Provably secure non interactive key distribution based on pairings

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1. Key agreement and identity based cryptosystems
2. Pairings and elliptic curves
3. Non interactive key exchange
4. Security proof

Key agreement and identity based cryptosystems

Key agreement issues

Diffie–Hellman (1976):

solves the key agreement/distribution problem

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What if Alice and Bob do not want extra communication?

Identity based cryptosystems (Shamir, 1984)

Alice

Public key: $A = H(\text{"Alice"})$

Bob

Public key: $B = H(\text{"Bob"})$

(H is a public hash function)

Identity based cryptosystems (Shamir, 1984)

Private Key Generator

Master key s (secret)

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Public key: $A = H(\text{"Alice"})$

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Master key s (secret)

$K_A = f(A, s)$

$K_B = f(B, s)$

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Public key: $A = H(\text{"Alice"})$

Private key: K_A

Bob

Public key: $B = H(\text{"Bob"})$

Private key: K_B

(H is a public hash function)

Previous work

Identity based cryptography:

- Shamir, 1984

Identity based cryptosystems:

- Maurer–Yacobi, 1992
- Kügler, 1998
- Cocks, 2001

Pairings and elliptic curves

Pairings

$(G,+)$, $(G',+)$, $(V,\times)$ groups of prime order ℓ

$$e : G \times G' \to V$$

Needed properties:

- bilinearity: $e(aP, bQ) = e(P,Q)^{ab}$
- non-degeneracy: $\exists P, Q$ s.t. $e(P,Q) \neq 1_V$
- computability

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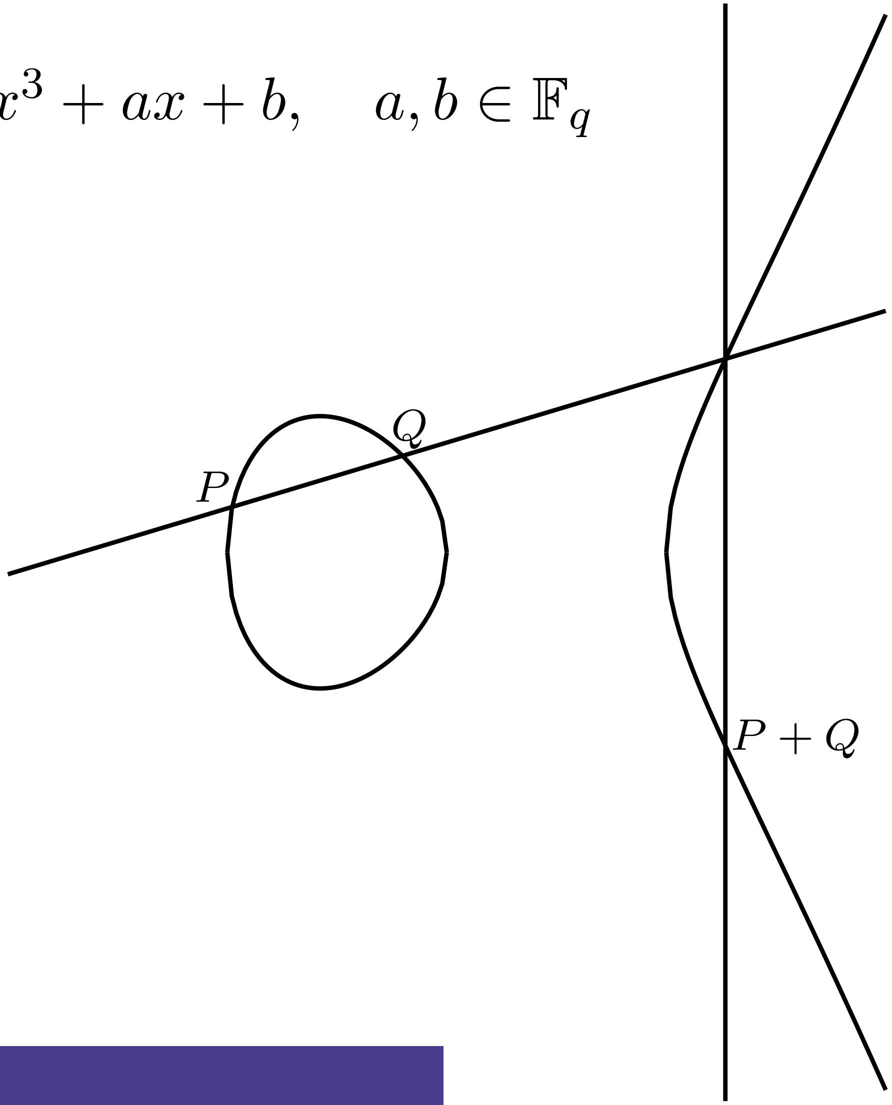
$$\left.\begin{array}{r} H : \{0,1\}^* \to G \\ H' : \{0,1\}^* \to G' \end{array}\right\} \text{hash functions}$$

Elliptic curves

$$E : y^2 = x^3 + ax + b, \quad a, b \in \mathbb{F}_q$$



Weil and Tate pairings on elliptic curves

ℓ prime, $\ell \mid \#E$ and $\ell \mid q^k - 1$,

$\Rightarrow \exists P \in E(\mathbb{F}_q), Q \in E(\mathbb{F}_{q^k})$ that generate $E[\ell]$

Weil and Tate pairings:

$$e : \langle P \rangle \times \langle Q \rangle \to \mu_\ell \subseteq \mathbb{F}_{q^k}^\times$$

- bilinear
- non-degenerate
- efficiently computable if k is small

Previous work

Use of pairings in cryptosystems:

- Cryptosystems based on pairings (Sakaï–Ohgishi–Kasahara, 2000)
- Tripartite Diffie–Hellman (Joux, 2000)
- Identity based encryption (Boneh–Franklin, 2001)

Non interactive key exchange

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Alice

$P_A = H(\text{“Alice”})$

$Q_A = H'(\text{“Alice”})$

Bob

$P_B = H(\text{“Bob”})$

$Q_B = H'(\text{“Bob”})$

Non interactive key exchange

Private Key Generator

Master key $s \in_R [0, \ell - 1]$

sP_A, sQ_A

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$e(sP_B, Q_A)$

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$$e(P_B, sQ_A) = e(P_B, Q_A)^s = e(sP_B, Q_A)$$

Security proof

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- DLP in G, G', V hard (otherwise master key s)
- Generalized bilinear Diffie–Hellman problem (GBDH)

  given P, Q, aP, bQ, cP, cQ, compute $e(P,Q)^{abc}$

  (otherwise shared keys)

  $\Rightarrow$ the pairing inversion problem

  given P and $e(P,Q)$, compute Q

  should be hard

Attack scenario (computational version)

- System parameters are known.
- Private key queries (adaptive):
  Attacker $\mathcal{C}$ chooses identities ID_i.
  $\mathcal{C}$ receives private keys $sH(\mathrm{ID}_i)$, $sH'(\mathrm{ID}_i)$.
- Guess:
  $\mathcal{C}$ chooses two identities $\notin \{\mathrm{ID}_i\}$
  and outputs their shared key.

Equivalence of GBDH and security

random oracle model

If an attacker $\mathcal{C}$ is able to forge a valid guess
with N private key queries, in time t and
with probability ε,

then GBDH problem can be solved
in time $t' = \mathrm{poly}(t)$ and
with probability $O(\varepsilon/(1+N)^2)$.

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random oracle model

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We want to design an algorithm $\mathcal{G}$ that obtains

$$e(P,Q)^{abc} \text{ from } (P,Q,aP,bQ,cP,cQ)$$

using the attacker $\mathcal{C}$

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- $\mathcal{G}$ plays the role of the PKG for $\mathcal{C}$ (simulates a master key c)

Solving GBDH from the attacker $\mathcal{C}$

- Evaluation of H and H' at ID
  - $\mathcal{G}$ picks $k, k' \in_R [0, \ell - 1]$ and sets
  - $H(\mathrm{ID}) = kP$ (prob. δ) or $H(\mathrm{ID}) = kaP$ (prob. $1 - \delta$)
  - $H'(\mathrm{ID}) = k'Q$ (prob. δ) or $H'(\mathrm{ID}) = k'bQ$ (prob. $1 - \delta$)

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- Private key queries:

  when $\mathcal{C}$ asks for the private key for ID
  - if $H(\mathrm{ID}) = kP$ and $H'(\mathrm{ID}) = k'Q$ (prob. δ^2),

    $\mathcal{G}$ returns $(kcP, k'cQ) = (cH(\mathrm{ID}), cH'(\mathrm{ID}))$
  - otherwise $\mathcal{G}$ fails

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Solving GBDH from the attacker $\mathcal{C}$

- if $\mathcal{C}$ outputs a correct guess (prob. $\geq \varepsilon$)

$$\mathrm{ID}_1,\ \mathrm{ID}_2,\ e(H(\mathrm{ID}_1), H'(\mathrm{ID}_2))^c$$

  - if $H(\mathrm{ID}_1) = kaP$ and $H'(\mathrm{ID}_2) = k'bQ$ (prob. $(1-\delta)^2$)
    then $e(H(\mathrm{ID}_1), H'(\mathrm{ID}_2))^c = e(P, Q)^{kk'abc}$
    so $\mathcal{G}$ can output $e(P, Q)^{abc}$
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Global success probability: $p \geq ((1-\delta)\delta^N)^2\varepsilon$
Setting $\delta = N/(1+N)$ we get

$$p \geq \frac{\varepsilon}{2(\exp(1)(1+N))^2}$$

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- distribution of the master key and "hierarchization" are possible (Gentry–Silverberg)

Conclusions and outlook

- efficient non interactive key distribution scheme
- security proof:
  reduction to a well identified computational problem (GBDH)
  (random oracle model)
- distribution of the master key and "hierarchization" are possible (Gentry–Silverberg)
- not limited to supersingular curves anymore
  ordinary curves with small k
  (Miyaji–Nakabayashi–Takano, Barreto–Lynn–Scott, D.–Enge–Morain)