Stable cycles for Autonomous Underwater Vehicles navigation

Quentin Brateau, Fabrice Le Bars, Luc Jaulin

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ENSTA Bretagne

Context

Research laboratory

- ENSTA Bretagne, UMR 6285, Lab-STICC, IAO, ROBEX

Supervisors

- Luc Jaulin
- Fabrice Le Bars

Funding

- AID funding: Jean-Daniel Masson

AUV

- Control of torpedo-like AUV
- Riptide's micro-uuv

Environment

- Constrained environment
- Pool, harbor, ...

Goals

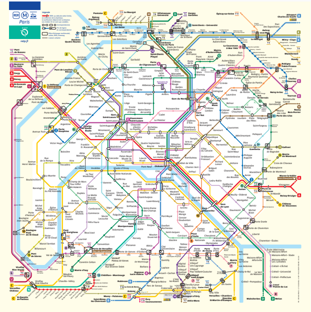
- Reactivity
- Manoeuvrability

Figure 1: Harbor and Riptide in the ENSTA Bretagne pool

Introduction

Figure 2: Earth-orbiting positioning and communications satellite

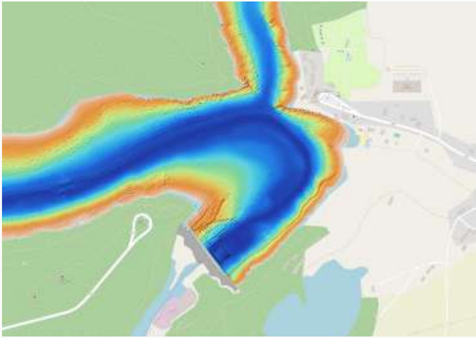
(a) Metric map - OpenStreetMap [1]



(b) Topological map - RATP [2]

Figure 3: Paris metric and topological map

[1] https://www.openstreetmap.org
[2] https://www.ratp.fr/plan-metro



(a) Guerlédan bathymetric map

(b) Ping2 Blureobotics [1]

Figure 4: Digital elevation model and depth sensing

[1] https://bluerobotics.com/

Dynamical system

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \quad (1)$$

Flow Function

$$\forall(\mathbf{x}_0, t1, t2) \in \mathcal{S} \times \mathcal{T}^2$$

$$\begin{aligned} \boldsymbol{\varphi}_{\mathbf{u}}(\mathbf{x}_0, t_1) &= \mathbf{x}(t_1) \\ \boldsymbol{\varphi}_{\mathbf{u}}(\mathbf{x}_0, 0) &= \mathbf{x}_0 \\ \boldsymbol{\varphi}_{\mathbf{u}}(\boldsymbol{\varphi}_{\mathbf{u}}(\mathbf{x}_0, t1), t2) &= \boldsymbol{\varphi}_{\mathbf{u}}(\mathbf{x}_0, t1 + t2) \end{aligned} \quad (2)$$

Dubins car

$$\mathbf{f}(\mathbf{x}, u) = \begin{bmatrix} V \cdot \cos(\theta) \\ V \cdot \sin(\theta) \\ u \end{bmatrix} \quad (3)$$

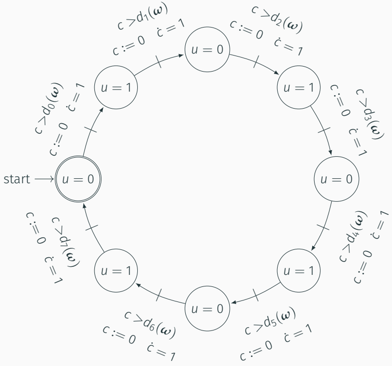


Figure 5: Square cyclic timed automata

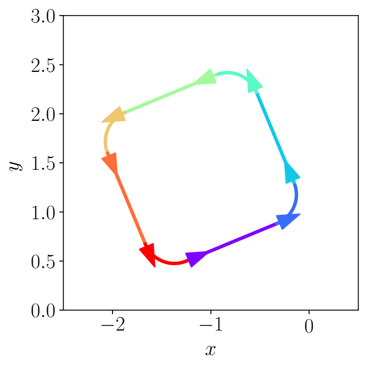


Figure 6: Square Cycle

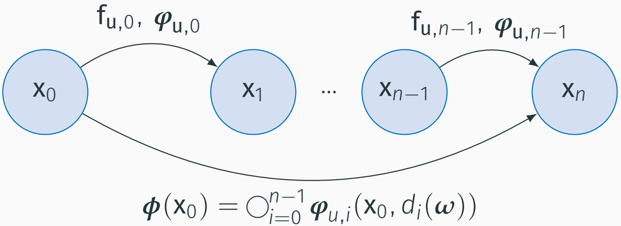


Figure 7: Composition of flow functions over a cycle

Cyclic Period

$$T(\boldsymbol{\omega}) = \sum_{i=0}^{n-1} d_i(\boldsymbol{\omega}) \tag{4}$$

Synchronization Condition

$$\boldsymbol{\phi}(\mathbf{x}(t)) \triangleq \mathbf{x}(t + T(\boldsymbol{\omega})) = \mathbf{x}(t) \tag{5}$$

Cycle abstraction

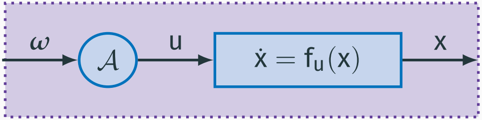


Figure 8: Block diagram of the robot controlled by a timed automata

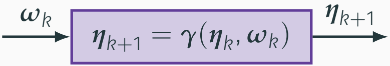


Figure 9: Block diagram of the controlled cycle

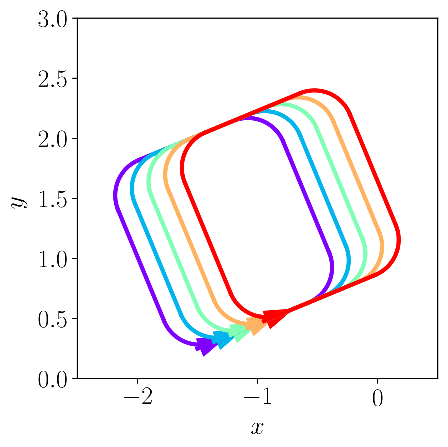


Figure 10: Cycle move with $\omega_0 = -0.15$

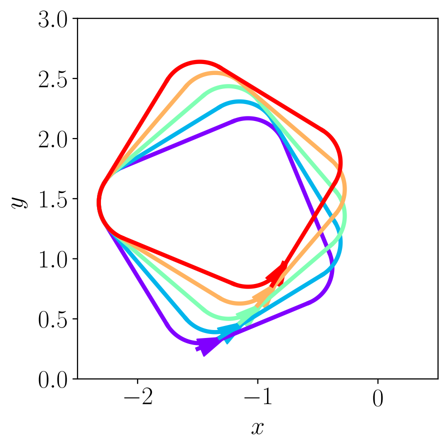


Figure 10: Cycle move with $\omega_1 = 0.1$

Cycle move through iterations

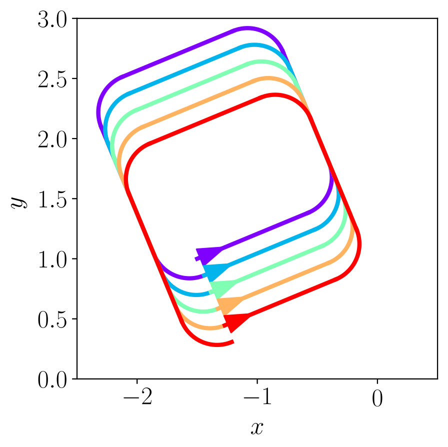


Figure 10: Cycle move with $\omega_2 = 0.15$

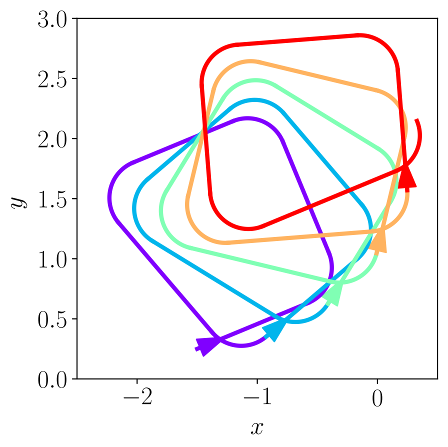


Figure 10: Control of the cycle with $\boldsymbol{\omega} = \begin{bmatrix} -0.1 & 0.2 & 0.2 \end{bmatrix}^T$

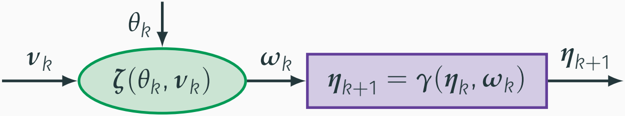


Figure 11: Block diagram of the controlled cycle

$$\zeta(\theta_k, \boldsymbol{\nu}_k) = \begin{bmatrix} -cos(\theta_k) & 0 & sin(\theta_k) \\ -sin(\theta_k) & 0 & -cos(\theta_k) \\ 0 & 1 & 0 \end{bmatrix} \cdot \boldsymbol{\nu}_k \quad (6)$$

Cycle world move through iterations

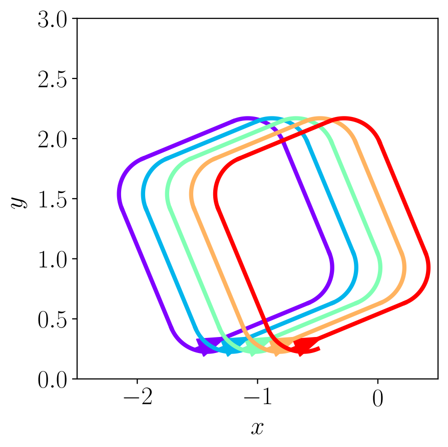


Figure 12: Cycle move with $\nu_0 = 0.2$

Cycle world move through iterations

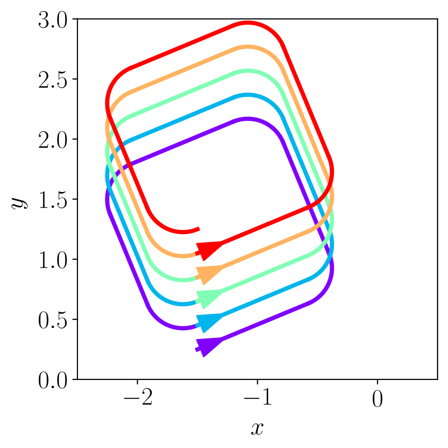


Figure 12: Cycle move with $\nu_1 = 0.2$

Cycle world move through iterations

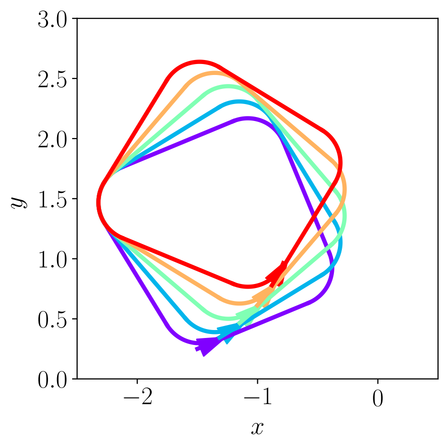


Figure 12: Cycle move with $\nu_2 = 0.1$

Adding measurements

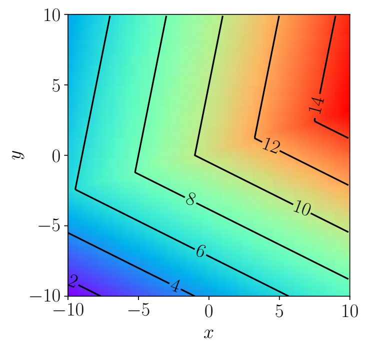


Figure 13: Bathymetric map

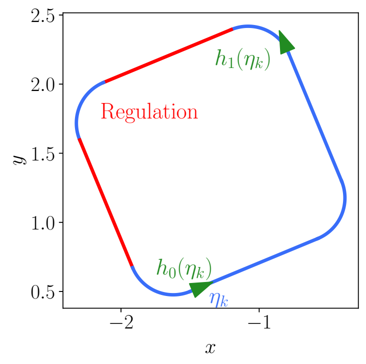


Figure 14: Measurements position

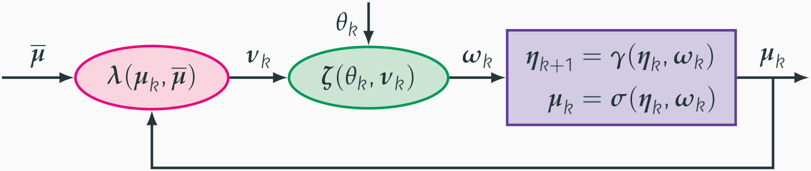


Figure 15: Block diagram of the autonomous system

$$\lambda(\mu_k, \overline{\mu}) = K \cdot \arctan\left(\frac{\overline{\mu} - \mu_k}{r}\right) \quad (7)$$

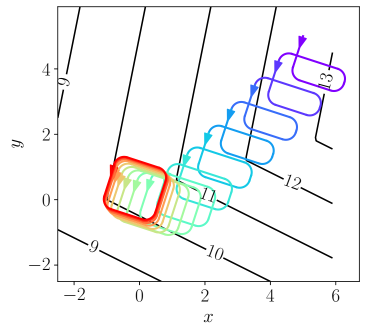


Figure 16: Measurements simulation

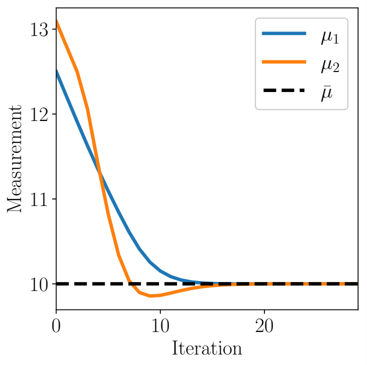


Figure 17: Measurements results

Trials

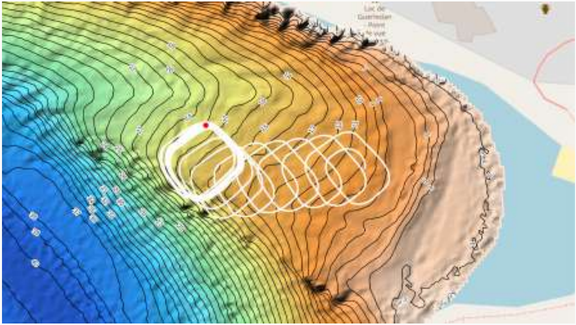


Figure 18: Guerlédan's Lake Stable Cycle Trial [a]

[a] https://www.youtube.com/watch?v=MDJ6iHYhxyM

Cycle navigation

Cycle element

Given a topological space X, $\pi(X, x_0)$ is the pointed space at x_0

$$\forall \gamma \in \pi(X, x_0), \begin{cases} \gamma : [0,1] \to X \\ \gamma(0) = \gamma(1) = x_0 \end{cases} \tag{8}$$

Concatenation and Cycle group

$\forall (\gamma_0, \gamma_1) \in \pi(X, x_0)^2,$

$$\gamma_0 \cdot \gamma_1 = \begin{cases} \gamma_0(t), & 0 \leq t < \frac{1}{2} \\ \gamma_1(t), & \frac{1}{2} \leq t \leq 1 \end{cases} \tag{9}$$

Figure 19: Cycles $(\gamma_0, \gamma_1) \in \pi(X, x_0)$

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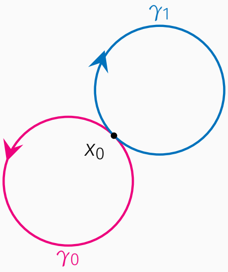

Figure 19: Cycles $(\gamma_0, \gamma_1) \in \pi(X, x_0)$

Cycle switch

- Start from a stabilized cycle
- Reach the capture basin of the next cycle

Relationship between cycles

- Define a binary relationship operator $\mathcal{R}$ between elements of cycle group

$\mathcal{R}$ is a Preorder

- **Reflexivity**: $\gamma_0 \mathcal{R} \gamma_0$
- **Transitivity**: $\gamma_0 \mathcal{R} \gamma_1 \wedge \gamma_1 \mathcal{R} \gamma_2 \implies \gamma_0 \mathcal{R} \gamma_2$

Cycle switch

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Reflexivity

$\forall i \in [\![0, 4]\!], r_i = id$

Transitivity

$\forall i \in [\![4, 11]\!], r_i$

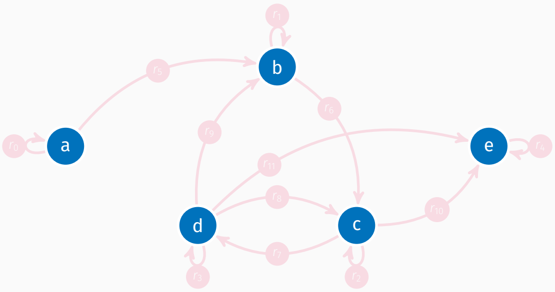

Figure 20: Graph of $\mathcal{R}$ relationship

Reflexivity

$\forall i \in [\![0, 4]\!], r_i = id$

Transitivity

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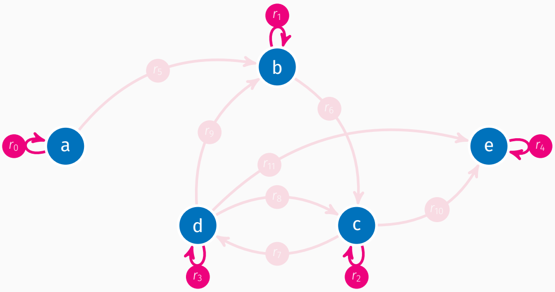

Figure 20: Graph of $\mathcal{R}$ relationship

Cycle navigation example

Reflexivity
$\forall i \in [\![0, 4]\!], r_i = id$

Transitivity
$\forall i \in [\![4, 11]\!], r_i$

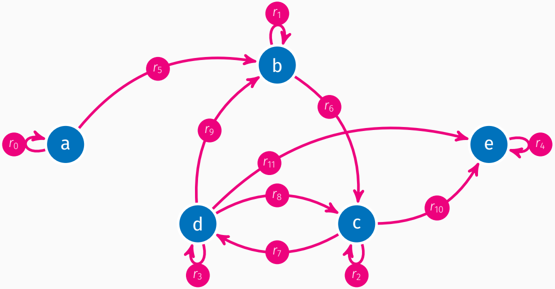

Figure 20: Graph of $\mathcal{R}$ relationship

Starting cycles

- Unable to reach once leave
- Let robot stabilize before starting

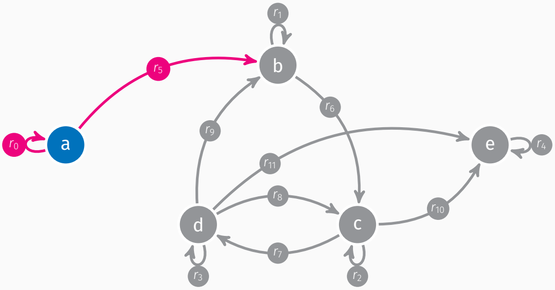

Figure 21: Starting Cycles

Navigating cycles

- Used for navigation
- $r_8 = r_9 \cdot r_6$

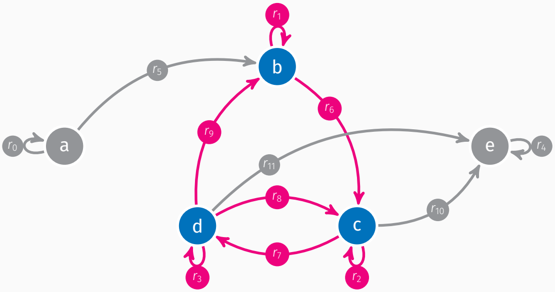

Figure 21: Navigating Cycles

Recovering cycles

- Unable to leave once reached
- Let system be recovered at known location

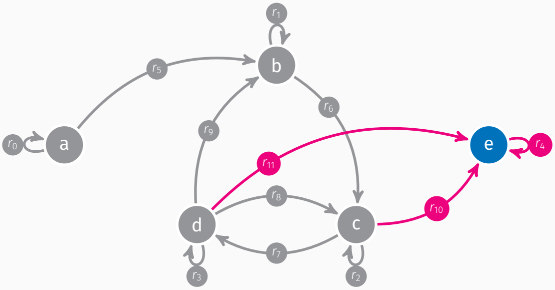

Figure 21: Recovering Cycles

Guerlédan’s lake Cycle Switch

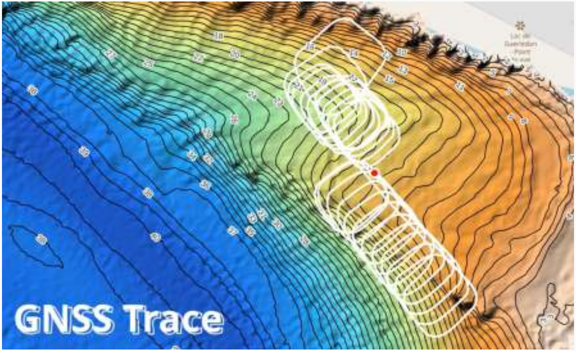

Figure 22: Guerlédan’s Lake Cycle Switch Trial [a]

[a] https://www.youtube.com/watch?v=MDJ6iHYhxyM

Conclusion

Contributions

- Introduction of stable cycles
- Cycle switch for navigation without getting lost
- Validation of this concept in simulation and real environment

Perspectives

- Test with underwater robots
- Complex mission involving many Cycle Switches

Questions?

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