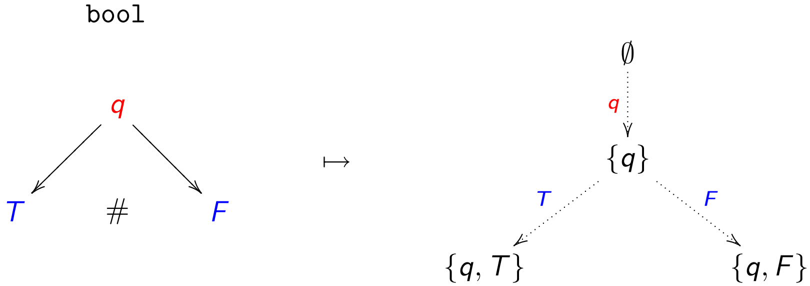
$m \quad n$ vs



Definition

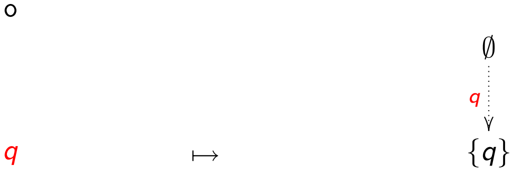
asynchronous graph = graph + homotopy tiles

The asynchronous graph of a game



- **Position**: downward-closed set of compatible moves.
- **Play**: path from the initial position $\emptyset$.

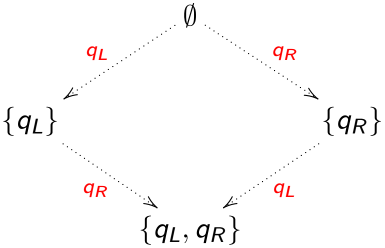
The asynchronous graph of a game



The asynchronous graph of a game

$o \otimes o$

q_L $\qquad$ q_R $\qquad \mapsto$



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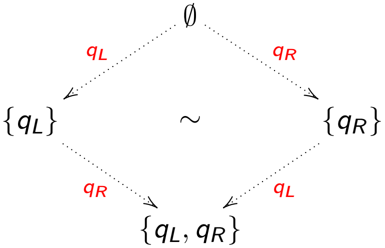
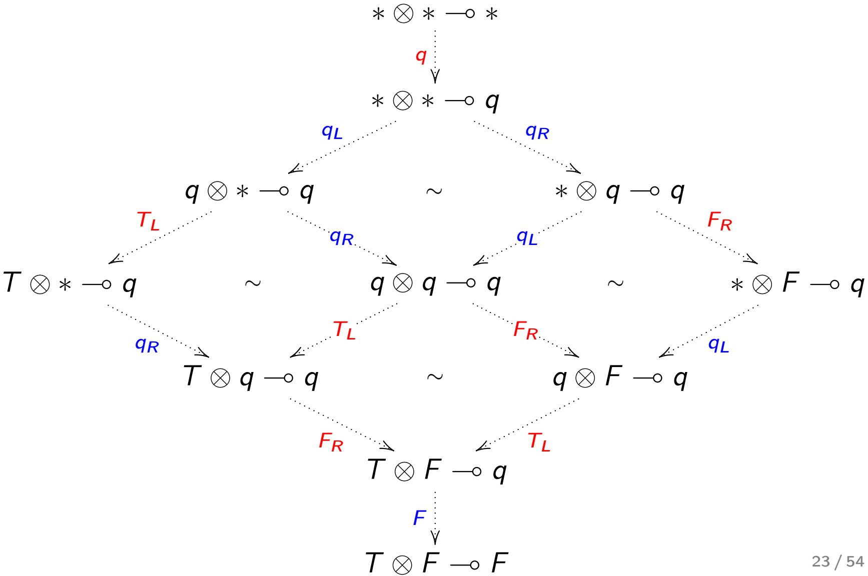
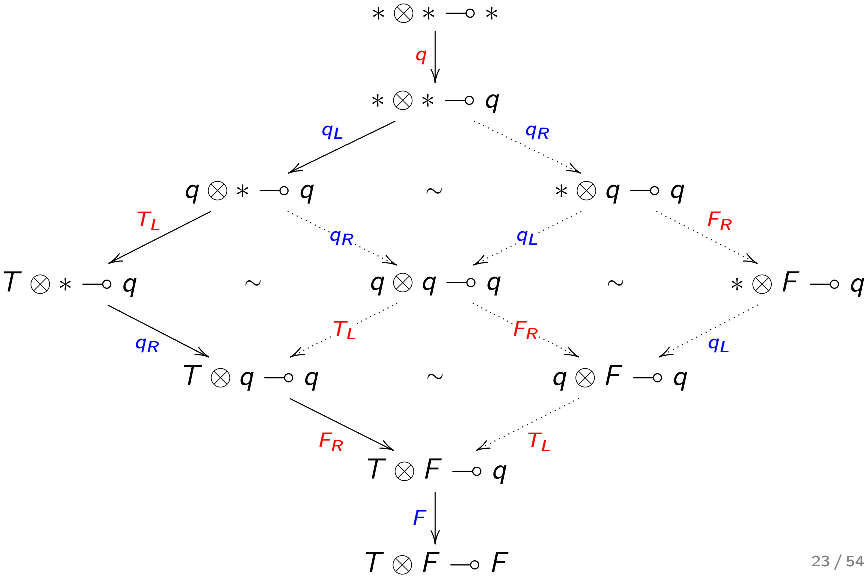


Strategies of conjunction

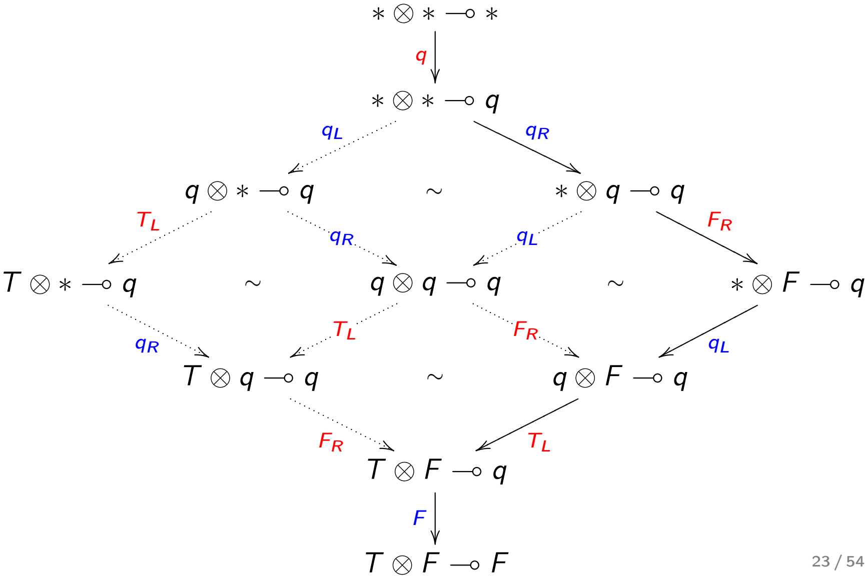
The game `bool` $\otimes$ `bool` $\multimap$ `bool` contains eight subgraphs:



Left implementation of conjunction:

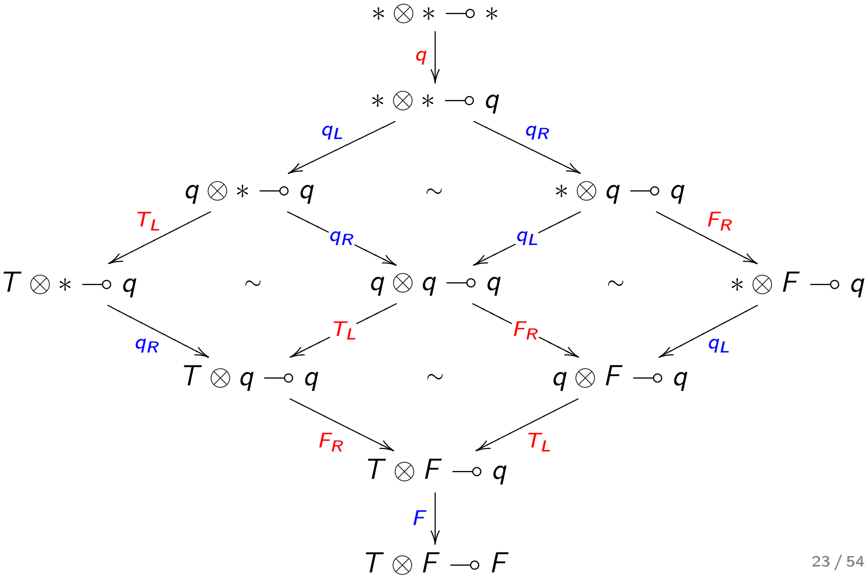


Right implementation of conjunction:



Strategies of conjunction

Parallel implementation of conjunction:



We want to be non-alternating!

- We consider here MALL formulas:

$$\frac{\vdash \Gamma, A, B}{\vdash \Gamma, A \parr B}(\parr) \qquad \frac{\vdash \Gamma_1, A \qquad \vdash \Gamma_2, B}{\vdash \Gamma_1, \Gamma_2, A \otimes B}(\otimes)$$

$$\frac{\vdash \Gamma, A \qquad \vdash \Gamma, B}{\vdash \Gamma, A \,\&\, B}(\&) \qquad \frac{\vdash \Gamma, A}{\vdash \Gamma, A \oplus B}(\oplus)$$

MALL with lifts

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- with explicit lifts:

$$\frac{\vdash \Gamma, A}{\vdash \Gamma, \forall x.A}(\uparrow) \qquad \frac{\vdash [t/x]\Gamma, A}{\vdash \Gamma, \exists x.A}(\downarrow)$$

MALL with lifts

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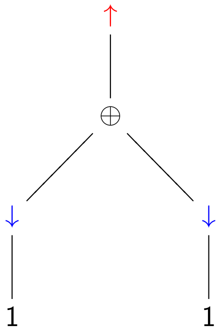
- with explicit lifts:

$$\frac{\vdash \Gamma, A}{\vdash \Gamma, {\uparrow} A}(\uparrow) \qquad \frac{\vdash \Gamma, A}{\vdash \Gamma, {\downarrow} A}(\downarrow)$$

From formulas to games

The formula corresponding to booleans is

$$\mathbb{B} \quad = \quad \uparrow(\downarrow 1 \oplus \downarrow 1)$$



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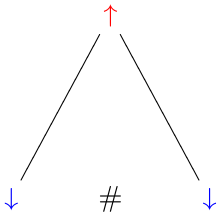

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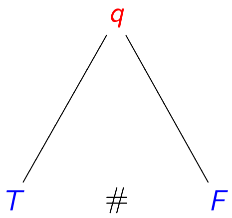

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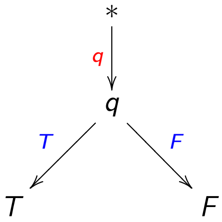

From proofs to strategies

The game associated to $\uparrow A$ is of the form

From proofs to strategies

The game associated to $\uparrow A \otimes \uparrow B = \uparrow A \parr \uparrow B$ is of the form

From proofs to strategies

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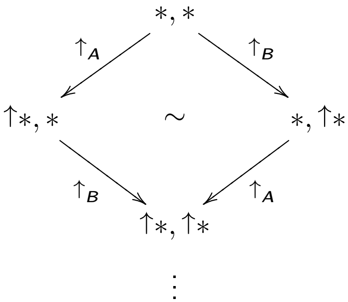
The corresponding asynchronous graph contains



From proofs to strategies

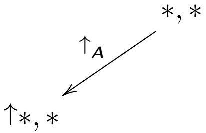
Three proofs of $\uparrow A \parr \uparrow B$:

$$*, *$$

$$\overline{\vdash \uparrow A, \uparrow B}$$

Three proofs of $\uparrow A \parr \uparrow B$:

$$\dfrac{\overline{\vdash A, \uparrow B}}{\vdash \uparrow A, \uparrow B}(\uparrow)$$



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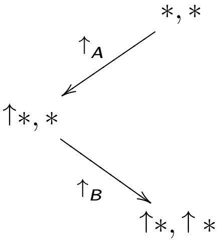

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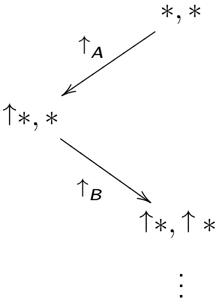
$$\dfrac{\dfrac{\dfrac{\vdots}{\vdash A, B}}{\vdash A, \uparrow B}(\uparrow)}{\vdash \uparrow A, \uparrow B}(\uparrow)$$



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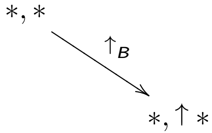

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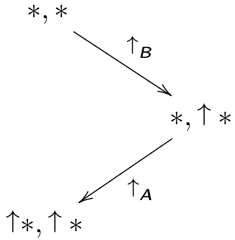

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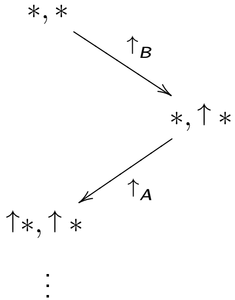

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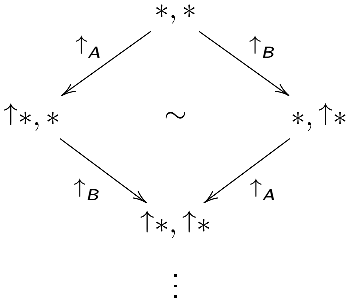

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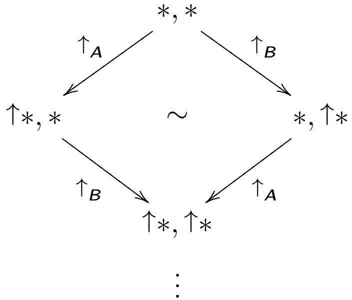
$$\dfrac{\dfrac{\vdots}{\vdash A, B}}{\vdash \forall x.A, \forall y.B}(\uparrow, \uparrow)$$

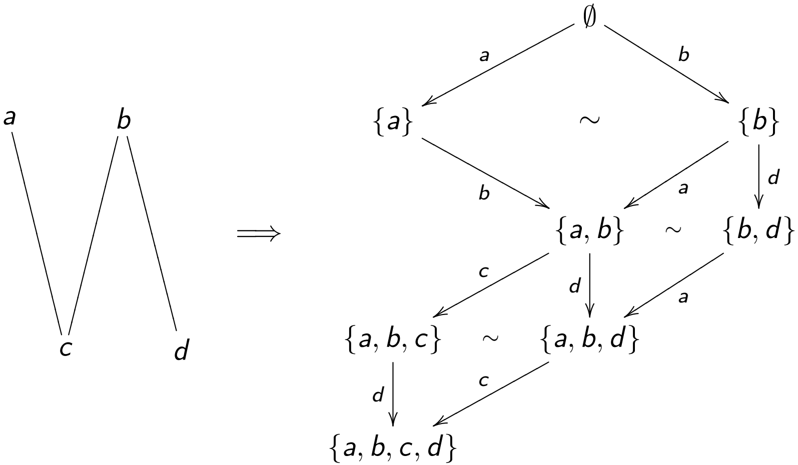


Proofs explore formulas

play	$=$	exploration of the formula
proof	$=$	exploration strategy

From sequentiality to causality

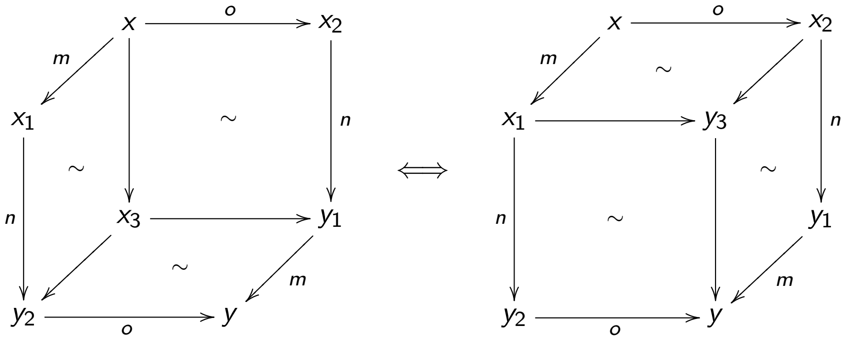
A game induces an asynchronous graph:



From sequentiality to causality

Conversely, one needs the Cube Property

The Cube Property



Theorem

Homotopy classes of paths are generated by a partial order on moves.

Asynchronous games

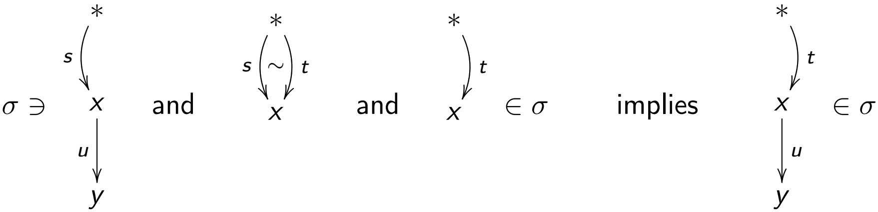
Definition
An **asynchronous game** is a pointed asynchronous graph satisfying the Cube Property.

Definition
A **strategy** $\sigma : A$ is a prefix closed set of plays on the asynchronous graph A.

How do we characterize “good” strategies?

Definition

A strategy σ is **positional** when its plays form a subgraph of the game:



Ingenuous strategies

We consider strategies which

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Ingenuous strategies

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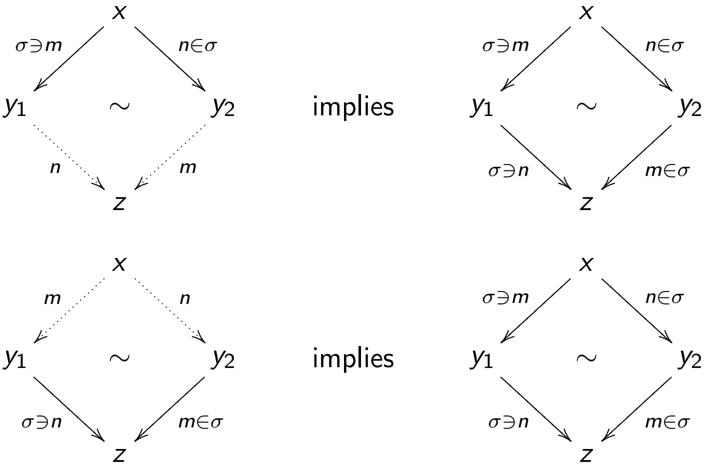
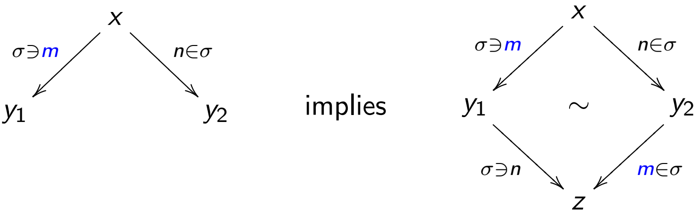


Ingenuous strategies

We consider strategies which

1. are **positional**,
2. satisfy the **Cube Property**,
3. satisfy . . .
4. are **deterministic**:



where m is a Proponent move.

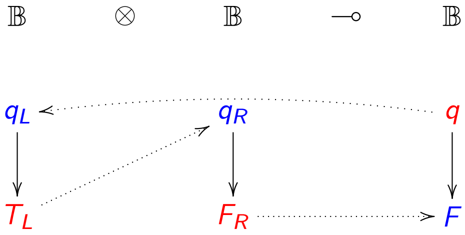
Ingenuous strategies

The game $\mathbb{B} \otimes \mathbb{B} \multimap \mathbb{B}$:

$\mathbb{B}$	$\otimes$	$\mathbb{B}$	$\multimap$	$\mathbb{B}$
q_L		q_R		q
$\downarrow$		$\downarrow$		$\downarrow$
T_L		F_R		F

Ingenuous strategies

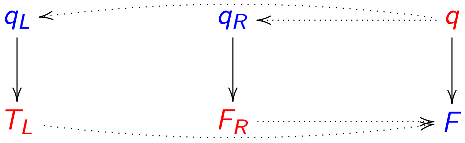
The left conjunction



Ingenuous strategies

The parallel conjunction

$$\mathbb{B} \quad \otimes \quad \mathbb{B} \quad \multimap \quad \mathbb{B}$$



A model of MLL

Property

Asynchronous games and strategies form a $*$-autonomous category (which is compact closed).

This category still has “too many” strategies!

$$A \otimes B \quad = \quad A \parr B$$

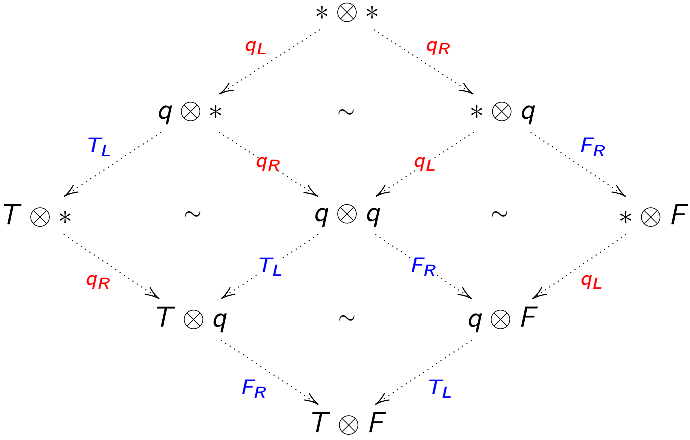
Halting positions

In the spirit of the relational model, a strategy σ should be characterized by its set σ° of halting positions.

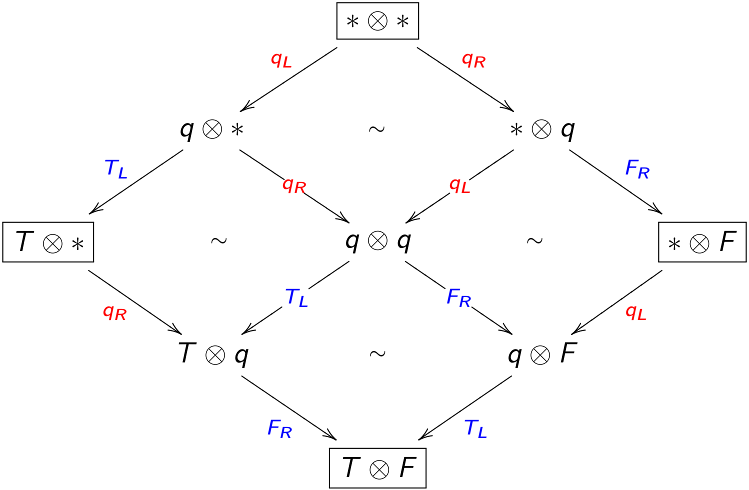
Definition

A **halting position** of a strategy σ is a position x such that there is no Player move $m : x \longrightarrow y$ that σ can play.

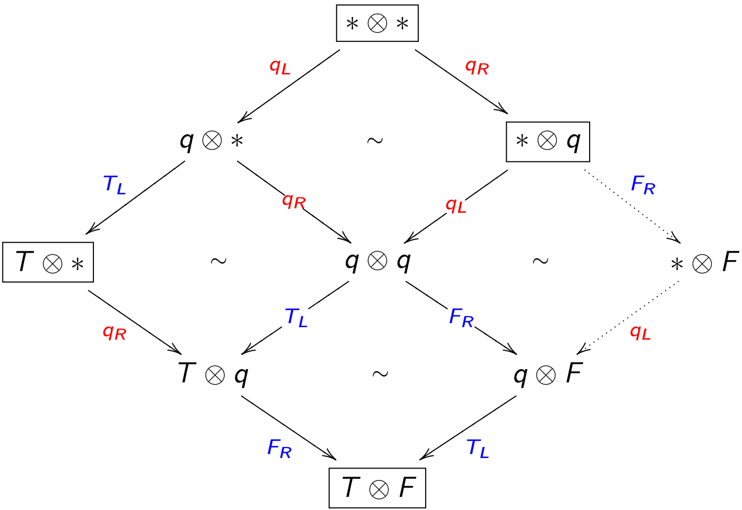
The game bool $\otimes$ bool contains the subgraph:



The pair true ⊗ false:

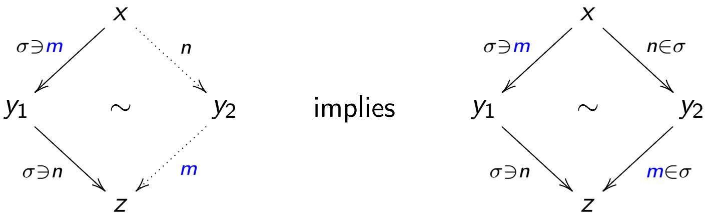


The left biased pair `true` $\oslash$ `false`:



Definition

An ingenuous strategy σ is **courteous** when it satisfies



where m is a Player move.

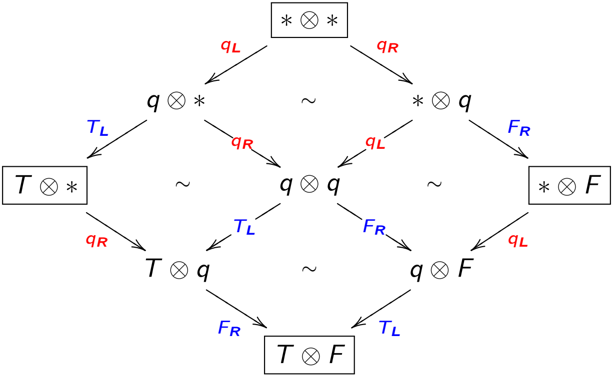
Theorem

A courteous ingenuous strategy σ is characterized by its set σ° of halting positions.

Concurrent strategies

The halting positions of such a strategy $\sigma : A$ are precisely the fixpoints of a **closure operator** on the positions of A.

- We thus recover the model of **concurrent strategies**.
- A semantical counterpart of the **focalization** property: strategies can play all their Player moves in one "cluster" of moves.

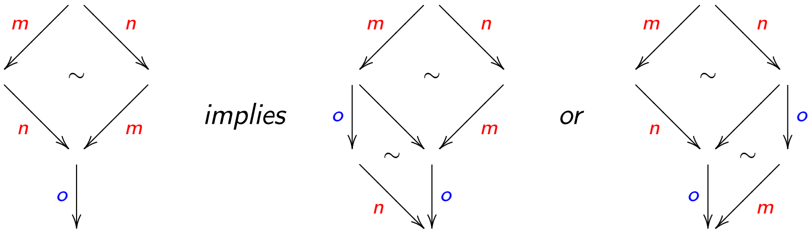


Innocence

Theorem

An innocent strategy $\sigma : A$ is a strategy which is

1. *ingenuous*
2. *courteous*
3. *receptive: if $s \in \sigma$ and $s \cdot m$ is a play then $s \cdot m \in \sigma$*
4. *sequential*

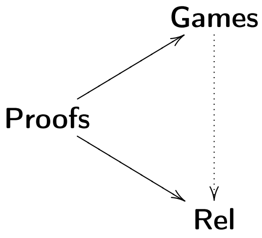


+ dual property

Sequentiality, which is an asynchronous counterpart of alternation, schedules composition and ensures that it will perform correctly.

Without sequentiality...

The operation $(-)^{\circ}$ from the category of games
and courteous ingenuous strategies
to the category of relations is not functorial!

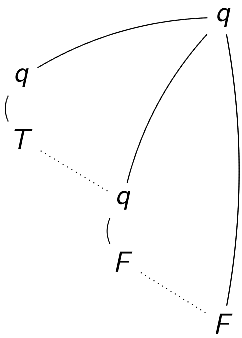


This mismatch is essentially due to **deadlock** situations occurring during the interaction.

The scheduling criterion

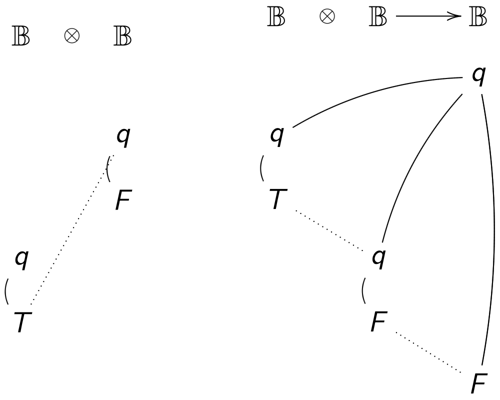
the left conjunction:

$$\mathbb{B} \quad \otimes \quad \mathbb{B} \longrightarrow \mathbb{B}$$



The scheduling criterion

The right boolean composed with the left conjunction:



The scheduling criterion

Two kinds of tensors: $\otimes$ and $\parr$.

$$\mathbb{B} \otimes \mathbb{B} \multimap \mathbb{B} \quad = \quad \mathbb{B}^* \parr \mathbb{B}^* \parr \mathbb{B}$$

The scheduling criterion

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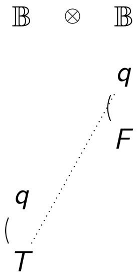
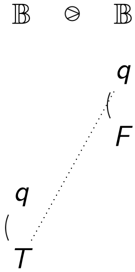


Two kinds of tensors: $\otimes$ and $\invamp$.



The scheduling criterion

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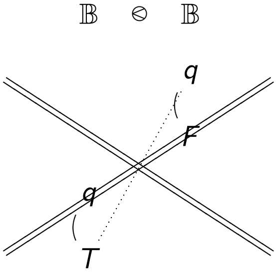

Functoriality

Theorem

Strategies which are

- *ingenuous*
- *courteous*
- *receptive*
- *and satisfy the scheduling criterion*

compose and satisfy

$$(\sigma;\tau)^\circ \quad = \quad \sigma^\circ;\tau^\circ$$

Full completeness

The criterion only detects *oriented cycles*.

$$(A^* \otimes B^*) \parr (B \otimes A)$$

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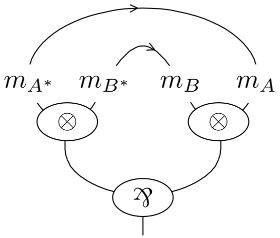

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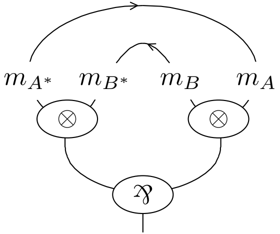
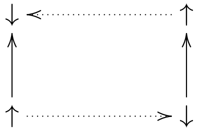


Full completeness for MLL + MIX

If we implement atoms by

Full completeness for MLL + MIX

If we implement axioms by



we get a fully complete model of MLL + MIX, similar to the AJ model.

Conclusion

We have:

- a game semantics adapted to concurrency
- an unifying framework in which we recover
  - innocent strategies
  - game semantics
  - concurrent games
  - the relational model
  - event structure semantics

In the future:

- extend this model (exponentials in particular)
- a local presentation of the correctness criterion
- typing of concurrent processes (CCS without deadlocks)

Thanks!