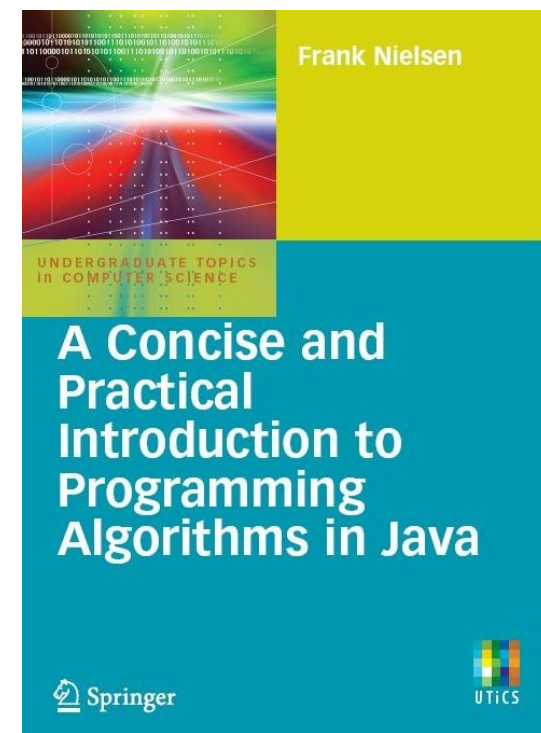


A Concise and Practical Introduction to Programming Algorithms in Java



Chapter 9: Combinatorial optimization algorithms

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Agenda

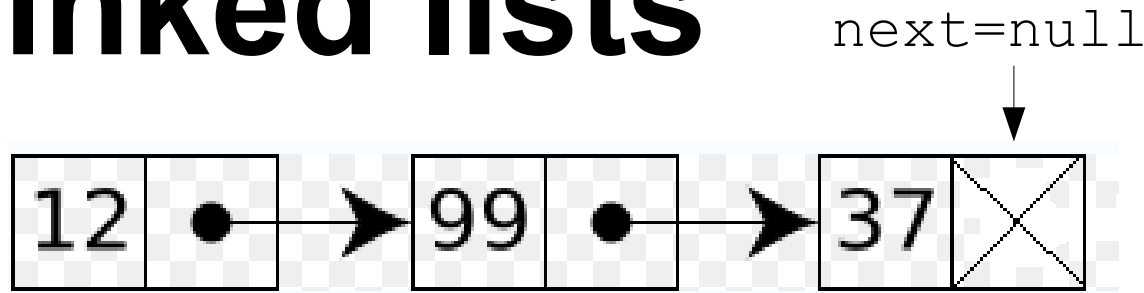


A few algorithms and paradigms:

- I/ **Exhaustive search**
- II/ **Greedy algorithm** (set cover problems)
- III/ **Dynamic programming** (knapsack)

+ ...Merging two lists...

Linked lists



```
public class List
{
    int container;
    List next;    // a reference to a cell of a list

    // Constructor List(head, tail)
    // Build a cell so that
    //           the reference to the next cell is the tail
    List(int element, List tail)
    {
        this.container=element;
        this.next=tail;
    }
}
```

Merging ordered linked lists

- Two *linked lists* u and v of increasing integers
- Build a *new linked list* in increasing order...
- ...using only cells of u and v (no cell creation, *new*)

For example:

U	3-->6-->8-->null
V	2-->4-->5-->7-->9-->null
Merge(U,V)	2-->3-->4-->5-->6-->7-->8-->9-->null

Linked lists

```
class List
{
    int container;
    List next;

    // Constructor List(head, tail)
    List(int element, List tail)
    {
        this.container=element;
        this.next=tail;
    }
}
```

```
List insert(int el)    // insert element at the head of the list
{
    return new List(el,this);
}
```

```
void Display()
{
    List u=this;

    while(u!=null)
    {
        System.out.print(u.container+"-->");
        u=u.next;
    }
    System.out.println("null");
}
```

```
U   | 3-->6-->8-->null
V   | 2-->4-->5-->7-->9-->null
```



```

class MergeList
{
//
// Merge two ordered lists
//
static List mergeRec(List u, List v)
{
if (u==null) return v;
if (v==null) return u;

if (u.container < v.container)
{
// Recycle cells/ no new
u.next=mergeRec(u.next,v);
return u;
}
else
{
// Recycle cells/no new
v.next=mergeRec(u,v.next);
return v;
}
}
}

```

```

public static void main(String [] args)
{
List u=new List(8,null);
u=u.insert(6);u=u.insert(3);
u.Display();

List v=new List(9,null);
v=v.insert(7);v=v.insert(5);
v=v.insert(4);v=v.insert(2);
v.Display();

List w=mergeRec(u,v);
w.Display();
}

```



```

static List sortRec(List u)
{
    int i,l=u.length(), lr;
    List l1, l2, split, psplit; // references to cells

    if (l<=1)
        return u;
    else
    {
        l1=u;

        psplit=split=u;
        i=0;lr=l/2;

        while (i<lr)
        {
            i++;
            psplit=split;
            split=split.next;}

        l2=split; // terminates with a null
        psplit.next=null;

        return mergeRec( sortRec(l1), sortRec(l2) );
    }
}

```

Sort in $O(n \log n)$ time:

- Split list in two halves
- Recursively apply sorting
- Merge ordered lists



```

public static void main(String [] args)
{

List u=new List(3,null);
u=u.insert(2);
u=u.insert(9);
u=u.insert(6);u=u.insert(1);
u=u.insert(15);u=u.insert(17);
u=u.insert(23);u=u.insert(21);
u=u.insert(19);u=u.insert(20);

u.Display();

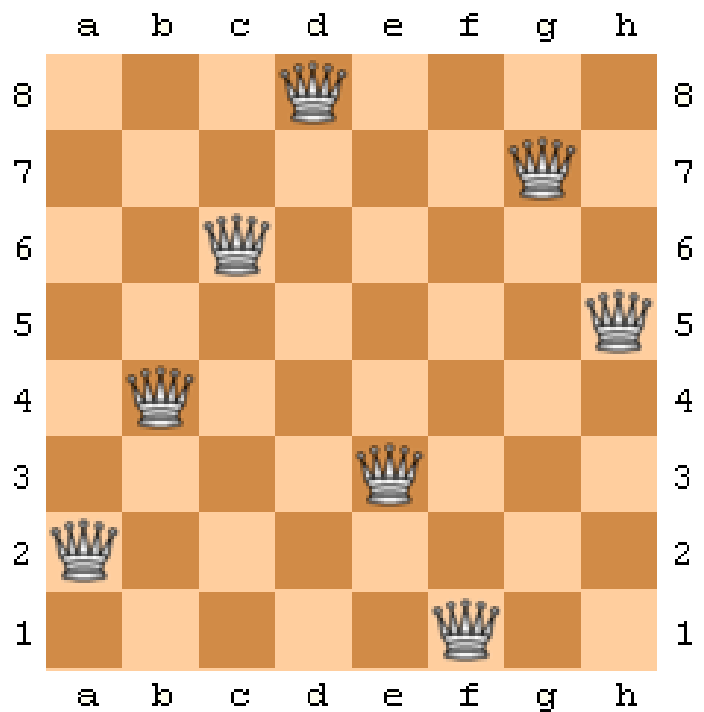
List sortu=sortRec(u);
System.out.println("Sorted linked list:");
sortu.Display();
}

```

20-->19-->21-->23-->17-->15-->1-->6-->9-->2-->3-->null
 Sorted linked list:
 1-->2-->3-->6-->9-->15-->17-->19-->20-->21-->23-->null

Exhaustive search (Brute force search)

The Eight Queens puzzle:



Max Bezzel (1848, chess player)
Find safe positions of 8 queens
on a 8x8 chessboard

- 92 distinct solutions
- 12 non-naive distinct solutions (rotation/symmetry)
- Good exercise for *designing* algorithms
- Generalize to n-queens

Exhaustive search & Backtracking

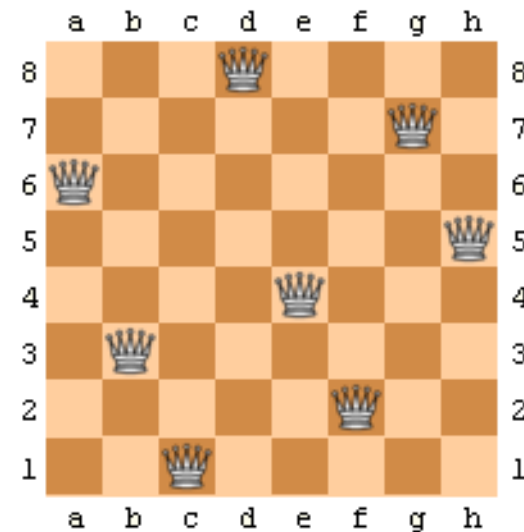
Brute force (naive) algorithm:

Check all $64 \times 63 \times \dots \times 57/8! = 283,274,583,552$?! solutions...

Easy to check that a **configuration** is not safe
(*check horizontal/vertical/diagonal lines*)

→ Two queens cannot be on the same line...

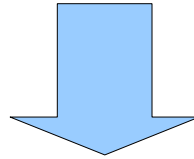
Therefore, incrementally place queen i ($0 \dots 7$)
on the i -th row, on the first **free** column



If there is no more free columns left, then **backtrack...**

Exhaustive search & Backtracking

Incrementally place queen i ($0 \dots 7$) on the first **free** column
If there is no more free columns left, then **backtrack:**



Consider the previous queen position and increment its column position, etc., etc., etc.

... until we find a solution
(=reach a successful location for queen indexed 7)

`queen`: 1D Array that specifies the column position
Queen i is located at position $(i, \text{queen}[i])$
(with i ranging from 0 to 7)

`search`: Static function that returns a/all solution(s).

```
// Are queen (i1,j1) and queen (i2,j2) safe ?
static boolean WrongPos(int i1, int j1, int i2, int j2)
{ // same row?, same col?, same diag?
return (i1==i2 ||
        j1==j2 ||
        Math.abs(i1-i2) == Math.abs(j1-j2));
}
```

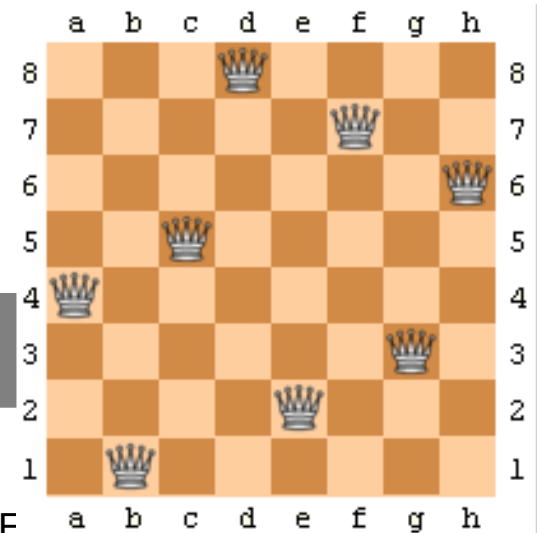
```
// Place safely queen i at column j?
static boolean FreeMove(int i, int j)
{
boolean result=true;

for(int k=0;k<i;k++)
    result=result&&!WrongPos(i,j,k,queen[k]);

return result;
}
```

Check for the queens
placed so far
on the chessboard

Static functions to check for collisions



```

static boolean search(int row)
{
    boolean result=false;

    if (row==n)
    { // Terminal case
        DisplayChessboard();
        nbsol++;
    }
    else
    { // Exhaustive search
        int j=0;

```

```

        while(!result && j<n)
        {
            if (FreeMove(row,j))
            {
                queen[row]=j;
                result=search(row+1); // RECURSION/BACKTRACK
            }
            j++; // explore all columns
        }
    }

```

Increment the number
of found solutions
(static class variable)

```

    return result;

```

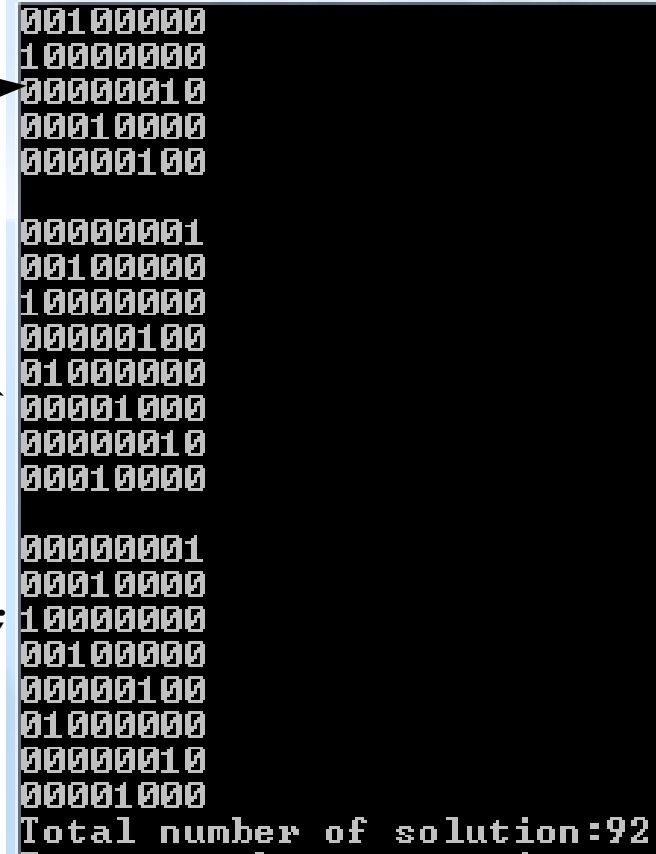


```
static final int n=8;
static int [] queen=new int[n];
static int nbsol;
```

```
static void DisplayChessboard()
{
    int i,j;

    System.out.println("");

    for(i=0;i<n;i++)
    {
        for(j=0;j<n;j++)
        {
            if (queen[i]!=j) System.out.print("0");
            else System.out.print("1");
        }
        System.out.println("");
    }
}
```



```
00100000
10000000
00000010
00010000
00000100

00000001
00100000
10000000
00000100
01000000
00001000
00000010
00010000

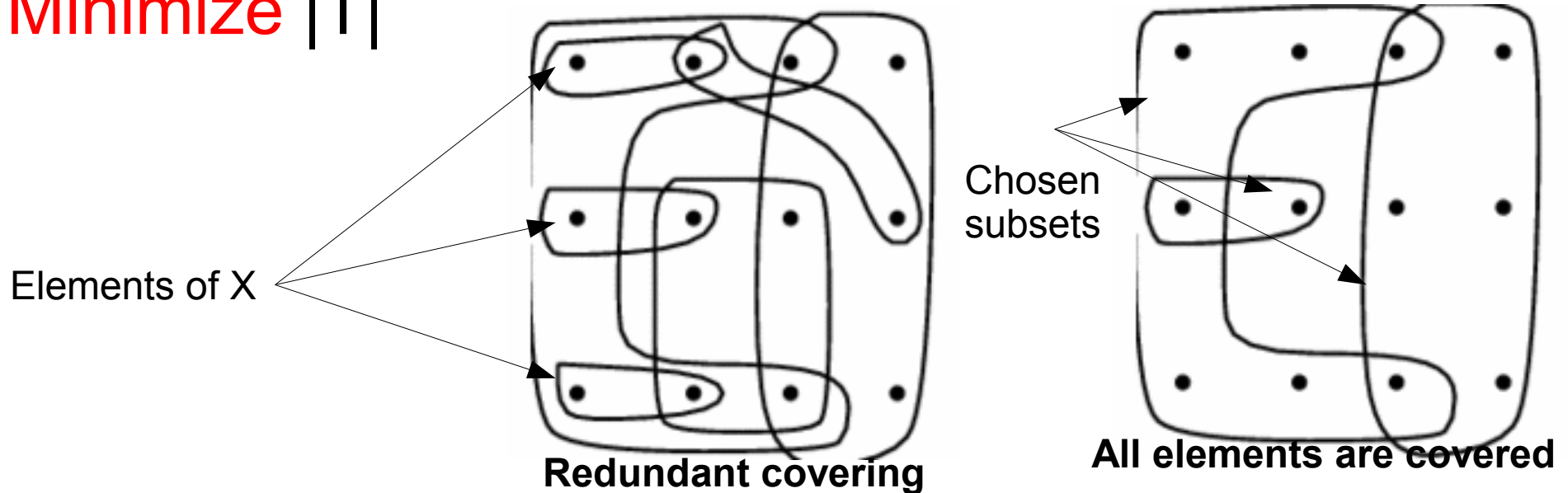
00000001
00010000
10000000
00100000
00000100
01000000
00000010
00001000
Total number of solution:92
```

```
public static void main(String [] arguments)
{
    nbsol=0;
    search(0); // Call exhaustive search procedure
    System.out.println("Total number of solutions:"+nbsol);
}
```



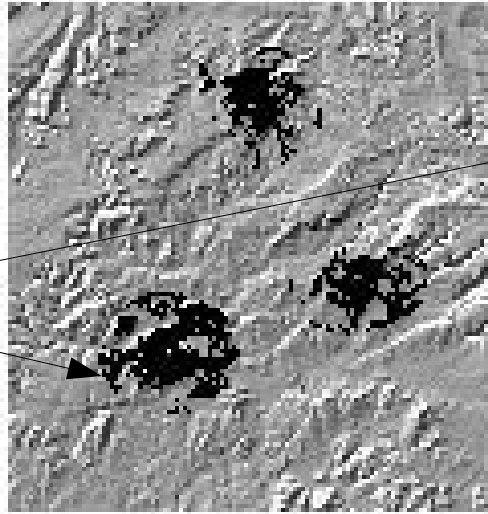
Optimization: Set Cover Problem (SCP)

- Given a graph:
 - A **finite set** $X = \{1, \dots, n\}$
 - A **collection of subsets** of S : $S_1, S_2, \dots, S_m$
- **Problem:**
 - Find a subset T of $\{1, \dots, m\}$ such that $\bigcup_{j \in T} S_j = X$
 - **Minimize** $|T|$

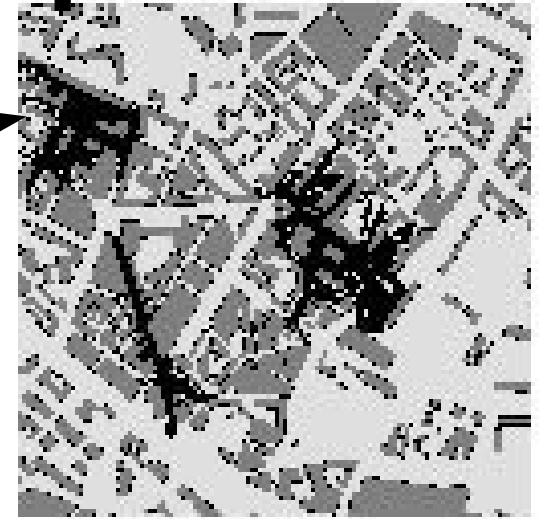


Applications of set cover problems

Wave propagation patterns
of a single antenna



Mountains



City

- Choose base stations for quality of service (cell phone network, etc)
- Discretize terrain using a **regular grid** (set X)
- Each position of antenna $\rightarrow$ subset of covered grid areas (S)
- **Minimize the number of chosen antennas** (X)

- Given a graph:
 - A **finite set** $X = \{1, \dots, n\}$ (=regular grid elements)
 - A **collection of subsets** of S (=antenna patterns)

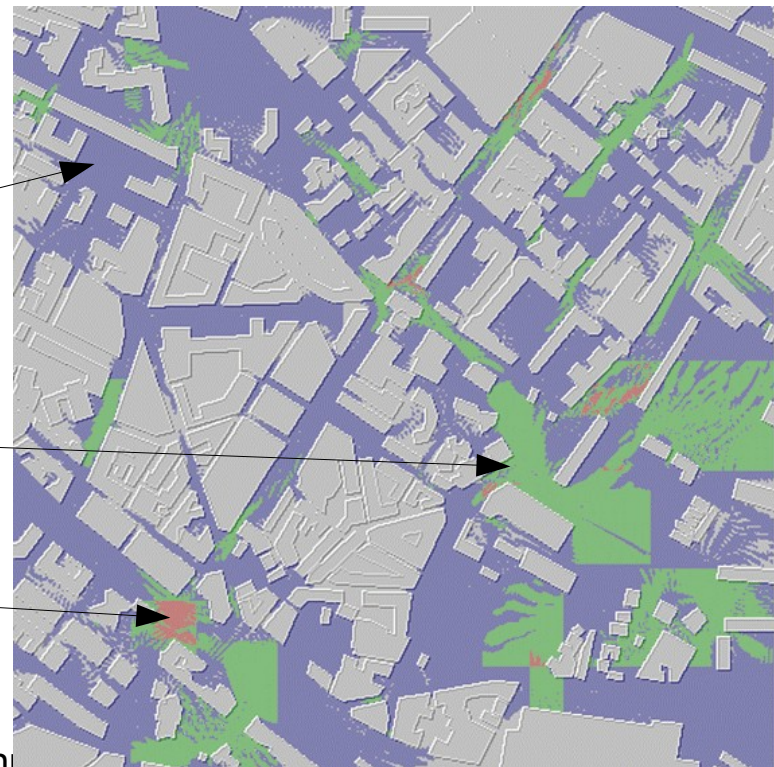
$$S_1, S_2, \dots, S_m$$
- **Problem:**
 - Find a subset T of $\{1, \dots, m\}$ (=subset of antennas)

such that $\bigcup_{j \in T} S_j = X$
 - **Minimize** $|T|$

Covered once

Covered twice

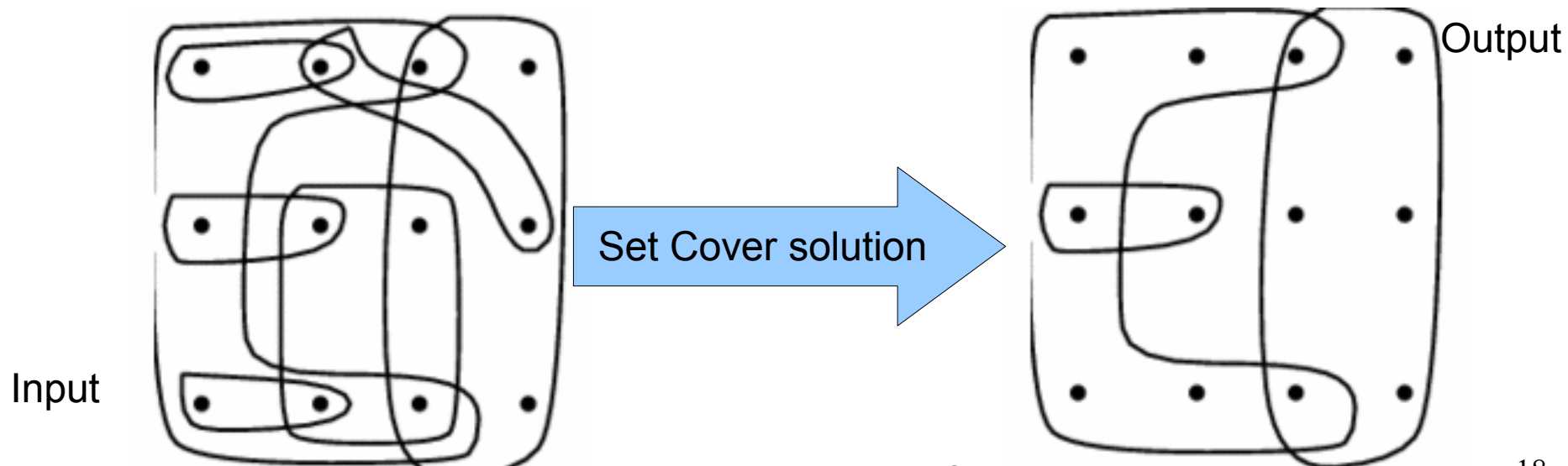
Covered thrice



A greedy algorithm (algorithme glouton)

GREEDY-SET-COVER(X,S)

```
1  $M \leftarrow X$     // all elements must be covered
2  $C \leftarrow \emptyset$  // chosen subsets
3 while  $M \neq \emptyset$  do
4     select an  $Q \in S$  that maximizes  $|Q \cap M|$ 
5      $M \leftarrow M - Q$ 
6      $C \leftarrow C \cup \{Q\}$ 
7 return  $C$ 
```



Visualizing a « range set »

Elements:

$X = \{1, \dots, 6\}$

Subsets:

$S_1 = \{1, 2, 4\}$

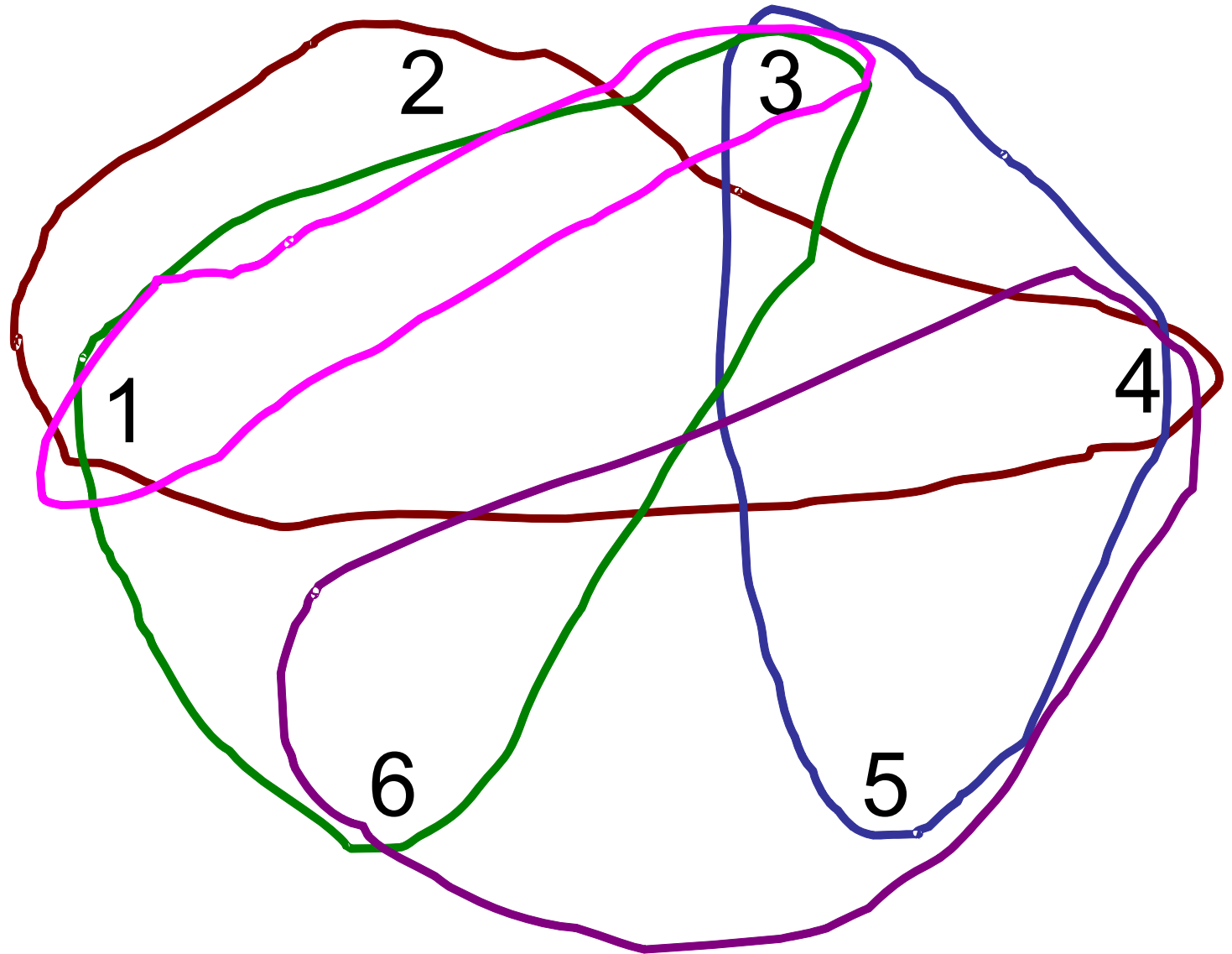
$S_2 = \{3, 4, 5\}$

$S_3 = \{1, 3, 6\}$

$S_4 = \{2, 3, 5\}$

$S_5 = \{4, 5, 6\}$

$S_6 = \{1, 3\}$



Data-structure for the set cover problem

$$X = \{1, \dots, 6\}$$

$$S_1 = \{1, 2, 4\}$$

$$S_2 = \{3, 4, 5\}$$

$$S_3 = \{1, 3, 6\}$$

$$S_4 = \{2, 3, 5\}$$

$$S_5 = \{4, 5, 6\}$$

$$S_6 = \{1, 3\}$$

M=6 SUBSETS

	1	2	3	4	5	6	
	1	1	0	1	0	0	S_1
	0	0	1	1	1	0	S_2
	1	0	1	0	0	1	S_3
	0	1	1	0	1	0	S_4
	0	0	0	1	1	1	S_5
	1	0	1	0	0	0	S_6
	↑	↑	↑	↑	↑	↑	

N=6 ELEMENTS

Incidence matrix: boolean matrix

```

class SetCover
{
int nbelements;
int nbsubsets;
boolean [][] incidenceMatrix;

//Constructor
SetCover(int nn, int mm)
{
this.nbelements=nn; this.nbsubsets=mm;

incidenceMatrix=new boolean[nbsubsets][nbelements];

for(int i=0;i<nbsubsets;i++)
    for(int j=0;j<nbelements;j++)
        incidenceMatrix[i][j]=false;
}

void SetSubsets(int [][] array) // Set incidence matrix
{for(int j=0;j<array.length;j++)
    {for(int i=0;i<array[j].length;i++)
        incidenceMatrix[j][array[j][i]]=true;
    }
}
}

```



```

void Display()
{
    for(int i=0;i<nbsubsets;i++){

        for(int j=0;j<nbelements;j++)
            if (incidenceMatrix[i][j]) System.out.print("1");
            else System.out.print("0");
            System.out.println("");
        }
    }
}

```

```

public static void main(String [] args)
{
    int [][] subsets={{0,1,3},{2,3,4}, {0,2,5},{1,2,4},{3,4,5},{0,2}};

    SetCover setcover=new SetCover(6,6);
    setcover.SetSubsets(subsets);

    System.out.println("Set cover problem:");
    setcover.Display();
}

```

```

Set cover problem:
110100
001110
101001
011010
000111
101000

```



```

static boolean [] GreedySCP(SetCover problem)
{
boolean [] result=new boolean[problem.nbsubsets];
int cover=0; int select;

for(int i=0;i<problem.nbsubsets;i++)    // initially no subsets
    result[i]=false;

while(cover!=problem.nbelements)
{
// Choose largest not-yet covered subset
select=problem.LargestSubset();
result[select]=true;

// Update covered matrix
cover+=problem.Cover(select);

// Update incidence matrix
problem.Update(select);

System.out.println("Selected "+select+" Number of covered
elements="+cover);
problem.Display();
}

return result;
}

```

Greedy algorithm



```
// Number of covered element by subset i
int Cover(int i)
{
    int nbEl=0;

    for(int j=0;j<nbelements;j++)
        if (incidenceMatrix[i][j]) ++nbEl;

    return nbEl;
}

// Report the current largest subset
int LargestSubset()
{
    int i, nbEl, max, select;

    max=-1;select=-1;

    for(i=0;i<nbsubsets;i++)
    {
        nbEl=Cover(i);
        if (nbEl>max) {max=nbEl; select=i;}
    }

    return select;
}
```

Methods of class SetCover



Methods of class SetCover

```
// Update the incidence matrix
void Update(int sel)
{
    int i,j;

    for (i=0; i<nbsubsets; i++)
    {
        if (i!=sel) //use sel below so don't modify it
        {
            for (j=0; j<nbelements; j++)
                if (incidenceMatrix[sel][j])
                    incidenceMatrix[i][j]=false;
        }
    }
    // Remove the chosen subset as well
    for (j=0; j<nbelements; j++)
        incidenceMatrix[sel][j]=false;
}
```



```

public static void main(String [] args)
{
int [] [] subsets ={{0,1,3},{2,3,4},
                    {0,2,5},{1,2,4},
                    {3,4,5},{0,2}};

SetCover setcover=new SetCover(6,6);

setcover.SetSubsets(subsets);

System.out.println("Set cover problem:");
setcover.Display();

boolean [] solution=GreedySCP(setcover);

System.out.print("Solution:");
for(int i=0;i<setcover.nbsubsets;i++)
if (solution[i]) System.out.print(" "+i);
System.out.println("");
}

```

Set cover problem:

110100

001110

101001

011010

000111

101000

Selected 0 Number of covered elements=3

000000

001010

001001

001010

000011

001000

Selected 1 Number of covered elements=5

000000

000000

000001

000000

000001

000000

Selected 2 Number of covered elements=6

000000

000000

000000

000000

000000

000000

Solution: 0 1 2



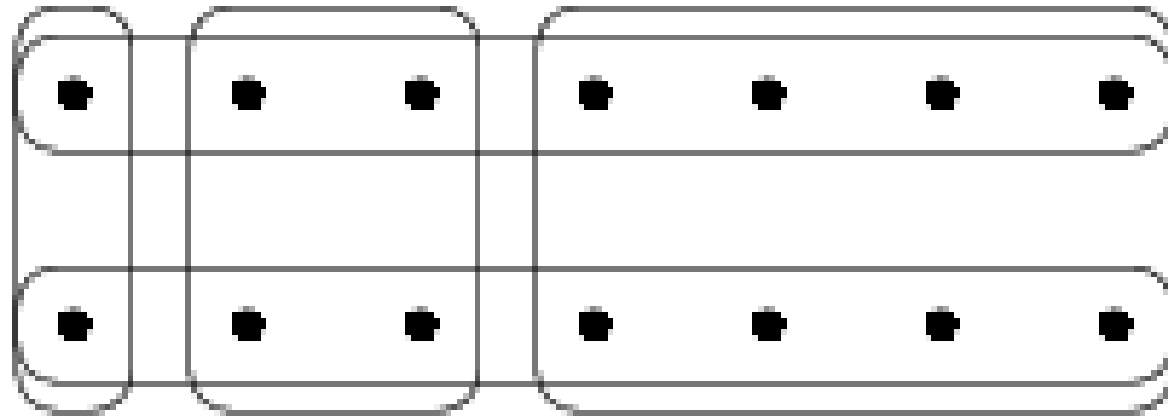
Optimization bounds for greedy algorithm

$$C_{\text{opt}} \leq C_{\text{greedy}} \leq \text{ApproximationFactor} \times C_{\text{opt}}$$

Upper bound:

Approximation factor is at most $H(n) \leq \log(n)$

2 sets is optimal solution
but greedy chooses 3 sets here



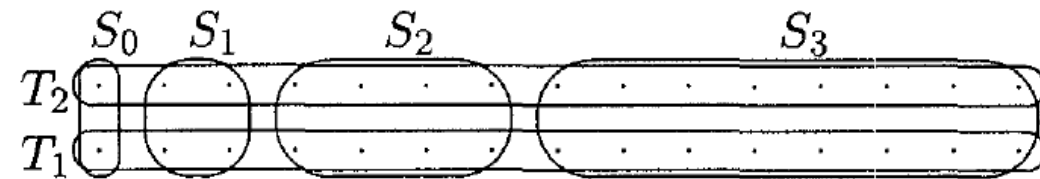
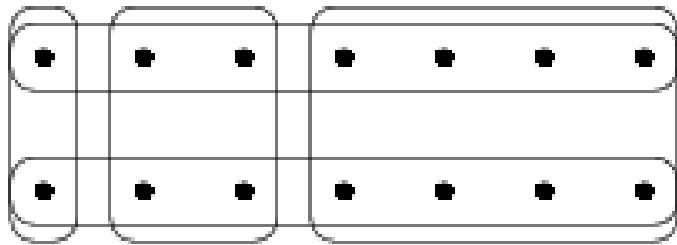
Lower bound:

Approximation factor can be as big as $\Omega(\log(n))$

Difficult to approximate: cannot beat $(1-\epsilon)\text{Opt}$ unless $P=NP$



```
int [][] subsets={{0,1,2,3,4,5,6},{7,8,9,10,11,12,13},
                 {0,7},{1,2,8,9},{3,4,5,6,10,11,12,13}};
SetCover setcover=new SetCover(14,5);
```



Etc...

Easy to build generic examples
where greedy does not behave well
with $O(\log n)$ approximation ratio

Set cover problem:

11111110000000

00000001111111

10000001000000

01100000110000

00011110001111

Selected 4 Number of covered elements=8

11100000000000

00000001110000

10000001000000

01100000110000

00000000000000

Selected 3 Number of covered elements=12

10000000000000

00000001000000

10000001000000

00000000000000

00000000000000

Selected 2 Number of covered elements=14

00000000000000

00000000000000

00000000000000

00000000000000

00000000000000

Solution: 2 3 4



Knapsack problem (sac a dos)

(First version)

Given:

- A set of n Objects $O_1, \dots, O_n$ with corresponding weights $W_1, \dots, W_n$
- And a bag (**knapsack**) of **capacity** W

Find:

All the ways of choosing objects to **fully fill** the bag

Filling the knapsack

Need to enumerate all possibilities:

n objects $\Rightarrow 2^n$ choices

(2, 4, 8, 16, 32, 64, ...)

How to program this?

n **is** a variable

(cannot fix the number of nest loops)

Need to enumerate all combinations:

= **Exhaustive search**

$$n=4$$
$$2^4=16$$

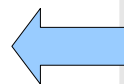
1	1	1	1
1	1	1	0
1	1	0	1
1	1	0	0
1	0	1	1
1	0	1	0
1	0	0	1
1	0	0	0
0	1	1	1
0	1	1	0
0	1	0	1
0	1	0	0
0	0	1	1
0	0	1	0
0	0	0	1
0	0	0	0



Enumerating: A recursive approach

```
static void Display(boolean [] tab)
{
    for(int i=0;i<tab.length;i++)
        if (tab[i]) System.out.print("1 ");
        else
            System.out.print("0 ");
    System.out.println("");
}
```

```
static void Enumerate(boolean [] selection, int pos)
{
    if (pos==selection.length-1)
        Display(selection);
    else
    {
        pos++;
        selection[pos]=true;
        Enumerate(selection,pos);
        selection[pos]=false;
        Enumerate(selection,pos);
    }
}
```



```
public static void main(String[] args)
{
    int n=4;
    int i;
    boolean [] select=new boolean[n];
    for(i=0;i<n;i++)
        select[i]=false;

    Enumerate(select,-1);
}
```



Fully filling the knapsack

```
final static int n=10; // 10 objects
static int [] weight={2,3,5,7,9,11,4,13,23,27};

static void SolveKnapSack(boolean [] chosen, int goal, int
i, int total)
{
    numbercall++; // keep track of total number of calls

    if ((i>=chosen.length)&&(total!=goal)) return;

    if (total==goal)
        { Display(chosen, goal);
          numbersol++; // total number of solutions
        }
    else
        {
            chosen[i]=true; // add item first
            SolveKnapSack(chosen,goal,i+1,total+weight[i]);
            chosen[i]=false; // and then remove it
            SolveKnapSack(chosen,goal,i+1,total);
        }
}
```




```

final static int n=10; // 10 objects
static int [] weight={2,3,5,7,9,11,4,13,23,27};

public static void main(String [] args)
{
    int totalweight=51;
    numbersol=0;
    numbercall=0;

    System.out.println("Knapsack:");
    boolean [] chosen=new boolean[n];

    SolveKnapSack(chosen, totalweight, 0, 0);
    System.out.println("Total number of
        solutions:"+numbersol);
    System.out.println(" #calls="+numbercall);
}

```

Knapsack:

2+3+5+7+11+23+0=51

2+5+7+9+11+4+13+0=51

2+5+4+13+27+0=51

2+7+11+4+27+0=51

2+9+4+13+23+0=51

2+9+13+27+0=51

3+5+7+9+4+23+0=51

3+5+7+9+27+0=51

3+5+7+13+23+0=51

3+5+9+11+23+0=51

7+4+13+27+0=51

9+11+4+27+0=51

11+4+13+23+0=51

11+13+27+0=51

Total number of solutions:14

#calls=2029



Exhaustive search: Branch & bound

```
static void SolveKnapSack(boolean [] chosen, int goal, int i,  
int total)  
{  
    numbercall++;
```

Stop recursion if we already
exceed the weight amount

```
if (total>goal) return; // cut
```

```
if ((i>=chosen.length)&&(total!=goal)) return;
```

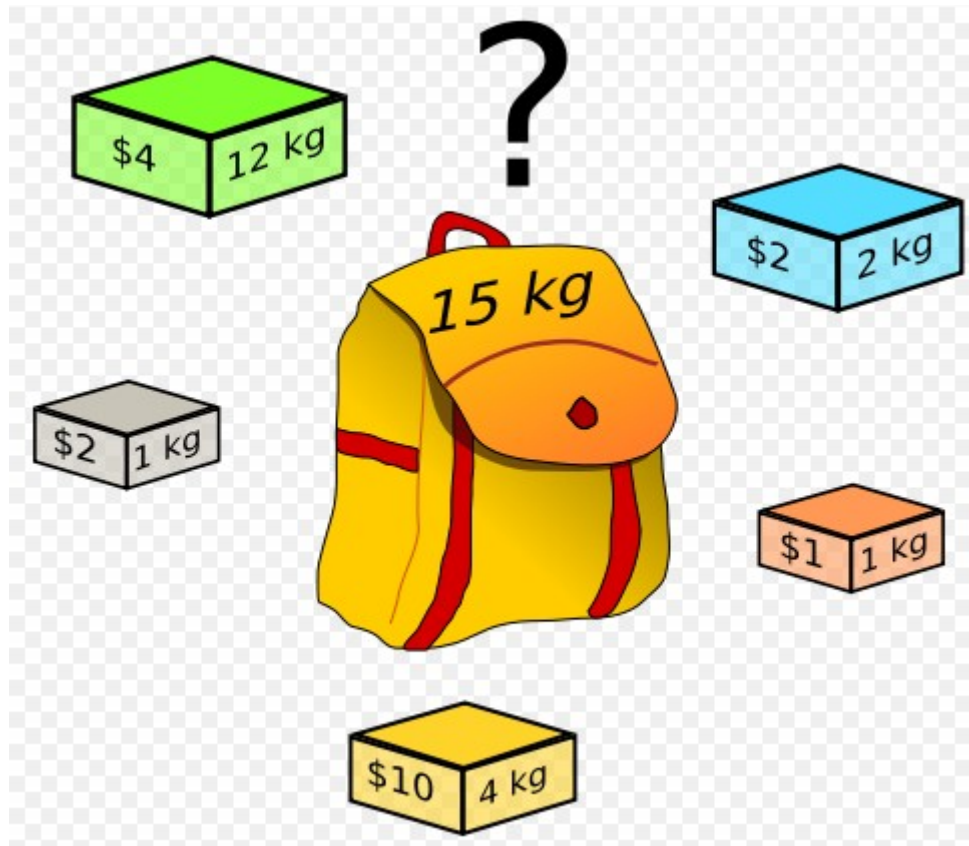
```
if (total==goal)  
{    Display(chosen, goal);  
    numbersol++;  
}
```

```
else  
{  
    chosen[i]=true;// add item first  
    SolveKnapSack(chosen,goal,i+1,total+weight[i]);  
    chosen[i]=false; // and then remove it  
    SolveKnapSack(chosen,goal,i+1,total);  
}
```

```
Knapsack:  
2+3+5+7+11+23+0=51  
2+5+7+9+11+4+13+0=51  
2+5+4+13+27+0=51  
2+7+11+4+27+0=51  
2+9+4+13+23+0=51  
2+9+13+27+0=51  
3+5+7+9+4+23+0=51  
3+5+7+9+27+0=51  
3+5+7+13+23+0=51  
3+5+9+11+23+0=51  
7+4+13+27+0=51  
9+11+4+27+0=51  
11+4+13+23+0=51  
11+13+27+0=51  
Total number of solutions:14 #calls=1791
```



Knapsack: Fundamental optimization problem



Given a bag capacity (15 kg),
maximize the **utility** (price) of selected objects
(NP-hard problem)

Knapsack optimization problem

Given $p_1, p_2, \dots, p_n$ $\longleftarrow$ weights

$a_1, a_2, \dots, a_n$ $\longleftarrow$ utility

P_{max} $\longleftarrow$ Maximum weight
(capacity of bag)

Optimize $\sum_{i \in I} a_i$

such that

$$\sum_{i \in I} p_i \leq P_{max}$$

(Maybe there exists several solutions)

Knapsack: Example

$P_{max} = 12$

8 objects

weight	2	3	5	2	4	6	3	1
utility	5	8	14	6	13	17	10	4

Dynamic programming (Knapsack)

Dynamic programming computes a table...
... from which a **solution** can be retrieved.

Requires a **relational equation** to deduce
solutions progressively.

Dynamic programming (Knapsack)

Let $u(i,j)$ be the maximum utility by taking objects in $\{1, \dots, i\}$ with total weight $\leq j$

- If $i=1$ then $u(i,j)=0$ for $j < p_1$
and $u(i,j)=A_1$ for $j \geq p_1$
- If $i > 1$
$$u(i,j) = \max(u(i-1,j) , u(i-1,j-p_i) + A_i)$$

Do not take object O_i

Take object O_i :

- gain A_i
- but leave room: p_i

weight	2	3	5	2	4	6	3	1	Pmax= 12
utility	5	8	14	6	13	17	10	4	

$$u(i,j)=\max(u(i-1,j) , u(i-1,j-P_i)+A_i)$$

$u_{i,j}$	0	1	2	3	4	5	6	7	8	9	10	11	12
$i = 1$	0	0	5	5	5	5	5	5	5	5	5	5	5
$i = 2$	0	0	5	8	8	13	13	13	13	13	13	13	13
$i = 3$	0	0	5	8	8	14	14	19	22	22	27	27	27
$i = 4$	0	0	6	8	11	14	14	20	22	25	28	28	33
$i = 5$	0	0	6	8	13	14	19	21	24	27	28	33	35
$i = 6$	0	0	6	8	13	14	19	21	24	27	30	33	36
$i = 7$	0	0	6	10	13	16	19	23	24	29	31	34	37
$i = 8$	0	4	6	10	14	17	20	23	27	29	33	35	38

Optimization result: Maximum utility, given Pmax

Reading *back*: Solution

↓			↓	↓			↓	↓
2	3	5	2	4	6	3	1	
5	8	14	6	13	17	10	4	

$u_{i,j}$	0	1	2	3	4	5	6	7	8	9	10	11	12	
$i = 1$	0	0	▲5	5	5	5	5	5	5	5	5	5	5	Choose (38=33+5, 2-2=0)
$i = 2$	0	0	▲5	8	8	13	13	13	13	13	13	13	13	5=5 do not choose
$i = 3$	0	0	▲5	8	▲8	14	14	19	22	22	27	27	27	5=5 do not choose
$i = 4$	0	0	6	8	▲11	14	14	20	22	25	28	28	33	Choose (33=27+6, 4-2=2)
$i = 5$	0	0	6	8	13	14	19	21	▲24	27	28	33	35	Choose (27=14+13, 8-4=4)
$i = 6$	0	0	6	8	13	14	19	21	▲24	27	30	▲33	36	24=24, do not choose
$i = 7$	0	0	6	10	13	16	19	23	24	29	31	▲34	37	Choose (14, 11-3=8)
$i = 8$	0	4	6	10	14	17	20	23	27	29	33	35	▲38	Choose (4, 11)

Choose (Utility=0, Pmax=12)

From the table, chosen solution: O8, O7, O5, O4, O1



```

static int nbObjects=8;
static int [] weight={2,3,5,2,4,6,3,1};
static int [] utility={5,8,14,6,13,17,10,4};
static int weightmax=12;
static int [] [] array;

static void SolveDP()
{
    int i,j;
    array=new int[nbObjects][weightmax+1];

    // initialize the first row
    for(j=0;j<=weightmax;j++)
        if (j<weight[0]) array[0][j]=0;
        else array[0][j]=utility[0];
    // for all other rows
    for(i=1;i<nbObjects;i++)
    {
        for(j=0;j<=weightmax;j++)
            if (j-weight[i]<0) array[i][j]=array[i-1][j];
            else
                array[i][j]=max( array[i-1][j],
                                array[i-1][j-weight[i]]+utility[i]);
    }
}

```



```

static void InterpretArray()
{
    int i,u,w;
    u=0;
    w=weightmax;

    for (i=nbObjects-1;i>=1;i--)
    {
        if (array[i][w]!=array[i-1][w])
            {System.out.print((i+1)+" ");
              w=w-weight[i];
              u=u+utility[i];
            }
    }

    if (array[0][w]!=0);
        {System.out.println("1");
          w=w-weight[0];
          u=u+utility[0];
        }

    System.out.println("Cross check:"+u+" remaining weight "+w);
}

```



```

public static void main(String[] args)
{
    System.out.println("Solving knapsack using the dynamic
                        programming paradigm.");
    SolveDP();
    Display();
    System.out.println("Reading solution:");
    InterpretArray();
}

```

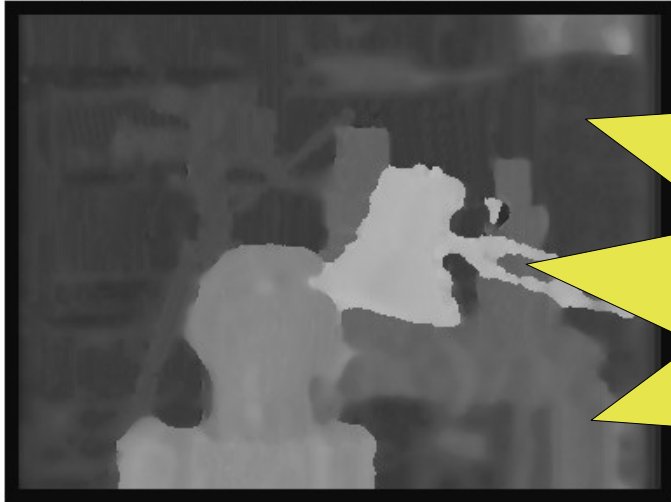
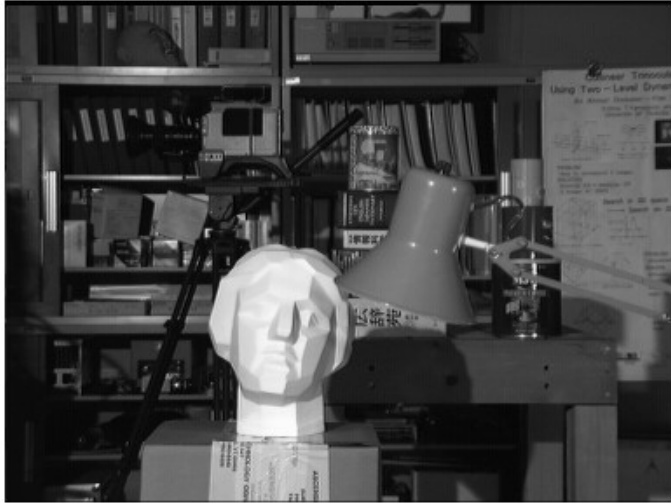
```

Solving knapsack using the dynamic programming paradigm.
0 0 5 5 5 5 5 5 5 5 5 5
0 0 5 8 8 13 13 13 13 13 13 13
0 0 5 8 8 14 14 19 22 22 27 27
0 0 6 8 11 14 14 20 22 25 28 28
0 0 6 8 13 14 19 21 24 27 28 33
0 0 6 8 13 14 19 21 24 27 30 33
0 0 6 10 13 16 19 23 24 29 31 34
0 4 6 10 14 17 20 23 27 29 33 35
Reading solution:
8 7 5 4 1
Cross check:38 remaining weight 0

```



Dynamic programming: binocular stereo matching



**Dynamic
Programming**

Benchmark:

<http://vision.middlebury.edu/~schar/stereo/web/results.php>



Optimization: A brief summary

- Exhaustive search: recursion but $O(2^n)$ complexity
- Can be improved by backtracking (cuts)
- Greedy algorithm: Polynomial $O(n^3)$
 - but yields an approximation
- Dynamic programming yields an exact solution
 - but requires $O(\text{weight} \times \text{objects})$ time
(weights should not be too big)

Last but not least: Java applets!

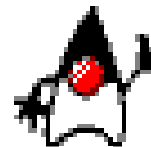
Applets are special java programs...
...that can run into your favorite Internet **browser**

You need to:

- (1) write a web page with `<APPLET>` `</APPLET>` tags
- (2) write and compile the Java applet code (`javac`)

Advantages of applets are:

- (1) to be accessible worldwide
- (2) to provide graphics



```

import java.awt.*;
import java.applet.*;

class Point2D
{double x,y;
Point2D()
{this.x=300.0*Math.random();
this.y=300.0*Math.random();}}

public class AppletINF311 extends Applet {
final static int n=100;
static Point2D [] set;

    public void init() {
        int i;
        set=new Point2D[n];
        for(i=0;i<n;i++)
            set[i]=new Point2D();}

    public void paint(Graphics g) {int i;

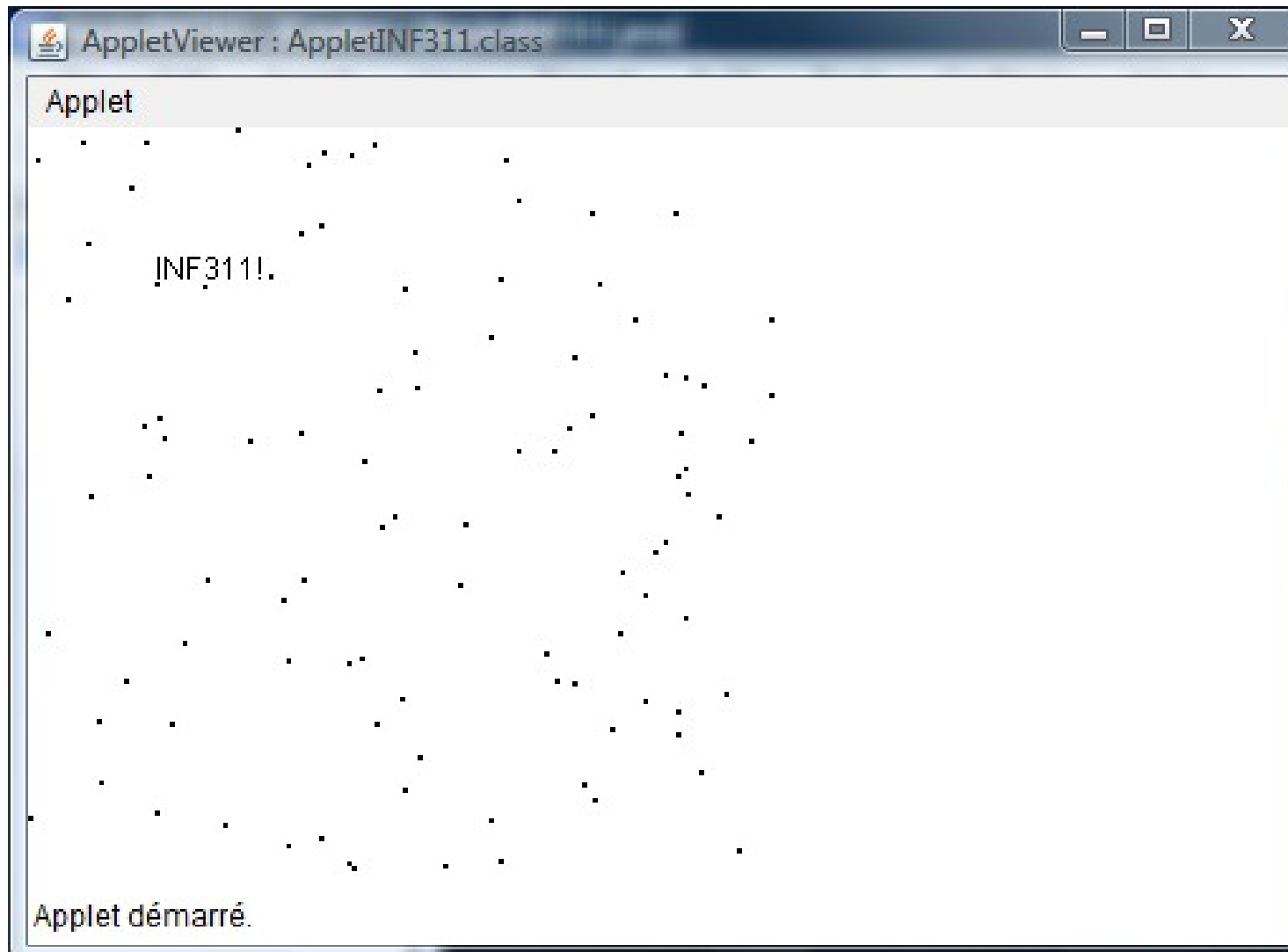
        for(i=0;i<n;i++)
        {
            int xi, yi;
            xi=(int)set[i].x; yi=(int)set[i].y;
            g.drawRect(xi, yi,1,1);
        }
        g.drawString("INF311!", 50, 60 );
    }
}

```



Java applets

```
<APPLET  
    code    = "AppletINF311.class"  
    width   = "500"  
    height  = "300"  
>  
</APPLET>
```



Java applets for online demos...



Hallucination (mirage)

Two technologies in use:

- Image **segmentation**
- Texture **synthesis** (hole filling)

Try it !!! <http://www.sonycs1.co.jp/person/nielsen/ClickRemoval/>

F. Nielsen, R. Nock ClickRemoval: interactive pinpoint image object removal. [ACM Multimedia 2005](#)

There is much more in Java!

JDK JRE	Java Language	Java Language								
	Tools & Tool APIs	java	javac	javadoc	apt	jar	javap	JPDA		jconsole
		Security	Int'l	RMI	IDL	Deploy	Monitoring	Troubleshoot	Scripting	JVM TI
	Deployment Technologies	Deployment			Java Web Start				Java Plug-in	
	User Interface Toolkits	AWT			Swing				Java 2D	
		Accessibility	Drag n Drop		Input Methods		Image I/O	Print Service		Sound
	Integration Libraries	IDL	JDBC™		JNDI™		RMI	RMI-IIOP		Scripting
	Other Base Libraries	Beans	Intl Support			I/O	JMX	JNI		Math
		Networking	Override Mechanism			Security	Serialization	Extension Mechanism		XML JAXP
	lang and util Base Libraries	lang and util	Collections		Concurrency Utilities		JAR	Logging	Management	
		Preferences API	Ref Objects		Reflection		Regular Expressions	Versioning	Zip	Instrument
	Java Virtual Machine	Java Hotspot™ Client VM					Java Hotspot™ Server VM			
	Platforms	Solaris™			Linux		Windows			Other
		Java SE API								

Java
SE
API

JDK™ 6 Documentation



A glimpse at object inheritance

All objects inherit from the topmost object: `Object`
...meaning some methods are already **predefined**

`java.lang`
Class `Object`

`java.lang.Object`

`public class Object`

Class `Object` is the root of the class hierarchy. Every class has the methods of this class.

Method Summary

protected <code>Object</code>	<code>clone()</code> Creates and returns a copy of this object.
boolean	<code>equals(Object obj)</code> Indicates whether some other object is "equal to" this one.
protected void	<code>finalize()</code> Called by the garbage collector on an object when garbage collection determines that there are no more references to the object.
<code>Class<?></code>	<code>getClass()</code> Returns the runtime class of this <code>Object</code> .
int	<code>hashCode()</code> Returns a hash code value for the object.
void	<code>notify()</code> Wakes up a single thread that is waiting on this object's monitor.
void	<code>notifyAll()</code> Wakes up all threads that are waiting on this object's monitor.
<code>String</code>	<code>toString()</code> Returns a string representation of the object.

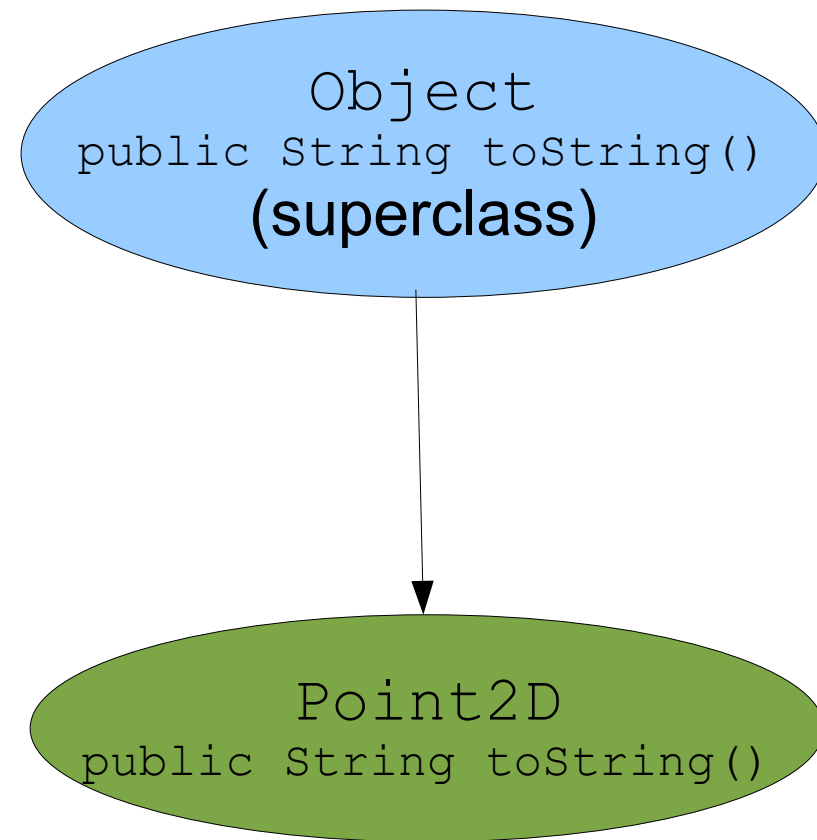
Can overwrite the
method `toString`



A glimpse at object inheritance

```
class Point2D
{double x,y;
// Constructor
Point2D(double xi, double yi)
{this.x=xi;this.y=yi; }
// Overrides default method
public String toString()
{return "["+x+" "+y+"]"; }
}
```

```
class SuperClass
{
public static void main(String [] a)
{
    Point2D myPoint=new Point2D(Math.PI, Math.E);
    System.out.println("Point:"+myPoint);
}
}
```



```
Point:[3.141592653589793 2.718281828459045]
```



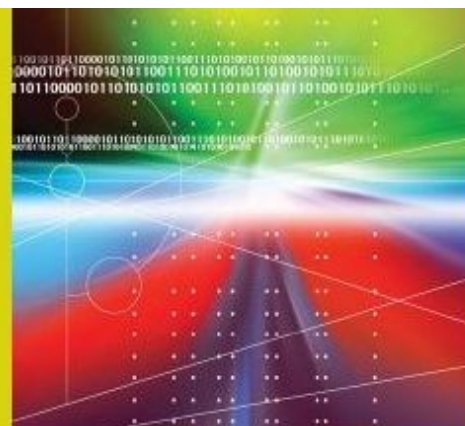


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The second part of the book focuses on data structures and algorithms, describing sequential and bisection search techniques and analysing their efficiency by using complexity analysis. Iterative and recursive sorting algorithms are discussed followed by linked lists and common insertion/deletion/merge operations that can be carried out on these. Abstract data structures are introduced along with how to program these in Java using object-orientation. The book closes with an introduction to more

evolved algorithmic tasks that tackle combinatorial optimisation problems.

Exercises are included at the end of each chapter in order for students to practice the concepts learned, and a final section contains an overall exam which allows them to evaluate how well they have assimilated the material covered in the book.

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ISBN 978-1-84882-338-9

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Algorithms in Java

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