

Generalized context-free grammars over categories and operads

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Ecole Polytechnique (LIX)

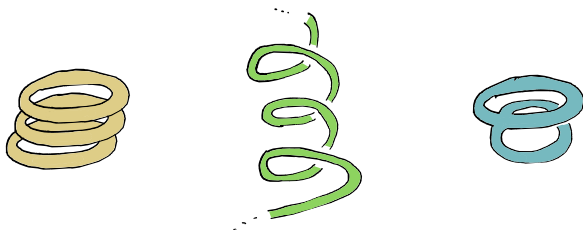
Workshop on Theory and Applications of Sheaf Theory
ESSLLI 2025, Bochum, 6 August 2025

¹Based on joint work with Paul-André Melliès. Main reference: *The categorical contours of the Chomsky-Schützenberger representation theorem*, LMCS 21:2, 2025.

0. Thinking fibrationally about deductive systems

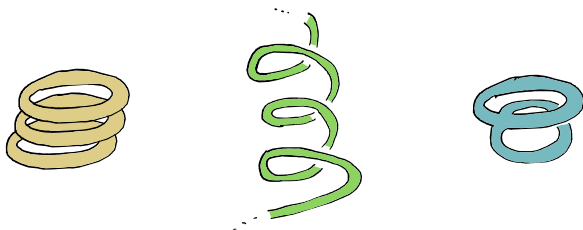
Topological intuitions

Imagine we are analyzing different spaces, more or less complicated.



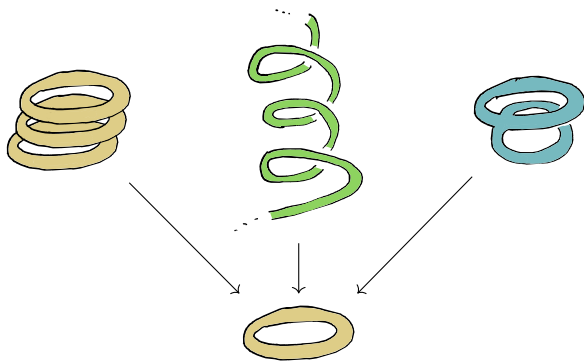
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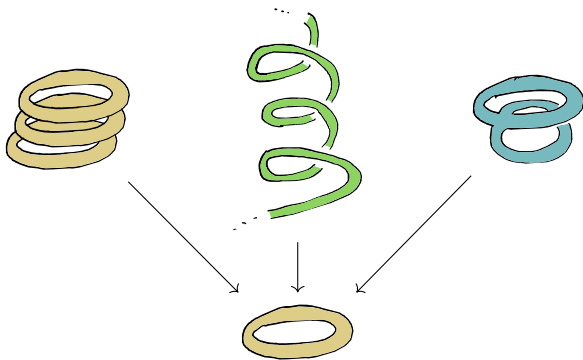
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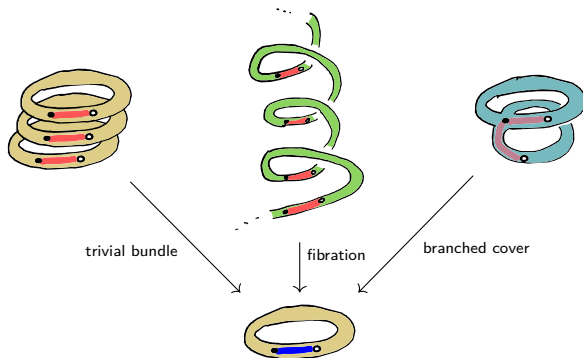
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Fibrational perspectives on deductive systems

Bill Lawvere (late 1960s): the structure of predicate logic and the nature of quantifiers may be clarified by organizing proofs into bundles lying over spaces of formulas.

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This idea may be extended to a variety of computational and deductive systems, including program logics, finite-state automata and context-free grammars.

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Proof, recognition, and parsing may be thus reduced to lifting problems.

Functors as type refinement systems: some terminology and notation

We like to think of a category $\mathcal{D}$ equipped with a functor $p : \mathcal{D} \rightarrow \mathcal{C}$ as defining a kind of abstract type system, or “type refinement system”. The idea is that the bundle $p : \mathcal{D} \rightarrow \mathcal{C}$ provides a source of *additional typing information* for the arrows of $\mathcal{C}$.

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A **typing judgment** is a triple (S, f, T) of an arrow $f : A \rightarrow B$ of $\mathcal{C}$ and a pair of objects $S \sqsubset A$ and $T \sqsubset B$ of $\mathcal{D}$ refining its domain and codomain.

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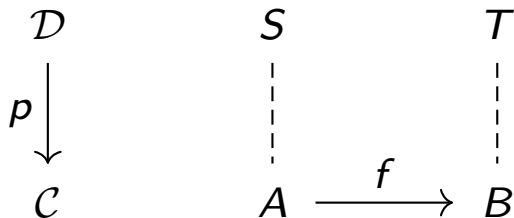
A **derivation** of a typing judgment (S, f, T) is an arrow $\alpha : S \rightarrow T$ of $\mathcal{D}$ s.t. $p(\alpha) = f$.

Functors as type refinement systems: some intuition

$$\begin{array}{ccc} \mathcal{D} & & \\ p \downarrow & & \\ \mathcal{C} & & A \xrightarrow{f} B \end{array}$$

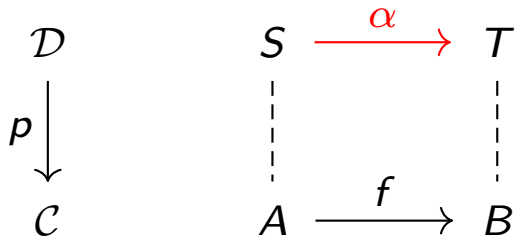
(f a program taking input of type A and producing output of type B)

Functors as type refinement systems: some intuition



(S and T predicates refining A and B)

Functors as type refinement systems: some intuition



(α a proof that f has a more refined type $S \rightarrow T$)

From type systems to languages

Given a functor $p : \mathcal{D} \rightarrow \mathcal{C}$ and objects $S \sqsubset A$ and $T \sqsubset B$, the **language of arrows** induced by (p, S, T) is defined as

$$\mathcal{L}_{p,S,T} := \{ p(\alpha) \mid \alpha : S \rightarrow T \} \subseteq \mathcal{C}(A, B).$$

We may think of $\mathcal{L}_{p,S,T}$ as the set of programs $f : A \rightarrow B$ such that (S, f, T) holds.

Ambiguity

A judgment (S, f, T) may in general have more than one derivation, in other words, a type refinement system may be *ambiguous*.

By definition, $p : \mathcal{D} \rightarrow \mathcal{C}$ is **unambiguous** iff p is faithful, i.e., for any pair of arrows $\alpha, \alpha' : S \rightarrow T$ with the same source and target, if $p(\alpha) = p(\alpha')$ then $\alpha = \alpha'$.

When p is potentially ambiguous, it can be interesting to consider $\mathcal{L}_{p,S,T}$ as a “proof-relevant language”, that is, as a bundle of homsets

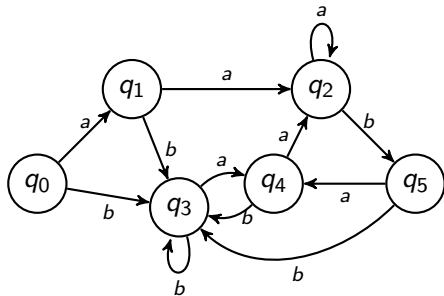
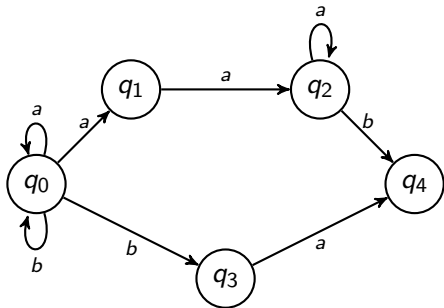
$$\mathcal{L}_{p,S,T} := \mathcal{D}(S, T) \longrightarrow \mathcal{C}(A, B).$$

1. Finite-state automata as functors

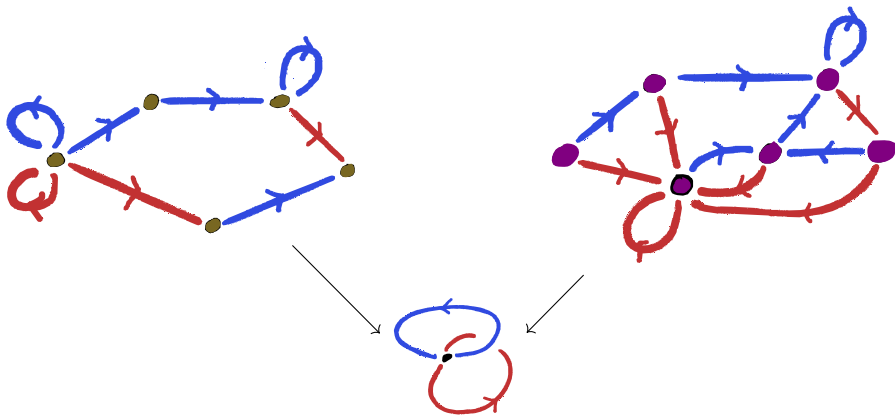
Automata as graph homomorphisms

The underlying transition graph of any NFA (without ϵ -transitions) over the alphabet Σ is entirely described by a graph homomorphism $\phi : \mathbb{G} \rightarrow \mathbb{B}_\Sigma$ into the *bouquet graph* with one node $*$ and a loop $a : * \rightarrow *$ for every $a \in \Sigma$.

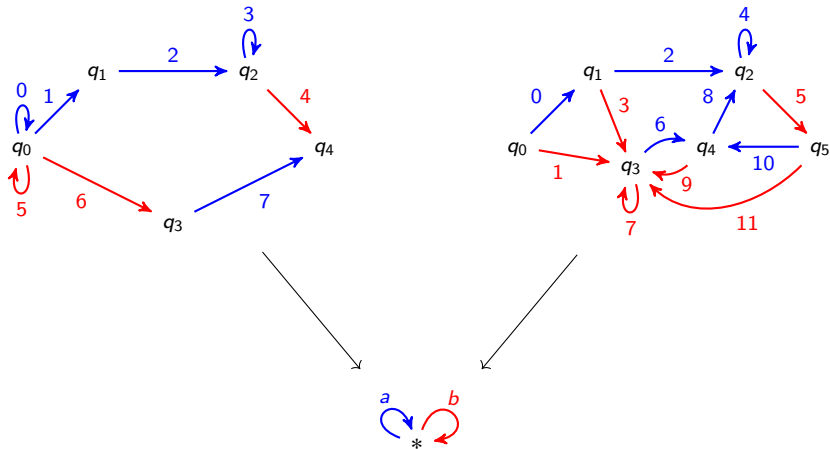
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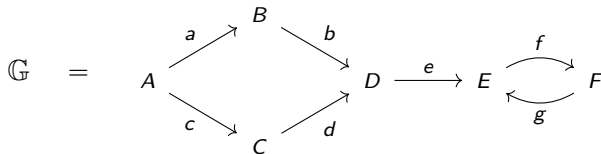


Automata as graph homomorphisms



Free categories

To any graph $\mathbb{G}$ is associated a **free category** $\mathcal{F}\mathbb{G}$ whose objects are nodes and whose arrows are paths. For example, the free category over



has $\text{hom}(A, D) = \{ ab, cd \}$ and $\text{hom}(E, E) = (fg)^*$.

Universal property of free categories: any functor $\mathcal{F}\mathbb{G} \rightarrow \mathcal{C}$ into a category $\mathcal{C}$ is uniquely determined by a graph homomorphism $\mathbb{G} \rightarrow \mathcal{C}$ into the underlying graph of $\mathcal{C}$.

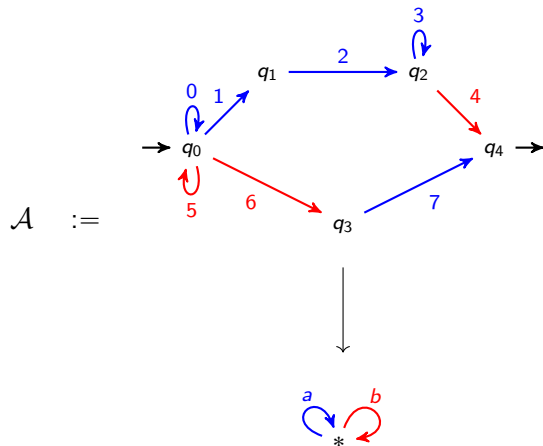
Recognition as a path lifting problem

Any word $w \in \Sigma^*$ corresponds to a *path* in $\mathbb{B}_\Sigma$, i.e., to an arrow $* \rightarrow *$ in $\mathcal{F}\mathbb{B}_\Sigma$.

Any graph homomorphism $\phi : \mathbb{G} \rightarrow \mathbb{H}$ induces a corresponding functor between free categories $\mathcal{F}\phi : \mathcal{F}\mathbb{G} \rightarrow \mathcal{F}\mathbb{H}$, sending paths in $\mathbb{G}$ to paths in $\mathbb{H}$ of the same length.

Let $\mathcal{A}$ be an NFA with transition graph $\phi : \mathbb{G} \rightarrow \mathbb{B}_\Sigma$ and associated functor $p = \mathcal{F}\phi$. Then $\mathcal{A}$ accepts $w \in \Sigma^*$ just in case there is an arrow $\alpha : q_0 \rightarrow q_f$ in $\mathcal{F}\mathbb{G}$ such that $p(\alpha) = w$, from an initial state q_0 to an accepting state q_f .

Recognition as a path lifting problem

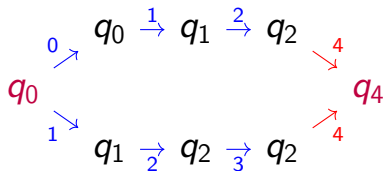
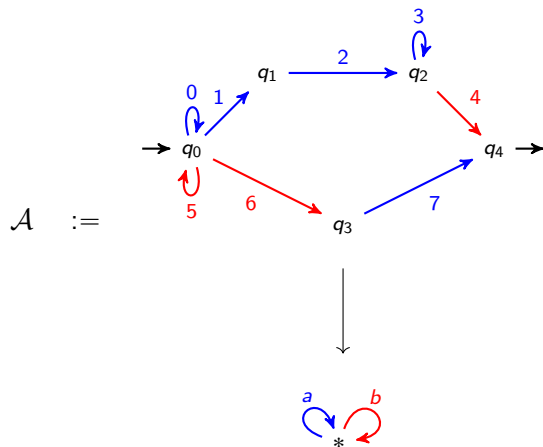


q_0

q_4

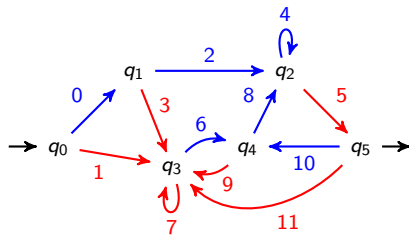


Recognition as a path lifting problem



Recognition as a path lifting problem

$\mathcal{A} :=$

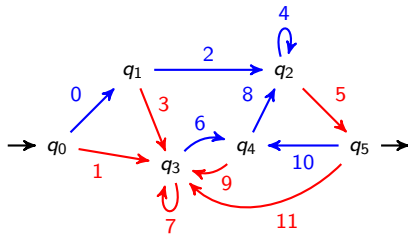


q_0



Recognition as a path lifting problem

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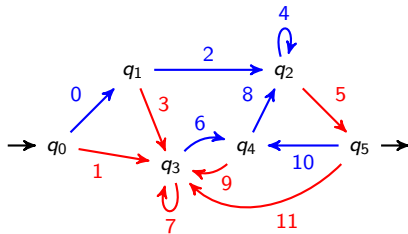
$$q_0 \xrightarrow{0} q_1$$



$$* \xrightarrow{a} * \xrightarrow{a} * \xrightarrow{a} * \xrightarrow{b} *$$

Recognition as a path lifting problem

$\mathcal{A} :=$

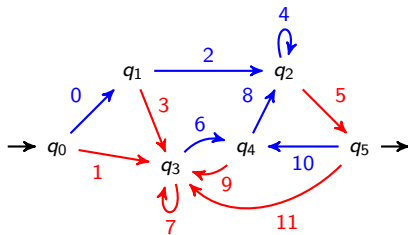


$$q_0 \xrightarrow{0} q_1 \xrightarrow{2} q_2$$

$$* \xrightarrow{a} * \xrightarrow{a} * \xrightarrow{a} * \xrightarrow{b} *$$

Recognition as a path lifting problem

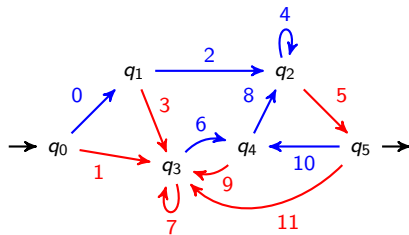
$\mathcal{A} :=$



$$q_0 \xrightarrow{0} q_1 \xrightarrow{2} q_2 \xrightarrow{4} q_2$$

$$* \xrightarrow{a} * \xrightarrow{a} * \xrightarrow{a} * \xrightarrow{b} *$$

Recognition as a path lifting problem



$\mathcal{A} :=$



$q_0 \xrightarrow{0} q_1 \xrightarrow{2} q_2 \xrightarrow{4} q_2 \xrightarrow{5} q_5$

$* \xrightarrow{a} * \xrightarrow{a} * \xrightarrow{a} * \xrightarrow{b} *$

Fibrational properties

A homomorphism of finite graphs $\phi : \mathbb{G} \rightarrow \mathbb{B}_\Sigma$ represents the transition graph of a *complete DFA* just in case the functor $\mathcal{F}\phi : \mathcal{F}\mathbb{G} \rightarrow \mathcal{F}\mathbb{B}_\Sigma$ is a *discrete opfibration*. Such functors are in 1-to-1 correspondence with covariant presheaves $\mathcal{F}\mathbb{B}_\Sigma \rightarrow \mathbf{FinSet}$.

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More generally, a functor $p : \mathcal{D} \rightarrow \mathcal{F}\mathbb{B}_\Sigma$ represents the transition graph of an NFA just in case it has *finite fibers* and *unique lifting of factorizations*.

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More generally, a functor $p : \mathcal{D} \rightarrow \mathcal{F}\mathbb{B}_\Sigma$ represents the transition graph of an NFA just in case it has *finite fibers* and *unique lifting of factorizations*. Such functors are in 1-to-1 correspondence with covariant presheaves valued in the *bicategory of spans of finite sets*, that is, (pseudo)functors $\mathcal{F}\mathbb{B}_\Sigma \rightarrow \mathbf{FinSpan}$.

2. Context-free grammars as functors

From categories to operads

An operad (aka multicategory) is like a category where arrows can take multiple objects as input.

$$f : A_1, \dots, A_n \rightarrow B \quad (n \geq 0)$$

(We work with *planar* operads, which have a linear order on inputs, and no exchange.)

Any functor of operads $p : \mathcal{D} \rightarrow \mathcal{O}$ equipped with an object $S \sqsubset A$ induces a **language of constants** defined by

$$\mathcal{L}_{p,S} := \{ p(\alpha) \mid \alpha : S \} \subseteq \mathcal{O}(\cdot; A).$$

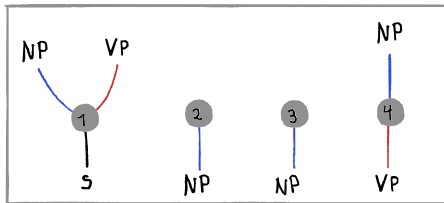
$S \rightarrow NP VP$

$NP \rightarrow \text{linguists}$

$NP \rightarrow \text{sheaves}$

$VP \rightarrow \text{love } NP$

Parsing as a lifting problem



1: $S \rightarrow NP VP$
2: $NP \rightarrow linguists$
3: $NP \rightarrow sheaves$
4: $VP \rightarrow love NP$

$\varepsilon \sqcup \varepsilon$ linguists sheaves love $\sqcup \varepsilon$

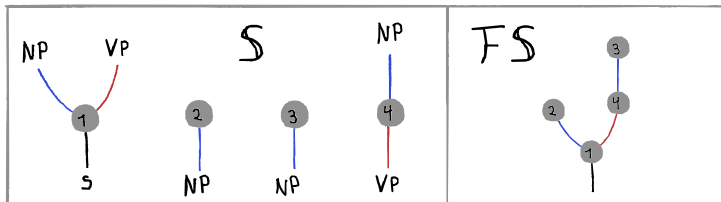
Parsing as a lifting problem

- 1: $S \rightarrow NP VP$
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- 3: $NP \rightarrow sheaves$
- 4: $VP \rightarrow love NP$

FS

P

$W\Sigma$



$\varepsilon \cup \varepsilon$ linguists sheaves love $\cup \varepsilon$

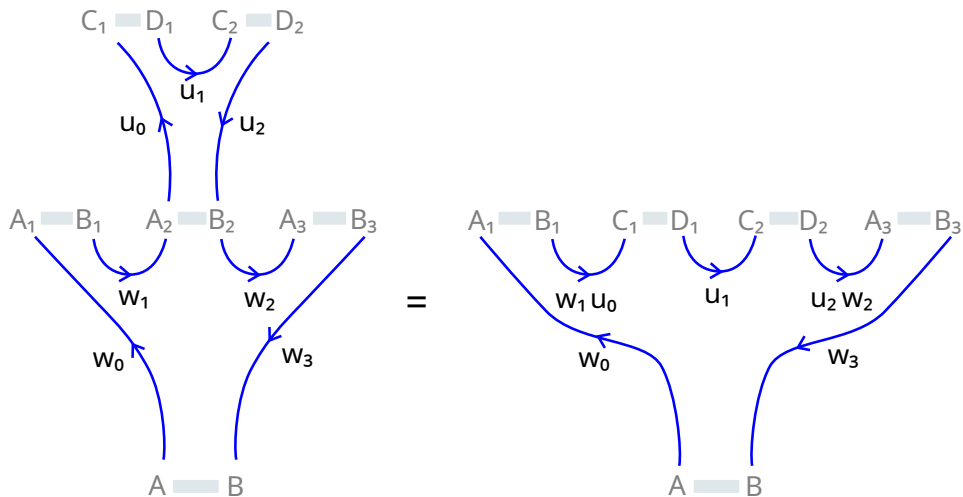
$\varepsilon \cup \varepsilon \circ (\text{linguists}, \text{love} \cup \varepsilon \circ \text{sheaves})$
 $= \text{linguists} \cup \text{love} \cup \text{sheaves}$

The splicing construction

For any category $\mathcal{C}$, the **operad of spliced arrows** $\mathcal{WC}$ is defined as follows:

- ▶ objects are pairs (A, B) of objects of $\mathcal{C}$;
- ▶ n -operations $f : (A_1, B_1), \dots, (A_n, B_n) \rightarrow (A, B)$ given by sequences $f = w_0 - w_1 - \dots - w_n$ of $n + 1$ arrows $w_i : B_i \rightarrow A_{i+1}$ in $\mathcal{C}$ ($B_0 = A, A_{n+1} = B$);
- ▶ composition performed by “splicing into the gaps”...

The splicing construction



$$(w_0 - w_1 - w_2 - w_3) \circ_1 (u_0 - u_1 - u_2) = w_0 - w_1 u_0 - u_1 - u_2 w_2 - w_3$$

Context-free grammar over a category

A **CFG over a category** $\mathcal{C}$ is a tuple $G = (\dot{\mathbb{S}}, p)$ consisting of a pointed finite species $\dot{\mathbb{S}} = (\mathbb{S}, S \in \mathbb{S})$ and a functor $p : \mathcal{FS} \rightarrow \mathcal{WC}$. The context-free language generated by G is the set $\mathcal{L}_G := \mathcal{L}_{p,S} = \{ p(\alpha) \mid \alpha : S \} \subseteq \mathcal{C}(A, B)$ where $p(S) = (A, B)$.

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Example: let $\mathbb{B}_\Sigma^{\hat{\$}} := \perp \xrightarrow{\hat{}} * \xrightarrow{\$} \top$. A CFG over $\mathcal{FB}_\Sigma^{\hat{\$}}$ can have productions that are only applicable at the beginning/end of the string.

3. Generalized NFAs and CFGs over operads

Definitions

Let $\mathcal{O}$ be any operad.

A **NFA over** $\mathcal{O}$ is a tuple $M = (\dot{\mathcal{Q}}, p)$ of a pointed operad $(\mathcal{Q}, q_r \in \mathcal{Q})$ and a functor $p : \mathcal{Q} \rightarrow \mathcal{O}$ satisfying the finite fiber and ULF properties.

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A **CFG over** $\mathcal{O}$ is a tuple $G = (\dot{\mathbb{S}}, p)$ of a pointed finite species $\dot{\mathbb{S}} = (\mathbb{S}, S \in \mathbb{S})$ and an arbitrary functor $p : \mathcal{F}\mathbb{S} \rightarrow \mathcal{O}$.

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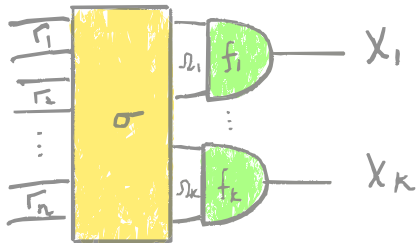
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Example: k -multiple and k -parallel CFGs in the sense of Seki et al. (1991) are CFGs over free semi-cartesian / free cartesian operads. . .

A few constructions on operads

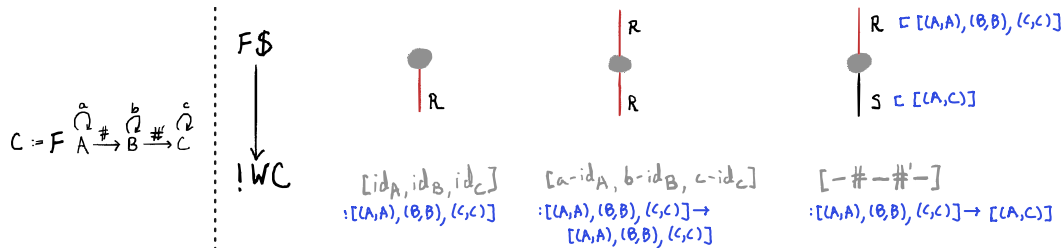
Given an operad $\mathcal{O}$, the *free symmetric monoidal operad* over $\mathcal{O}$ is the operad $!_{\text{sym}}\mathcal{O}$ whose objects are lists $[X_1, \dots, X_k]$ of objects of $\mathcal{O}$, and whose n -ary operations $[\Gamma_1], \dots, [\Gamma_n] \rightarrow [X_1, \dots, X_k]$ are pairs $([f_1, \dots, f_k], \sigma)$ of a list of operations $f_1 : \Omega_1 \rightarrow X_1, \dots, f_k : \Omega_k \rightarrow X_k$ and a bijection $\sigma : \Omega_1, \dots, \Omega_k \xrightarrow{\sim} \Gamma_1, \dots, \Gamma_n$.



The free semi-cartesian monoidal operad $!_{\text{aff}}\mathcal{O}$ / free cartesian monoidal operad $!_{\text{cart}}\mathcal{O}$ are defined in the same way but letting σ be any injection / function.

Generating mildly context-sensitive languages

The following 3-mCFG generates the language $a^n \# b^n \#' c^n$.



Closure properties of generalized CFLs

1. If $\mathcal{L}_1, \dots, \mathcal{L}_k \subseteq \mathcal{O}(A)$ are context-free, then so is their union $\bigcup_{i=1}^k \mathcal{L}_i \subseteq \mathcal{O}(A)$.
2. If $\mathcal{L}_1 \subseteq \mathcal{O}(A_1), \dots, \mathcal{L}_n \subseteq \mathcal{O}(A_n)$ are CF, and if $f : A_1, \dots, A_n \rightarrow A$ is an operation of $\mathcal{O}$, then $\{f(u_1, \dots, u_n) \mid u_1 \in \mathcal{L}_1, \dots, u_n \in \mathcal{L}_n\} \subseteq \mathcal{O}(A)$ is CF.
3. If $\mathcal{L} \subseteq \mathcal{O}(A)$ is CF and $F : \mathcal{O} \rightarrow \mathcal{P}$ is a functor of operads, then the functorial image $F(\mathcal{L}) \subseteq \mathcal{P}(F(A))$ is also CF.
4. If $\mathcal{L} \subseteq \mathcal{O}(A)$ is CF and $\mathcal{R} \subseteq \mathcal{O}(A)$ is regular, then $\mathcal{L} \cap \mathcal{R} \subseteq \mathcal{O}(A)$ is CF.

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1. If $\mathcal{L}_1, \dots, \mathcal{L}_k \subseteq \mathcal{O}(A)$ are context-free, then so is their union $\bigcup_{i=1}^k \mathcal{L}_i \subseteq \mathcal{O}(A)$.
2. If $\mathcal{L}_1 \subseteq \mathcal{O}(A_1), \dots, \mathcal{L}_n \subseteq \mathcal{O}(A_n)$ are CF, and if $f : A_1, \dots, A_n \rightarrow A$ is an operation of $\mathcal{O}$, then $\{f(u_1, \dots, u_n) \mid u_1 \in \mathcal{L}_1, \dots, u_n \in \mathcal{L}_n\} \subseteq \mathcal{O}(A)$ is CF.
3. If $\mathcal{L} \subseteq \mathcal{O}(A)$ is CF and $F : \mathcal{O} \rightarrow \mathcal{P}$ is a functor of operads, then the functorial image $F(\mathcal{L}) \subseteq \mathcal{P}(F(A))$ is also CF.
4. If $\mathcal{L} \subseteq \mathcal{O}(A)$ is CF and $\mathcal{R} \subseteq \mathcal{O}(A)$ is regular, then $\mathcal{L} \cap \mathcal{R} \subseteq \mathcal{O}(A)$ is CF.

(4) is a corollary of the following pullback construction: given a gCFG $G = (\dot{S}, p_G)$ and a gNFA $M = (\dot{Q}, p_M)$ with $p_G(S) = p_M(q_r)$, we can construct a gCFG $M^{-1}(G)$ over $\dot{Q}$ generating the language $p_M^{-1}(\mathcal{L}_G) \cap \dot{Q}(q_r)$. (cf. the Bar-Hillel construction)

4. Perspectives

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A long term goal is to transfer knowledge between category theory, automata theory, parsing & linguistics, type systems...

**Very selected bibliography
(see [MZ25] for more)**

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