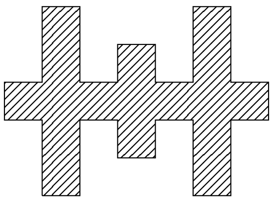
2.3.1: Concurrency

tuesday, the 3^{rd} of march 2020
duration: 3h

All the programs under consideration are supposed to be conservative.

Exercise 1: Denote the following (hatched) isothetic region by X:



1) Write a program whose forbidden region is X.

2) Draw the deadlock attractor of this program.

3) What are the maximal blocks of the forbidden region.

4) Give the prime factorization of the geometric model
(i.e. the complement of the hatched region).

5) Draw the category of components of the geometric model.

Exercise 2: Give the prime decomposition of the following homogeneous languages
(in the first one, the underlying alphabet is $\{\mathtt{0},\mathtt{1}\}$, in the second one it is $\{\mathtt{0},\mathtt{1},\ldots,\mathtt{9},\mathtt{A},\mathtt{B},\ldots,\mathtt{F}\}$)

0111	1908
0001	5B4A
1001	7968
0010	1B0A
0100	1F0E
0000	7D6C
1000	3928
1010	3B2A
0011	5D4C
0101	3F2E
1111	5F4E
1011	1D0C
1101	7B6A
1110	5948
0110	3D2C
1100	7F6E

Exercise 3: Assume that `a` and `b` are two synchronization barriers or arity 1 (each can stop a single process). Consider the 3 following programs (each being made of two processes):

`W(a)W(a)|W(a)W(a)` `W(a)W(b)|W(a)W(b)` `W(a)W(b)|W(b)W(a)`

A) For each of the three programs:

1a) Give the list of the sequences of multi-instructions corresponding to its execution traces.
1b) For each sequence of multi-instructions corresponding to its execution traces, what are the compatible permutations?
1c) Are the two processes observationally independent? (Explain)

2a) Draw the forbidden region of the program.
2b) Give the list of the maximal cubes the forbidden region
2c) Are the two processes model independent? (Explain)

3) Give the category of components of the geometric model (Explain)

B) What if that the arities of barriers `a` and `b` are both set to 2 (i.e. each barrier can stop two processes)? (Explain how the answers to the questions from A are modified)

C) Let P be a program made of n processes in which only wait instructions (i.e. `W(_)`) appear.

1) Write a conjecture about the program P assuming that all the arities of the barriers (used in P) are exactly $n-1$. (Provide arguments)

2) Write a conjecture about the program P assuming that all the arities of the barriers (used in P) are greater or equal than n. (Provide arguments)

Exercise 4: Let X be a path-connected topological space, which means that any two points of X are connected by a continuous path on X. The space X is considered as a pospace in which $x \sqsubseteq x'$ means $x = x'$. Then SX is the *directed suspension* of X, i.e. the quotient

$$SX \quad = \quad \Big((X,=) \ \times \ ([0,1],\leqslant)\Big)/\sim$$

where $(x,t) \sim (x',t')$ means $t = t' = 0$ or $t = t' = 1$ or $(x,t) = (x',t')$. The equivalence classes of $(x,0)$ and $(x,1)$ are called the *south pole* and the *north pole* respectively, they are denoted by 0 and 1.

1) Describe the partial order on the directed suspension SX.

2) Prove that all the directed paths from the south pole to the north pole are weakly equivalent. (Given an explicit weakly directed homotopy)

3) Let h be a directed homotopy on SX such that $h(\_,0) = 0$ (south pole) and $h(\_,1) = 1$ (north pole). Prove that the image of h is contained in a subspace of the form $\{x\} \times [0,1]$ for a unique $x \in X$.

The set of directed paths from the south pole to the north pole is equipped with the compact-open topology, the resulting space is denoted by PSX. We write $\gamma \sim \gamma'$ when the directed paths γ and γ' have the same image. The *trace space* of SX is the quotient $PSX/\sim$.

4) Up to isomorphism, what is the trace space of the directed suspension SX. (Provide an explicit isomorphism and an intuitive explanation, technical details are not required).