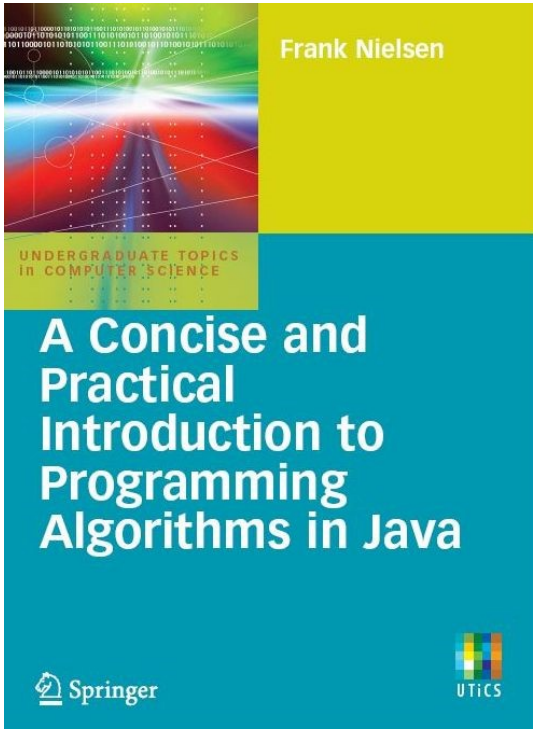
A Concise and Practical Introduction to Programming Algorithms in Java



Chapter 6: Searching and sorting

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Agenda

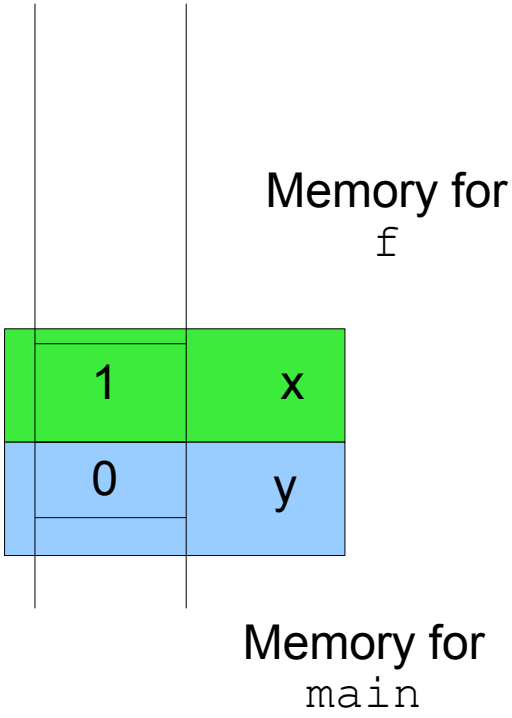
- Answering a few questions
- Searching (sequential, bisection)
- Sorting for faster searching
- Recursive sort
- Hashing

Swapping

Java is pass-by-value only.

Arrays and objects: pass-by-reference (=memory `value')

```
class FunctionCallSample
{

public static void f(double x)
{
x=1;
System.out.println(x);//1
return;
}

public static void main(String[] args)
{
int y=0;
f(y);
System.out.println(y);//y is still 0
}}
```



Swapping & Strings

```
public static void Swap(String p, String q)
   {  String tmp;
      tmp=p;
      p=q;
      q=tmp;}

public static void Swap(String [] t, int i, int j)
   {  String tmp;
      tmp=t[i];
      t[i]=t[j];
      t[j]=tmp;}
```

```
String s1="toto";
String s2="titi";
String [] s=new String[2];
s[0]=new String("toto");
s[1]=new String("titi");

System.out.println(s1+" "+s2);
Swap(s1,s2);
System.out.println(s1+" "+s2);
Swap(s,0,1);
System.out.println(s[0]+" "+s[1]);
```

Result

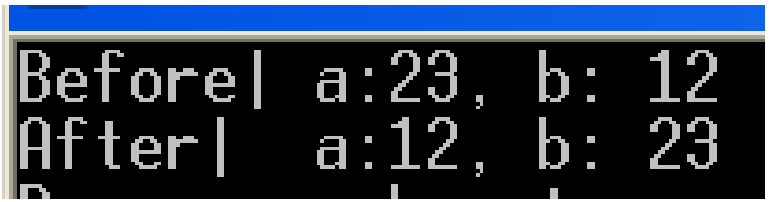
```
toto titi
toto titi
titi toto
```

Swapping using wrapped integers

Let us wrap integers into a tailored object named `IntObj`

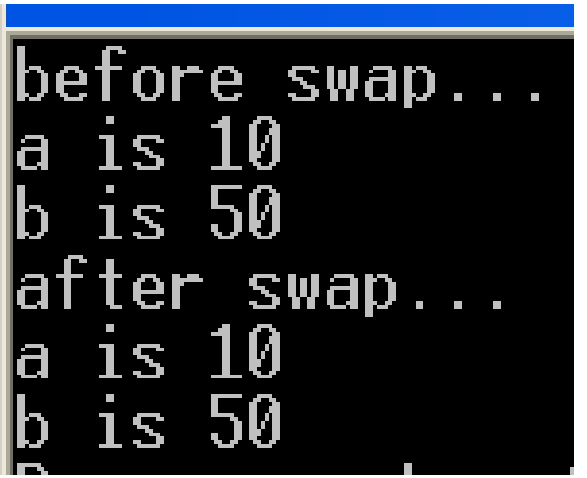
```
// Wrapping integers into an object

class IntObj {
   private int x;

   public IntObj(int x)
   { this.x = x; }

   public int getValue()
   { return this.x; }

   public void insertValue(int newx)
    { this.x = newx;}
}
```

```
public class SwapInt {

   // Pass-by-value (= pass-by-reference for objects)
   static void swap(IntObj p, IntObj q)
    {
      // interchange values inside objects
      int t = p.getValue();
      p.insertValue(q.getValue());
      q.insertValue(t);
   }

   public static void main(String[] args) {
      int a = 23, b = 12;

      System.out.println("Before| a:" + a + ", b: " + b);

      IntObj aObj = new IntObj(a);
      IntObj bObj = new IntObj(b);

      swap(aObj, bObj);

      a = aObj.getValue();
      b = bObj.getValue();

      System.out.println("After|  a:" + a + ", b: " + b);
   }
}
```



Java wrapper class for **int** is **Integer** *BUT* it doesn't allow you to alter the data field inside.

```
class SwapInteger
{
  public static void main(String args[])
  {
    Integer a, b;

    a = new Integer(10);
    b = new Integer(50);

    System.out.println("before swap...");
    System.out.println("a is " + a);
    System.out.println("b is " + b);
    swap(a, b); // References did not change (PASS-BY-VALUE)
    System.out.println("after swap...");
    System.out.println("a is " + a);
    System.out.println("b is " + b);
  }

  public static void swap(Integer a, Integer b)
  {
    Integer temp = a;
    a = b;
    b = temp;
  }
}
```



Searching: Problem statement

- Objects are accessed via a corresponding **key**
- Each object stores its key and additional fields
- One seeks for information stored in an object from its key
  (key= a handle)
- All objects are in the main memory (no external I/O)

More challenging problem:
Adding/removing or changing object attributes dynamically

Searching: Basic examples

Dictionary

Object:
(word, definition)

Key: word

```
class DictionaryEntry
{
String word;
String definition;
}
```

Dictionary

```
class DictionaryEntry
{
String word;
String definition;

DictionaryEntry(String w, String def)
{
this.word=new String(w); // Clone the strings
this.definition=new String(def);
}

}


class TestMyDictionary
{public static void main(String[] args){
   DictionaryEntry [] Dico =new DictionaryEntry[10];

   Dico[0]=new DictionaryEntry("INF311","Cours d'informatique");
   Dico[1]=new DictionaryEntry("ECO311","Cours d'economie");

   }
}
```

Searching: Basic examples

- Objects denote people: Each object represents an individual
- The key (handle) is the lastname, firstname, ...
- Additional information fields are:
  address, phone number, etc.

```
Firstname
Lastname
Phonenumber
int Age
```

```
class Individual {
String Firstname;
String Lastname;

String Address;
String Phonenumber;

int Age;
boolean Sex;
}
```

Object

Searching: Basic examples

- Objects denote 2D point sets: Each object is a 2D point
- The key (handle) is the name, or x- or y- ordinate, etc.
- Additional information fields are:
  color, name, etc.

x
y
name
color

```
class Color {int Red, Green, Blue;}

class Point2D
{
double x;
double y;

String name;
Color color;
}
```

Sequential (linear) search

- Objects are stored in an **array** (of objects):
  Any order is fine (= not necessarily sorted)
- To seek for an object, we browse sequentially the array until we find the object, or that we report that it is not inside
- Compare the keys of the query with object keys

```
class Person{
    String firstname, lastname; // key for searching
    String phone;  // additional fields go here

    // Constructor
    Person(String l, String f, String p)
    {firstname=f; lastname=l; phone=p;}

    };
```

```
static Person[] array=new Person[5];

static String LinearSearch(String lastname, String firstname)
    {

    for(int i=0;i<array.length;i++)
    {
       if ((lastname.equals(array[i].lastname) &&
          (firstname.equals(array[i].firstname))))
                {return array[i].phone;}
    }

    return "Not found in my dictionary";

}
```

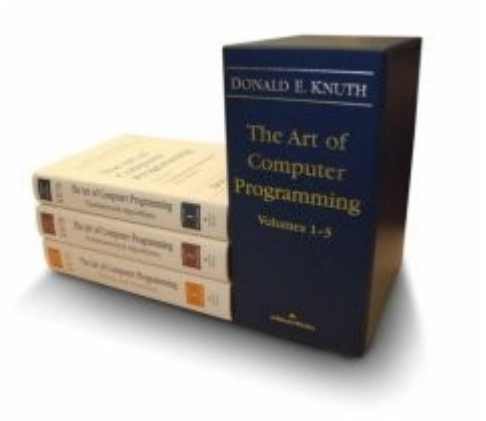
Array is class variable, here

```
class LinearSearch{
   static Person[] array=new Person[5];

   static String LinearSearch(String lastname, String firstname)
   {...}

   public static void main (String [] args)
   {
      array[0]=new Person("Nielsen","Frank","0169364089");
      array[1]=new Person("Nelson","Franck","04227745221");
      array[2]=new Person("Durand","Paul","0381846245");
      array[3]=new Person("Dupond","Jean","0256234512");
      array[4]=new Person("Tanaka","Ken","+81 3 1234 4567");

      System.out.println(LinearSearch("Durand","Paul"));
      System.out.println(LinearSearch("Tanaka","Ken"));
      System.out.println(LinearSearch("Durand","Jean"));
   }
}
```

```
0381846245
+81 3 1234 4567
Not found in my dictionary
Press any key to continue...
```

Complexity of sequential search

- In the **worst case**, we need to **scan the full array**
- On **average**, we scan **half the array size** (if object is inside)



1	a[0]
2	a[1]
4	a[2]
8	a[3]
16	a[4]

Donald Knuth. The Art of Computer Programming, Volume 3: Sorting and Searching, Third Edition. Addison-Wesley, 1997. ISBN 0-201-89685-0. Section 6.1: Sequential Searching, pp.396–408.

Adding an element (object)

- To add dynamically an element, we first build a bigger array
- Then we use an index for storing the position of the last stored element
- When added an element, we increment the position of the 'end of array'
- Of course, we should check whether the array there is overflow or not
- Notice that it is difficult to erase elements...

Adding an element to the corpus

```
public static final int  MAX_ELEMENTS=100;
static int nbelements=0;
static Person[] array=new Person[MAX_ELEMENTS];

static String LinearSearch(String lastname, String firstname)
{
for(int i=0;i<nbelements;i++)
{
   if ((lastname.equals(array[i].lastname) &&
      (firstname.equals(array[i].firstname))))
      return array[i].phone;
}
return "Not found in my dictionary";
}

static void AddElement(Person obj)
{if (nbelements<MAX_ELEMENTS)
   array[nbelements++]=obj; // At most MAX_ELEMENTS-1 here
   // nbelements is at most equal to MAX_ELEMENTS now
}
```

Adding elements to the corpus

```
public static void main (String [] args)
    {
        AddElement(new Person("Nielsen","Frank","0169334089"));
        AddElement(new Person("Nelson","Franck","04227745221"));
        AddElement(new Person("Durand","Paul","0381846245"));
        AddElement(new Person("Dupond","Jean","0256234512"));
        AddElement(new Person("Tanaka","Ken","+81 3 1234 4567"));

        System.out.println(LinearSearch("Durand","Paul"));
        System.out.println(LinearSearch("Tanaka","Ken"));
        System.out.println(LinearSearch("Durand","Jean"));
        AddElement(new Person("Durand", "Jean","0199989796"));
        System.out.println(LinearSearch("Durand","Jean"));

    }
```

```
0381846245
+81 3 1234 4567
Not found in my dictionary
0199989796
```

Check if a person already belongs to the array before adding it

Under or oversaturated memory

(not convenient for non-predictable array sizes)

Dichotomic search (in sorted arrays)

- Assume all elements (keys) are **totally sorted** in the array
- For example, in increasing order

  `array[0] < array[1] < ... <`

  (any lexicographic order would do)
- To find information, use a **dichotomic search** on the key
- Compare the key of the **middle element** with the query key:
  - If identical, we are done since we found it
  - If the key is greater, we search in the rightmost array part
  - If the key is less, we search in the leftmost array part
  - If the range [left,right] is <=1 and element not inside then...

    .....conclude it is not inside the array

Dichotomic search/Bisection search

```
static int Dichotomy(int [] array, int left, int right, int key)
   {
      if (left>right) return -1;

      int m=(left+right)/2; // !!! Euclidean division !!!

      if (array[m]==key) return m;
      else
      {
      if (array[m]<key) return Dichotomy(array,m+1, right, key);
            else  return Dichotomy(array,left,m-1, key);
      }
   }

static int DichotomicSearch(int [] array, int key)
   {
      return Dichotomy(array,0,array.length-1, key);
   }
```

Think recursion!

Result is the index of the element position
Does it always terminate? Why?

Dichotomic search/Bisection search

```
public static void main (String[] args)
    {
    int [] v={1, 6, 9, 12 , 45, 67, 76, 80, 95};

    System.out.println("Seeking for element 6: Position "+DichotomicSearch(v,6));
    System.out.println("Seeking for element 80: Position "+DichotomicSearch(v,80));
    System.out.println("Seeking for element 33: Position "+DichotomicSearch(v,33));
    }
```

```
1, 6, 9, 12 , 45, 67, 76, 80, 95
9 elements [0,8] m=4   6<45

1, 6, 9, 12
4 elements [0,3] m=1 6=6 Found it return 1
```

```
Seeking for element 6: Position 1
Seeking for element 80: Position 7
Seeking for element 33: Position -1
```

Complexity of bisection search

We halve by 2 the array range at every recursion call

n => n/2 => (n/2)/2 => ((n/2)/2)/2 => ...etc... => 1 or 0

How many recursion calls?

2**k=n => k=log(n)

(Execution stack (memory) is also of order log n)

Big O(.)-notation of ***ALGORITHMS*** & complexity

Requires a **logarithmic number of operations**: O(log n)

t(1024)=10,
t(2048)=11,
t(65556)=16

=> (exponentially more) Efficient compare to sequential search

!!! SORTING HELPS A LOT !!!

```
static int Dichotomy(int [] array, int left, int right, int key)
    {

        if (left>right)
                {throw new RuntimeException("Terminal state reached");
                          //return -1;
                          }

        int m=(left+right)/2;

        if (array[m]==key)
                {throw new RuntimeException("Terminal state reached");
                   //return m;
                   }
        else
        {
           if (array[m]<key) return Dichotomy(array,m+1, right, key);
                else  return Dichotomy(array,left,m-1, key);
        }
    }
```

```
Exception in thread "main" java.lang.RuntimeException: Terminal state reached
        at DichotomySearch.Dichotomy(dichotomysearch.java:14)
        at DichotomySearch.Dichotomy(dichotomysearch.java:20)
        at DichotomySearch.DichotomicSearch(dichotomysearch.java:27)
        at DichotomySearch.main(dichotomysearch.java:34)
```

Sorting: A fundamental procedure

- Given an (unsorted) array of elements
- We are asked to get a **sorted array** in, say **increasing order**
- The only primitive operations (basic operations) are
  - **comparisons** and
  - element **swappings**

```
static boolean GreaterThan(int a, int b)
{return (a>b);}

static void swap (int [] array, int i, int j)
{
int tmp=array[i];
array[i]=array[j];
array[j]=tmp;
}
```

Sorting: Sorting by selection

Many different sorting techniques (endless world)

- First, seek for the smallest element of the array (=SELECTION)
- Put it at the front (using a swapping operation)
- Iterate for the remaining part:

```
array[1], ..., array[n-1]
```

=> we seek for the smallest elements for all (sub)arrays

```
array[j], array[j+1], ..., array[n-1]
```

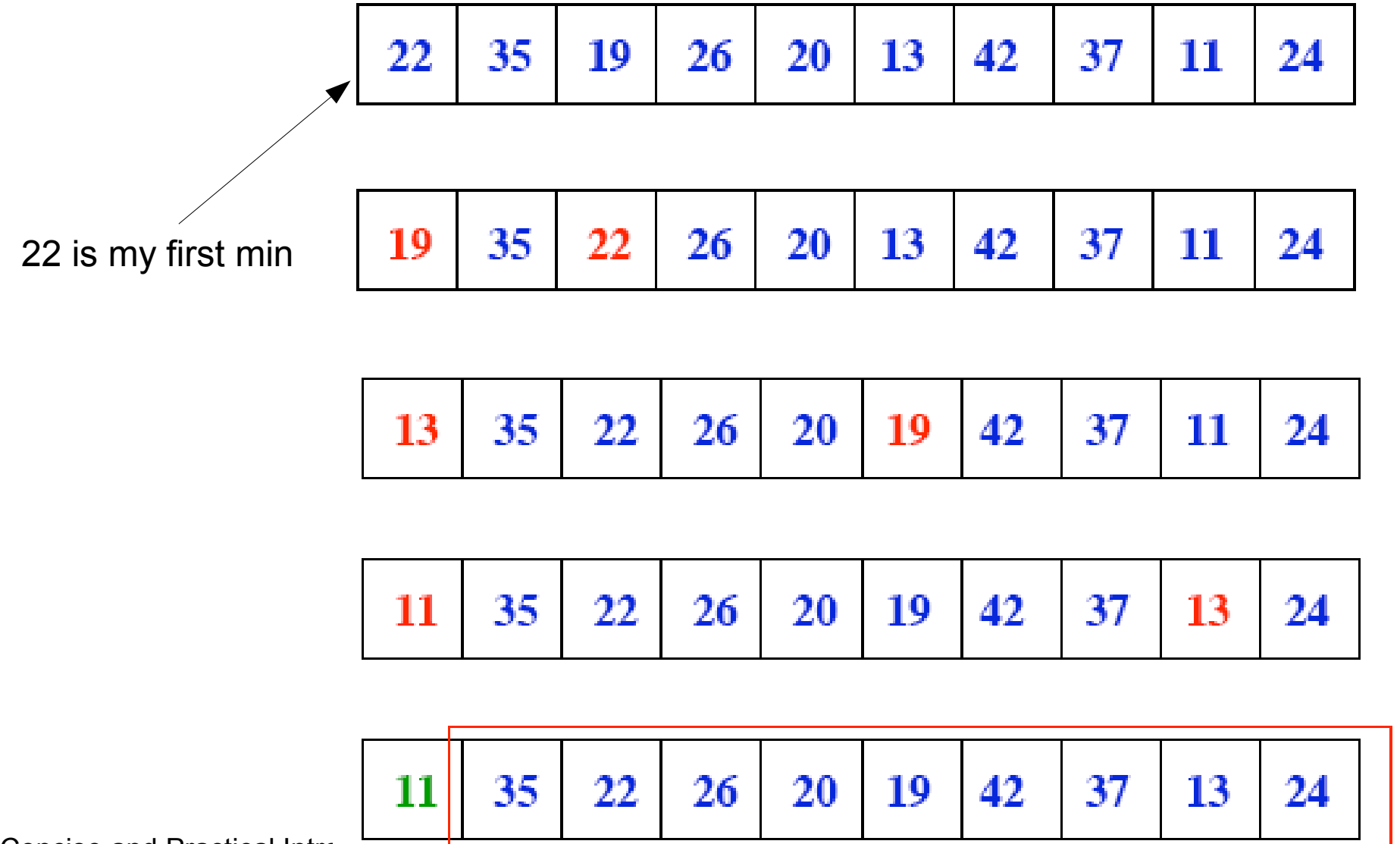
Sorting by selection

Pseudo-code: 2 NESTED LOOPS

```
for i ← 0 to n-2 do
    min ← i
    for j ← (i + 1) to n-1 do
        if A[j] < A[min]
            min ← j
    swap A[i] and A[min]
```

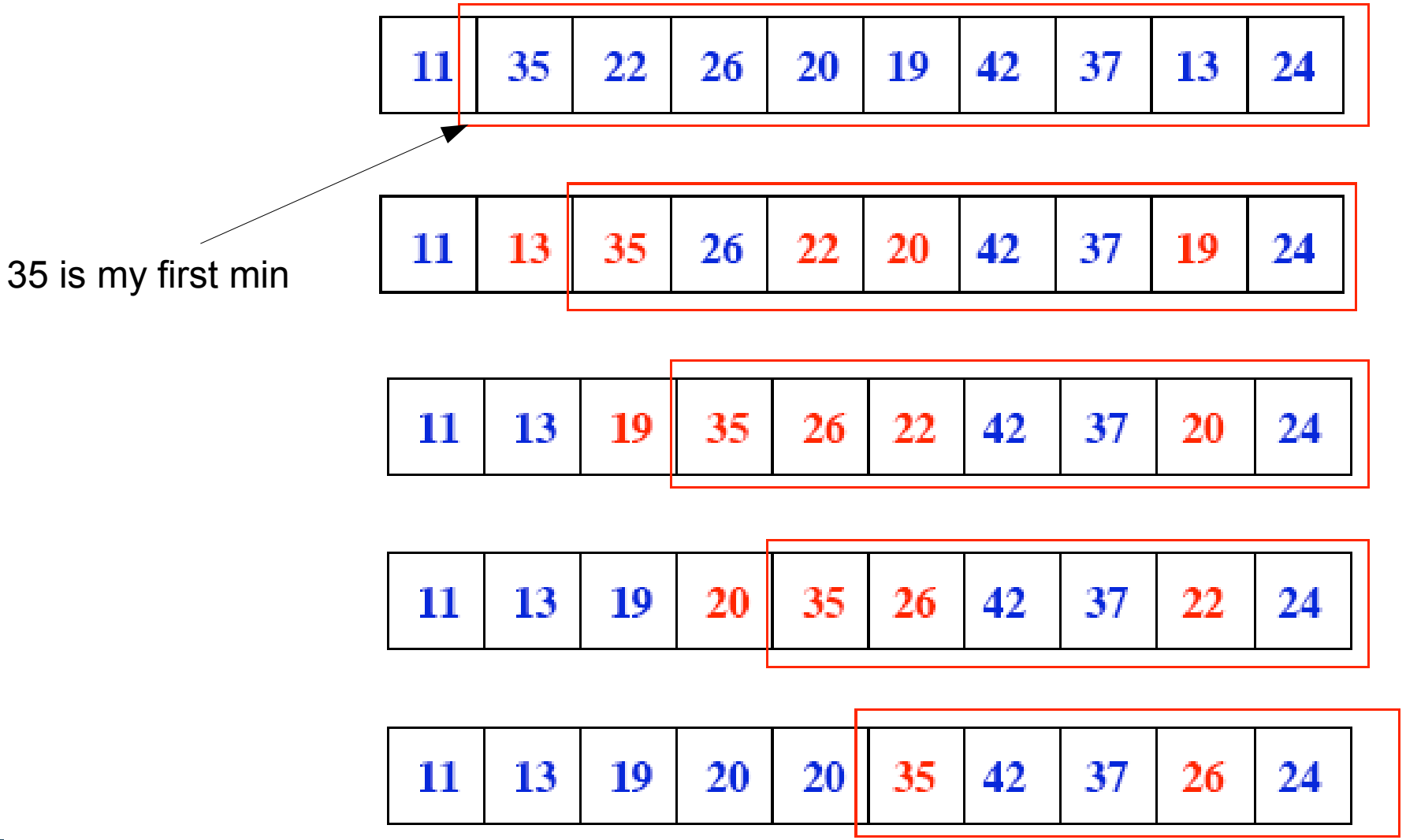
```
for i ← 0 to n-2 do
    min ← i
    for j ← (i + 1) to n-1 do
        if A[j] < A[min]
            min ← j
    swap A[i] and A[min]
```

The inner loop (i=0)



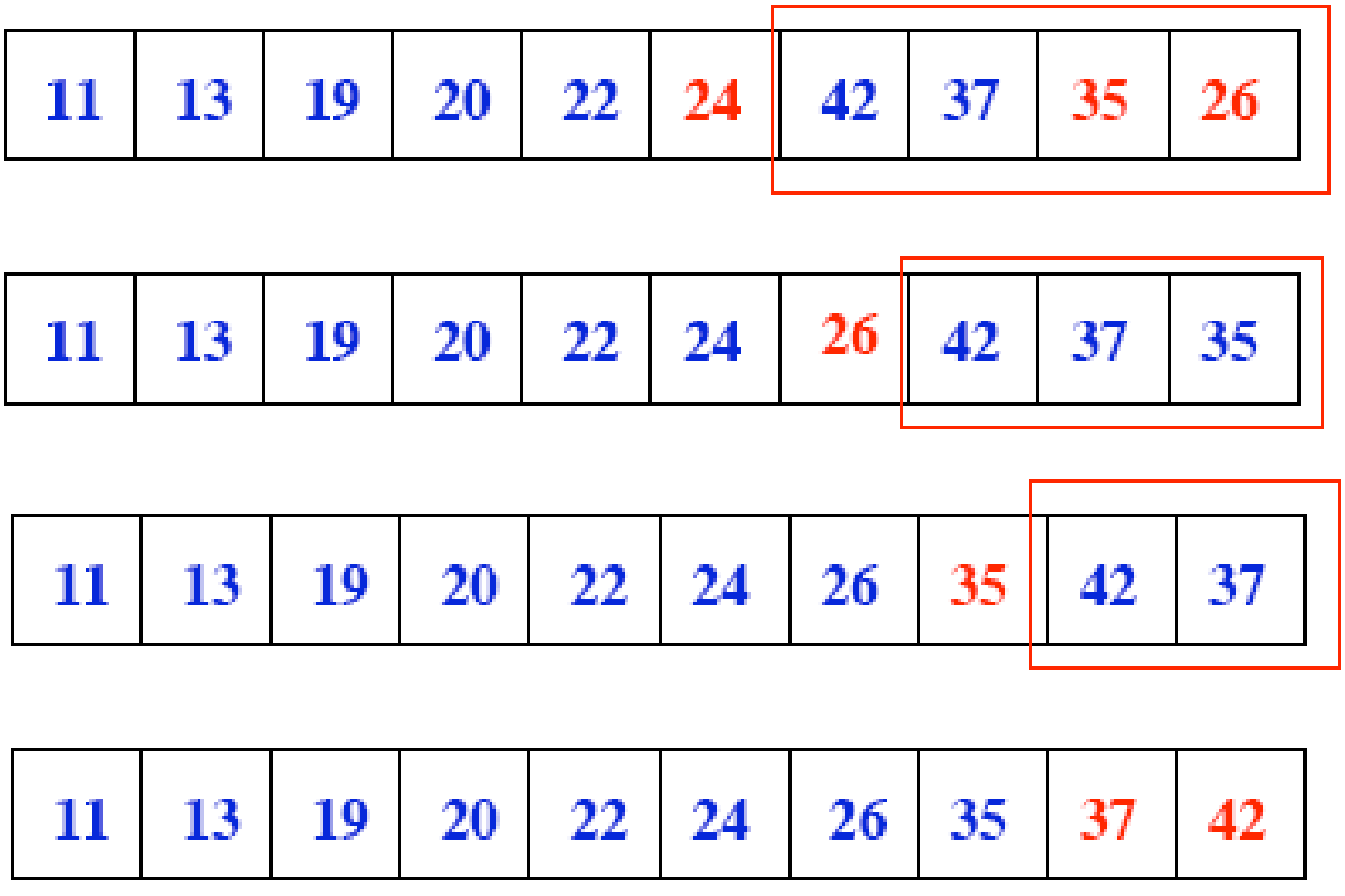
```
for i ← 0 to n-2 do
    min ← i
    for j ← (i + 1) to n-1 do
        if A[j] < A[min]
            min ← j
    swap A[i] and A[min]
```

The inner loop (i=1)



```
for i ← 0 to n-2 do
    min ← i
    for j ← (i + 1) to n-1 do
        if A[j] < A[min]
            min ← j
    swap A[i] and A[min]
```

The outer loop: min of respective subarrays



Sorting by selection

Two nested loops:

- Select and put the smallest element in front of the array (inner loop)
- Repeat for all subarrays (at the right)

```
static boolean GreaterThan(int a, int b)
{return (a>b);}

static void swap (int [] array, int i, int j)
{int tmp=array[i];array[i]=array[j];array[j]=tmp;}

static void SelectionSort(int [] array)
{
int n=array.length;

for(int i=0;i<n-1;i++){
   for(int j=i+1;j<n;j++){
      if (GreaterThan(array[i],array[j]))
       swap(array,i,j);}
       }
}
```

Sorting by selection

```
public  static void main(String[] args)
   {
      int [] array={22,35,19,26,20,13,42,37,11,24};

      SelectionSort(array);

      for(int i=0;i<array.length;i++)
         System.out.print(array[i]+" ");
         System.out.println("");
   }
```

```
11 13 19 20 22 24 26 35 37 42
```

From sorting integers to objects

Generic algorithms for sorting any kind (=type) of elements

- Array `array` contains objects of the same type
- Define a predicate that returns whether an object is greater than another one or not
- Adjust the swap procedure

In Java 1.5, there are generics, beyond the scope of this course

For example, sorting events

```
class EventObject
{
      int year, month, day;
      EventObject(int y, int m, int d)
      {year=y; month=m; day=d; }

      static void Display(EventObject obj)
      {System.out.println(obj.year+"/"+obj.month+"/"+obj.day); }
}
```

```
static boolean GreaterThan(EventObject a, EventObject b)
   {return (( a.year>b.year) ||
            ( (a.year==b.year) && (a.month>b.month)) ||
((a.year==b.year) && (a.month==b.month) && (a.day>b.day)) ) ;}

   static void swap (EventObject [] array, int i, int j)
   {
   EventObject tmp=array[i];
   array[i]=array[j];
   array[j]=tmp;
   }
```

sorting events

```
static void SelectionSort(EventObject [] array)
   {
   int n=array.length;

   for(int i=0;i<n-1;i++)
      for(int j=i+1;j<n;j++)
         if (GreaterThan(array[i],array[j]))
          swap(array,i,j);
   }

   public  static void main(String[] args)
   {
      EventObject [] array=new EventObject[5];

      array[0]=new EventObject(2008,06,01);
      array[1]=new EventObject(2005,04,03);
      array[2]=new EventObject(2005,05,27);
      array[3]=new EventObject(2005,04,01);
      array[4]=new EventObject(2005,04,15);

      SelectionSort(array);
      for(int i=0;i<array.length;i++)
         EventObject.Display(array[i]);
         System.out.println("");
   }
```

2005/4/1
2005/4/3
2005/4/15
2005/5/27
2008/6/1

Worst-case complexity of selection sort

Quadratic time: O(n^2)
= number of elementary operations
= number of comparisons+swap operations

...Try sorting a ***reversed*** sorted array...

16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1
1, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2
1, 2, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3
...
nb=2*array.length*(array.length-1)/2
(2 times n choose 2)

```
class SelectionSort
{
static int nboperations;

   static boolean GreaterThan(int a, int b)
   {nboperations++; return (a>b);}

   static void swap (int [] array, int i, int j)
   {
   nboperations++;
   int tmp=array[i];array[i]=array[j];array[j]=tmp;}

public  static void main(String[] args)
   {
      int [] array={16,15,14,13,12,11,10,9,8,7,6,5,4,3,2,1};
      nboperations=0;

      SelectionSort(array);


    System.out.println("Number of operations:"+nboperations);

    int nb=2*array.length*(array.length-1)/2;
    System.out.println("Number of operations:"+nb);
   }
```

- Classify the elements according to `array[0]` (the **pivot**):

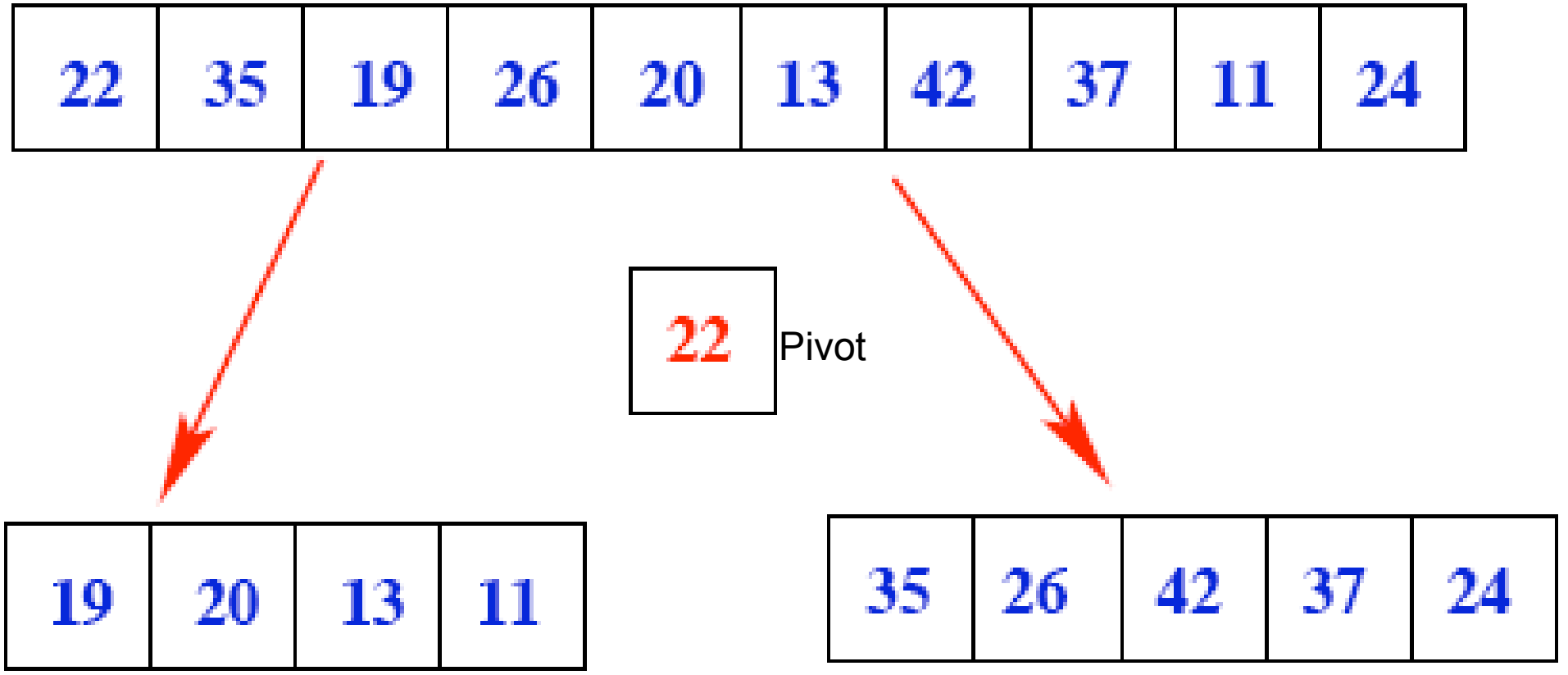
  - those *smaller than* the pivot: `arrayleft`
  - those *greater than* the pivot: `arrayright`
  - those *equal to* the pivot (in case of ties): `arraypivot`

- **Solve recursively** the sorting in `arrayleft` / `arrayright`, and recompose as

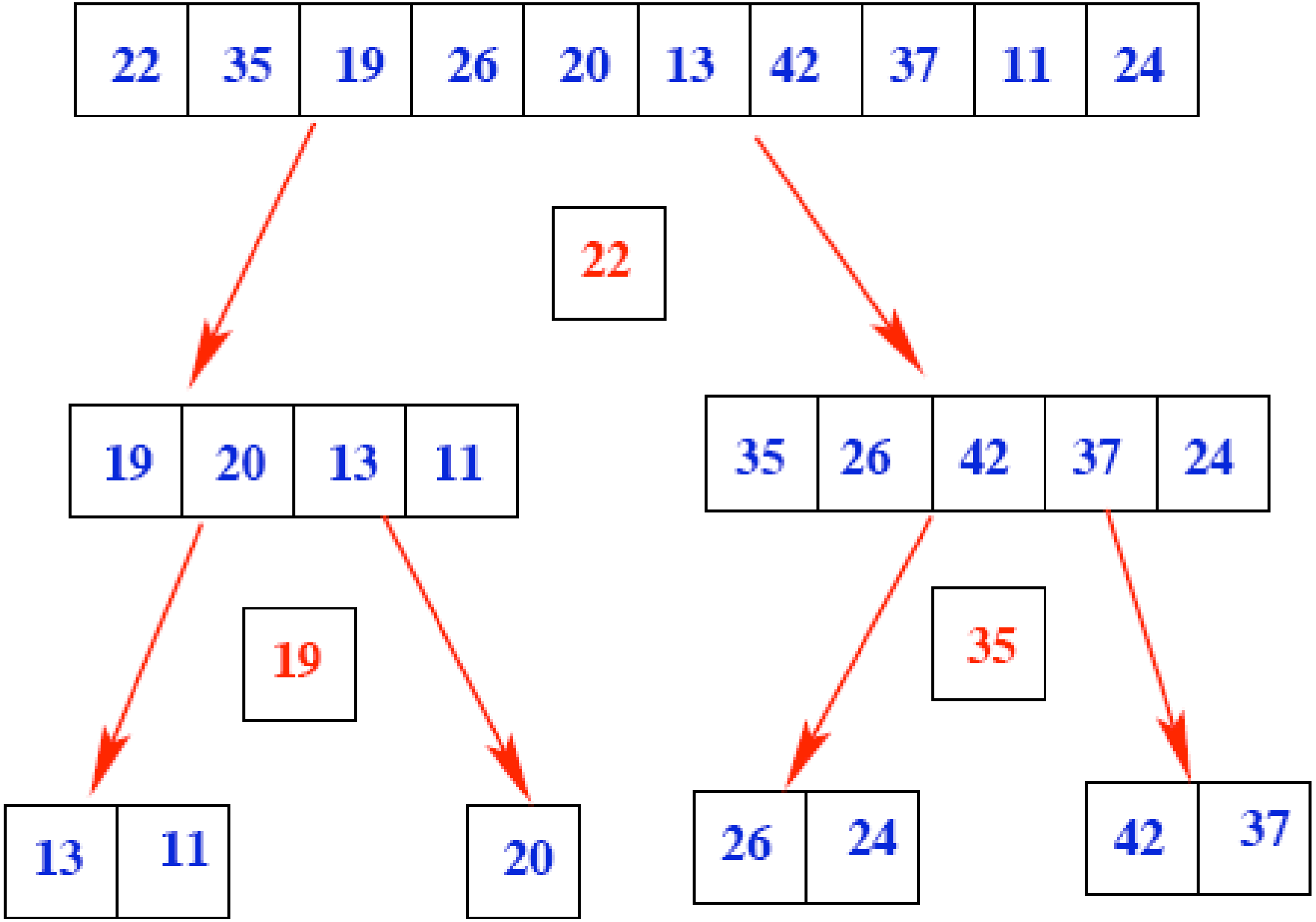
```
arrayleft arraypivot arrayright
```

- Note that a single element is sorted (=**terminal case**)

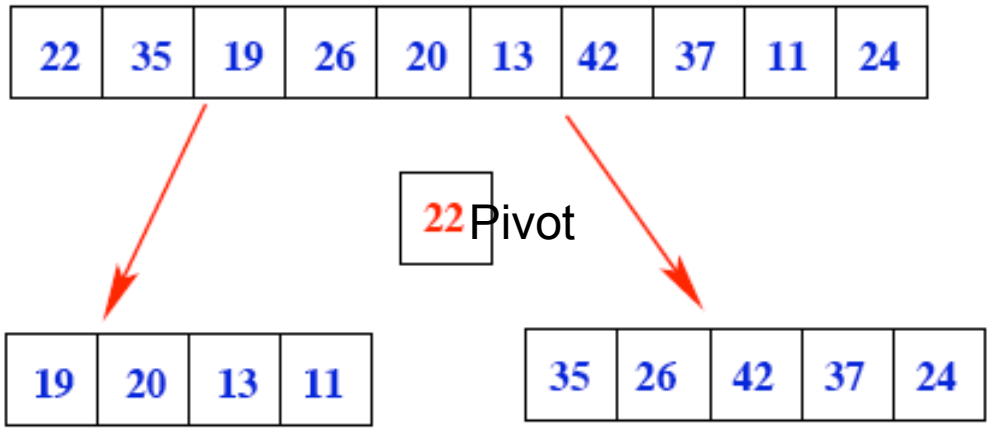
A recursive approach: Quicksort



Partition the array into 3 pieces


22 35 19 26 20 13 42 37 11 24
22
19 20 13 11
35 26 42 37 24
19
35
13 11
20
26 24
42 37

```
static int partition(int [] array, int left, int right)
   {
      int m=left;// pivot

      for(int i=left+1; i<=right;i++)
      {
         if (GreaterThan(array[left],array[i]))
         {
            swap(array,i,m+1);
            m++;// we extend left side array
            }
      }
         // place pivot now
         swap(array,left,m);

         return m;

   }
```



Sorting is **in-place**
(require no extre memory storage)

```
static void quicksort(int [] array, int left, int right)
   {
      if (right>left)
      {
         int pivot=partition(array,left,right);
         quicksort(array,left,pivot-1);
         quicksort(array,pivot+1,right);
      }
   }

static void QuickSort(int [] array)
   {
   quicksort(array,0, array.length-1);
   }
```

Quicksort: Analyzing the running time

- **Worst-case** running time is quadratic **O(n^2)**
- **Expected** running time is **O(n log n)**
- Best case is linear **O(n)** (all elements are identical)

http://www.cs.ubc.ca/~harrison/Java/sorting-demo.html

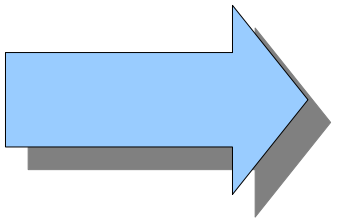
- **Lower bound** is n log n: Any algorithm sorting n numbers require n log n comparison operations

Las Vegas complexity analysis, tail bounds (how much deviation), etc...

Hashing: Principle and techniques

- Hashing: fundamental technique in computer science (see INF 421.a')
- Store object `x` in position `h(x) (int)` in an array
- Problem occurs if two objects `x` and `y` are stored on the same cell: **Collision.**
- Finding good hashing functions that minimize collisions, and adopting a good search policy in case of collisions are key problems of hashing.

```
int i;
array[i]
```



```
Object Obj=new Object();
int i;
i=h(Obj);// hashing function
array[i]
```

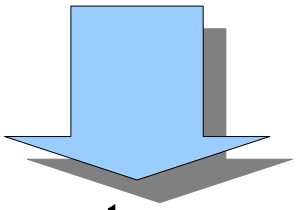
Hashing functions

- Given a universe X of keys and for any x in X,
  we need to find an integer h(x) between 0 and m

- Usually *easy to transform* the object into an integer:

For example, for strings just add the ASCII codes of characters

- The problem is then to transform
  a set of n (sparse) integers

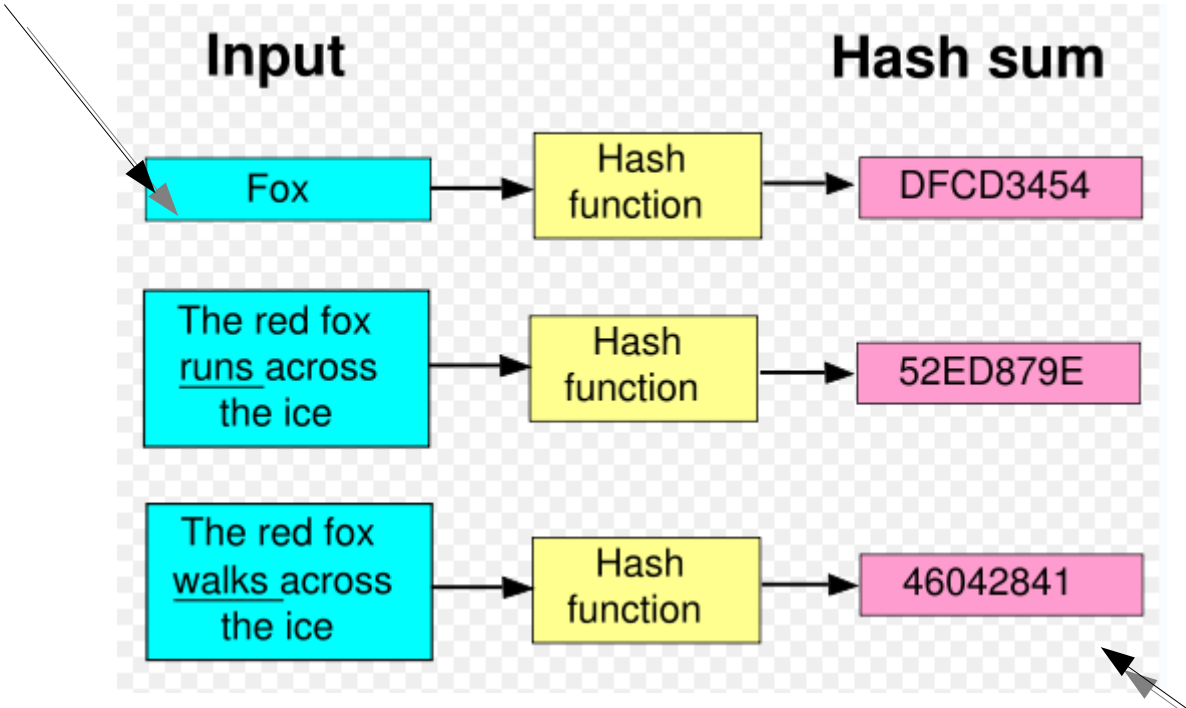


into a compact array of size m<<N.

(<< means much less than)

Hashing strings

String lengths
may vary much



http://en.wikipedia.org/wiki/Hash_function

Constant-size representation (compact)

Hashing functions

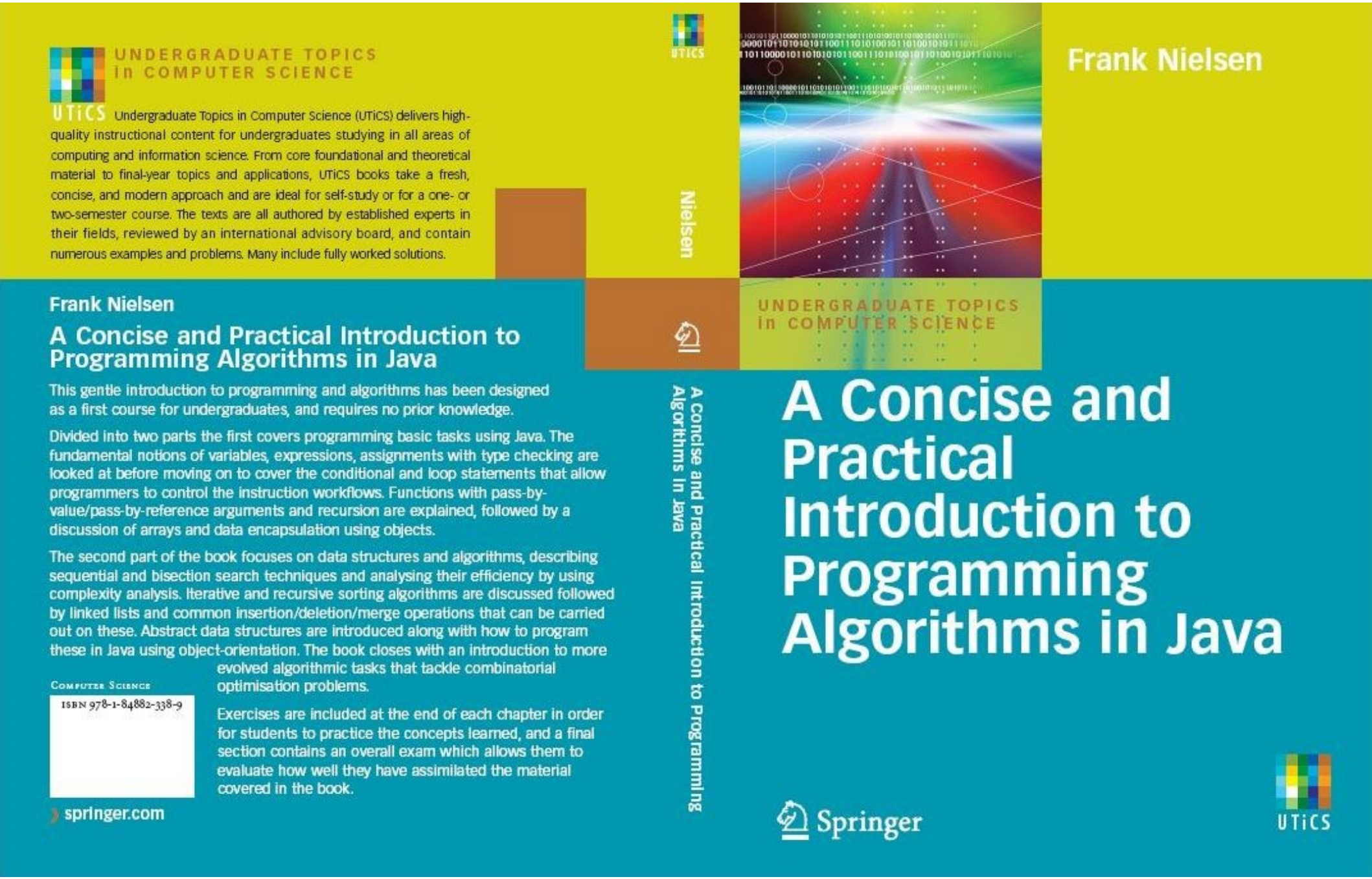
Key idea is to take the modulo operation

h(k) = k mod m where m is a **prime number**.

```
static int m=23;

   static int String2Integer(String s)
   {
      int result=0;

      for(int j=0;j<s.length();j++)
         result+=(int)s.charAt(j);

      return result;
   }

   // Note that m is a static variable
   static int HashFunction(int l)
   {return l%m;}
```

```
    public static void main (String[] args)
    {
    String [] animals={"cat","dog","parrot","horse","fish",
    "shark","pelican","tortoise", "whale", "lion",
    "flamingo", "cow", "snake", "spider", "bee", "peacock",
    "elephant", "butterfly"};

    int i;
    String [] HashTable=new String[m];

    for(i=0;i<m;i++)
       HashTable[i]=new String("-->");



    for(i=0;i<animals.length;i++)
       {int pos=HashFunction(String2Integer(animals[i]));
       HashTable[pos]+=(" "+animals[i]);
       }

    for(i=0;i<m;i++)
       System.out.println("Position "+i+"\t"+HashTable[i]);

}
```

```
Position 0     --> whale
Position 1     --> snake
Position 2     -->
Position 3     -->
Position 4     -->
Position 5     -->
Position 6     -->
Position 7     --> cow
Position 8     --> shark
Position 9     -->
Position 10     -->
Position 11     -->
Position 12     --> fish
Position 13     --> cat
Position 14     -->
Position 15     --> dog tortoise
Position 16     --> horse
Position 17     --> flamingo
Position 18     -->
Position 19     --> pelican
Position 20     --> parrot lion
Position 21     -->
Position 22     -->
```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