DISTRIBUTION OF THE LENGTH OF BRANCHES DURING A RANDOM SEARCH ON A RANDOM EUCLIDEAN GRAPH.

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ABSTRACT. RÉSUMÉ.

1. INTRODUCTION

1.1. **Definition of a random Euclidean graph.** To define a random Euclidean graph, we need to consider a set of points $\{T_i\}_{i\in\mathbb{Z}}$ thrown out on the plane in a *Poisson point process*. We speak about a point process, when we get an enumerable set of points thrown randomly on a certain space -usually the line, the plane, the sphere, ...-. Then for any borelian C - finite union of open sets - we define the quantity $N(C)$ as the number of points T_i in C: $N(C) = \sum_{i\in\mathbb{Z}} \mathbb{1}_C\{T_i\}$. From this definition $N(C)$ is a random integer variable. The point process is said to be of Poisson if for any finite familly $\{C_i\}$ of disjoint borelians, the random variables $N(C_i)$ are independant and follow a *Poisson law* of parameter $\lambda\mu(C_i)$, where $\mu(C_i)$ is the measure of Lebesgue of C_i - length for a segment, area for a surface, ...-:

$$\textbf{Poisson Law: } P(N(C) = k) = e^{-\lambda\mu(C)} \frac{(\lambda\mu(C))^k}{k!}$$

λ is said to be the *density* of the process.

To build a random Euclidean graph $\mathcal{G}(n,d)$, we need a set of $\{T_i\}_{i\in\mathbb{Z}}$ issued from a Poisson point process of density n. If $\mathcal{G}(n,d) = (V,E)$, then V is the set of points T_i which are in the unit square $\mathcal{U}$, and E is defined as follows: for $u,v \in V$, (u,v) is in E if and only if the Euclidean distance between u and v is less than d, ie:

$$\forall u,v \in V, \quad (u,v) \in E \iff d_2(u,v) \leq d,$$

with $d_2(u,v) = \sqrt{(v_x-u_x)^2 + (v_y-u_y)^2}$. In particular we don't know the number of vertices $|V|$ of $\mathcal{G}(n,d)$, but we know that this number is n in average. One can also define a random Euclidean graph with a *binomial point process*, meaning a fixed number of points are thrown out on the unit square $\mathcal{U}$. The drawback of this model is that the independance property is no more true. However, the probability distribution of the binomial point process defined on a surface S tends to the probability distribution of the Poisson point process, when the finite region S expands to an infinite one keeping $n/|S|$ constant. Moreover, the quantity $\lambda = n/|S|$ is the density of the limit Poisson distribution. In the following, we will assume that $\mathcal{G}(n,d)$ is a random Euclidean graph of parameters n and d issued from a Poisson point process.

1.2. **Random search on a graph.** Given a graph G, the first thing we can try to do is to explore it. Among all the classical searches - DFS (Depth First Search), BFS (Bright First Search), PFS (Priority First Search), etc. - the random search is the only one non deterministic. From a given vertex we could launch two searches and obtain two different search trees. Here comes our interest for this search. In our model, we define a general search as follows:

Definition 1. *Let $G = (V,E)$ be a graph; we say that $S = u_1 \dots u_k$ is a search on G if and only if the following properties hold:*

- $\forall\, 1 \leq i < j \leq k$, $u_i \neq u_j$.
- $\forall\, 1 \leq i \leq k$, $u_i \in D_i = \{v \in V | \exists j < i, (v,u_j) \in E\}$.