## DIRECTED ALGEBRAIC TOPOLOGY

## AND <br> CONCURRENCY

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MPRI : Concurrency (2.3.1)

- Lecture 1 -

2023-2024

## A QUICK OVERVIEW

## OF

## CONCURRENCY THEORY



Robin Milner
Communication
and Concurrency

Roberto Gorrieri

## HaNDBOOK

 OF Process Algebra
## Introduction to Concurrency Theory

Transition Systems and CCS

Edited by<br>J.A. Bergstra<br>A. Ponse<br>S.A. Smolka

| Handbook of Logic in Computer Science VOLUME 4 |
| :---: |
| S. ABRAMSKY, DOV M. GABBAY, and T. S. E. MAIBAUM |



## CONCURRENT PROGRAMMING

PRINCIPLES AND PRACTICE


## THE ORIGIN OF concubiant Programming

From Semaphores to Remote Procedure Calls
-7Mnentin

PER BRINCH HANSEII
Editor

## PARALLEL AUTOMATA META LANGUAGE

## Syntax

## Paradigm

Cooperating sequential processes, E. W. Dijkstra, 1965.
System deadlocks, E. G. Coffman, M. J. Elphick, and A. Shoshani, 1971.
The geometry of semaphore programs, S. D. Carson and P. F. Reynolds, 1987.

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- The Carson and Reynolds language is a restriction of Dijkstra's language:
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- var <identifier> $=<$ constant>
e.g. var $\mathrm{x}=0$
- proc <identifier> = <basic block>
- init <multiset of identifiers>
e.g. init a 2b 3c

Expressions and values

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| $v$ | content of $v \in \mathcal{V}$ | $x \in \mathbb{R}$ | constant |
| :---: | :--- | :--- | :--- |
| $\wedge$ | minimum | $\vee$ | maximum |
| + | addition | - | substraction |
| $*$ | multiplication | $/$ | division |
| $\leqslant$ | less or equal | $\geqslant$ | greater of equal |
| $<$ | strictly less | $>$ | strictly greater |
| $=$ | equal | $\neq$ | not equal |
| $\neg$ | complement | $\%$ | modulo |
| $\perp$ | bottom |  |  |


| nullary | unary |
| :---: | :---: |
| $\perp, x \in \mathbb{R}, v \in \mathcal{V}$ | $\neg$ |
| binary |  |
| $\wedge, \vee,+,-, *, /,<,>, \leqslant, \geqslant,=, \neq, \%$ |  |

Non branching instructions

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- ( $L$ ) enclose a list of instructions between parenthesis to make it a single instruction


## Branching

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The branching is provided by a kind of "match case like" instruction

$$
\left(L_{1}\right)+\left[e_{1}\right]+\left(L_{2}\right)+\left[e_{2}\right]+\cdots+\left(L_{n}\right)+\left[e_{n}\right]+\left(L_{n+1}\right)
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- If all the expressions evaluate to zero, then $L_{n+1}$ is triggered.


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Due to the branchings, basic blocks are actually trees.

Control Flow Graphs

## Control flow graphs and flowcharts

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- At the core of many softwares dealing with source code e.g. GCC (cf. "basic blocks"), LLVM, Frama-C.
- No such structure exist for parallel programs.


## Generators



## The Hasse-Syracuse algorithm in PAML

$$
\begin{aligned}
& \operatorname{var} x=7 \\
& \text { proc } p=()+[x=1]+J(q) \\
& \text { proc } q=(x:=x / 2)+[x \% 2=0]+(x:=3 * x+1) ; J(p) \\
& \text { init } p
\end{aligned}
$$

## Building the control flow graph

of the Hasse-Syracuse algorithm

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## Reducing the Control Flow Graph

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of the Hasse-Syracuse algorithm


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the current value of x is 7

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## An execution trace on a control flow graph

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the current value of x is 22

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the current value of x is 52

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Execution traces as paths over a control flow graph

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## Execution traces as paths over a control flow graph

- Any execution trace induces a path
- Some paths do not come from an execution trace
- Therefore the collection of path provides a (strict) overapproximation of the collection of execution traces


## Execution traces as paths over a control flow graph

- Any execution trace induces a path
- Some paths do not come from an execution trace
- Therefore the collection of path provides a (strict) overapproximation of the collection of execution traces
- The (infinite) collection of paths is entirely determined by the (finite) control flow graph

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One of the goal of the course it to provide such a structure for a large class of PAML programs.

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# Abstract Machine 

Abstract expressions

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- the other operators are interpreted as expected


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- The synchronisation primitives $P(s), V(s)$, and $W(b)$ for $s \in \mathcal{S}$ and $b \in \mathcal{B}$

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The arrows are interpreted as intermediate positions of the instruction pointer so a point on a control flow graph is either a vertex or an arrow.

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- The arity map $\alpha: \mathcal{S} \sqcup \mathcal{B} \rightarrow \mathbb{N} \cup\{\infty\}$.
- The tuple $\left(G_{1}, \ldots, G_{n}\right)$ of processes which are launched simultaneously at the beginning of each execution of the program.


## Points and multi-instructions

Higher Dimensional Transition Systems, G. L. Cattani and V. Sassone, 1996

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- A multi-instruction is a partial map $\mu:\{1, \ldots, n\} \rightarrow$ \{instructions $\}$.

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\begin{aligned}
\phi(s)= & |\sigma(s)| \\
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- for all $b \in \mathcal{B}, \operatorname{card}\{\mathrm{i} \in \operatorname{dom}(\mu) \mid \mu(\mathrm{i})=\mathrm{W}(\mathrm{b})\} \notin\{1, \ldots, \alpha(\mathrm{~b})\}$


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$$
i \mapsto \begin{cases}\sigma(s)(i)+1 & \text { if } i \in \operatorname{dom}(\mu) \text { and } \mu(i)=P(s) \\ \sigma(s)(i)-1 & \text { if } i \in \operatorname{dom}(\mu) \text { and } \mu(i)=V(s) \\ \sigma(s)(i) & \text { in all other cases }\end{cases}
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A sequence $\mu_{0}, \ldots, \mu_{q-1}$ of multi-intructions is said to be admissible at state $\sigma$ when for all $k \in\{0, \ldots, q-1\}$ the multi-instruction $\mu_{k}$ is admissible at state $\sigma \cdot \mu_{0} \cdots \mu_{k-1}$.

## Directed paths and sequences of multi-instructions

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A directed path $\gamma$ on $\left(G_{1}, \ldots, G_{n}\right)$ is a sequence $(\gamma(k))_{k \in\{0, \ldots, q\}}$ of points such that for all $k \in\{0, \ldots, q-1\}$ we have

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An admissible path is an execution trace when all the conditional branchings met along the way are respected: for all $k \in\{0, \ldots, q-2\}$ and all $i \in\{1, \ldots, n\}$ such that $\mu_{k}(i)$, which is by definition $\lambda_{i}\left(\gamma_{i}(k+1)\right)$, is a branching, we have

$$
\left(\mu_{k}(i)\right)\left(\sigma \cdot \mu_{0} \cdots \mu_{k-1}\right)=\gamma_{i}(k+2)
$$

## Concurrent access

```
var x = 0
proc p = x:=1
proc q = x:=2
init p q
```


## Admissible execution trace


the value of $x$ is 0

## Admissible execution trace


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## Admissible execution trace


the value of $x$ is 1

## Admissible execution trace


the value of $x$ is 2

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## Not admissible execution trace


the value of $x$ is 0

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the value of $x$ is 0

## Not admissible execution trace


the value of x is ?

## Lack of resources

```
sem 1 a
```

proc $p=P(a) ; V(a)$
init 2p

## Admissible concurrent execution trace



## Admissible concurrent execution trace



## Admissible concurrent execution trace



## Admissible concurrent execution trace



## Admissible concurrent execution trace



## Admissible concurrent execution trace



## Admissible concurrent execution trace



## Admissible concurrent execution trace


$๑$

## Admissible concurrent execution trace



## Admissible concurrent execution trace



## Admissible concurrent execution trace



## Admissible concurrent execution trace



## Not admissible concurrent execution trace



## Not admissible concurrent execution trace



## Not admissible concurrent execution trace



Not admissible concurrent execution trace


## Synchronisation

sync 1 b
proc $p=W(b)$
init 2p

## Concurrent execution trace

sync 1 b


## Concurrent execution trace

sync 1 b


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## Concurrent execution trace

sync 1 b


# Not admissible concurrent execution trace 



Not admissible concurrent execution trace
sync 1 b


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# Not admissible concurrent execution trace 

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Next goal

Encode admissibility into a model.

CONSERVATIVE PROGRAMS

Potential Functions

The potential functions of processes and programs

## The potential functions of processes and programs

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For all initial states $\sigma$, for all directed paths $\gamma, \gamma^{\prime}$ starting at the origin,

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In particular, the program $\Pi$ comes with a potential function

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## Conservative process



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- If the following holds for all ordered pairs of points ( $p, p^{\prime}$ ) such that $\partial^{-} p^{\prime}=p$ or $p^{\prime}=\partial^{+} p$, then $G$ is conservative, otherwise it is not.

$$
\pi\left(p^{\prime}\right)= \begin{cases}\pi(p) & \text { if } \partial^{-} p^{\prime}=p \\ \pi(p) \cdot \lambda\left(p^{\prime}\right) & \text { if } p^{\prime}=\partial^{+} p\end{cases}
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Discrete Models

The discrete model of a conservative program

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The forbidden set gathers all the conflicting, exhausting, and desynchronizing points.

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The discrete model is the complement of its forbidden set.
\{points of the program\} $\backslash\{$ forbidden points $\}$

## Discrete model



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## Discrete Model

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Main theorem of discrete models

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## Main theorem of discrete models

- Soundness: any directed path on a discrete model (i.e. which does not meet any forbidden point) is admissible.
- Completeness: for each admissible path which meets a forbidden point there exists a directed path which avoids them and such that both directed paths induce the same sequence of multi-instructions.


## Admissible execution trace


the value of $x$ is 0

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## Admissible execution trace


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Admissible execution trace avoiding forbidden points

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## Replacement




[^0]:    ${ }^{1}$ Portable Operating Systems Interface, X is a reference to Unix

[^1]:    ${ }^{1}$ Portable Operating Systems Interface, X is a reference to Unix

[^2]:    ${ }^{1}$ Portable Operating Systems Interface, X is a reference to Unix

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